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Method for the rapid estimation of spanwise loading of wings with camber and twist in subsonic attached flow

Associated software: ESDUpac A9510



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METHOD FOR THE RAPID ESTIMATION OF SPANWISE LOADING OF WINGS WITH CAMBER AND TWIST IN SUBSONIC ATTACHED FLOW

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METHOD FOR THE RAPID ESTIMATION OF SPANWISE LOADING OF WINGS WITH CAMBER AND TWIST IN SUBSONIC ATTACHED FLOW

1. NOTATION AND UNITS

		<i>SI</i>	<i>British</i>
a_1	lift-curve slope	degree ⁻¹	degree ⁻¹
A	aspect ratio (= $2s/\bar{c}$)		
A^*	effective aspect ratio, for wings with curved leading edges, see Equation (4.4) or (4.5)		
B_i	coefficients used in estimation of zero-lift angle for cambered wing section, see Equation (3.1)	degree	degree
c	local chord	m	ft
$\bar{c}$	geometric mean chord, $\int_0^1 c \, d\eta$	m	ft
c_r	root chord	m	ft
c_r^*	effective root chord for wings with curved or cranked trailing edges, see Equation	m	ft
C_{LL}	local lift coefficient		
C_{LLT}	local lift coefficient due to wing twist		
$\bar{C}_{LT}$	lift coefficient due to wing twist		
$(\bar{C}_{LT})_{int}$	value of $\bar{C}_{LT}$ obtained by integration of local loading, see Equation (5.8)		
$\bar{C}_{LTA}$	lift coefficient due to unit twist of type A	degree ⁻¹	degree ⁻¹
$\bar{C}_{LTB}$	lift coefficient due to unit twist of type B	degree ⁻¹	degree ⁻¹
$\bar{C}_L$	overall lift coefficient		
$\left. \begin{matrix} F(\eta, \bar{\eta}) \\ G(\eta) \\ H(\eta) \end{matrix} \right\}$	spanwise load functions, see Equation (4.3)		
K_A	effective twist parameter for twist of type A, see Equation (5.2)		

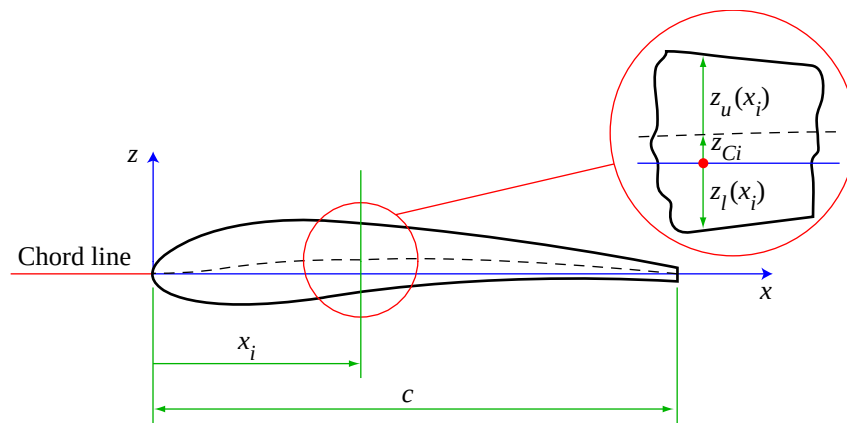
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K_B	effective twist parameter for twist of type B, see Equation (5.2)		
M	Mach number		
s	wing semi-span	m	ft
x_i	chordwise co-ordinate of the i 'th station measured from leading edge, see Sketch 1.1	m	ft
$x_T(\eta)$	distance of point on trailing edge measured in free-stream direction from root leading edge	m	ft
z_{Ci}	camber ordinate at x_i , see Equation (3.2)	m	ft
$z_l(x_i)$	lower-surface ordinate at x_i , measured from chord line, see Sketch 1.1	m	ft
$z_u(x_i)$	upper-surface ordinate at x_i , measured from chord line, see Sketch 1.1	m	ft
α	wing incidence measured from reference plane	degree	degree
α'	wing incidence measured from reference plane	radian	radian
α_C	incremental local twist angle for cambered wing section ($= -\alpha_0$)	degree	degree
α_E	effective local twist angle, see Section 5.2	degree	degree
α_G	local value of geometric twist	degree	degree
α_T	local value of total twist ($= \alpha_G + \alpha_C$)	degree	degree
α_{T0}	datum value of α_T , see Sketches 5.3 and 5.6	degree	degree
α_0	zero-lift angle of incidence for wing section, see Equation (3.1)	degree	degree
β	compressibility parameter, $(1 - M^2)^{1/2}$		
δ	wing twist angle relative to datum value, see Section 5	degree	degree
δ_A	root twist angle relative to datum value, α_{T0} , see Sketch 5.6	degree	degree
δ_B	tip twist angle relative to datum value, α_{T0} , see Sketch 5.6	degree	degree
η	spanwise distance from root as a fraction of semi-span		
$\bar{\eta}$	spanwise centre of pressure position, $\int_0^1 \sigma \eta d\eta$		

$\left. \begin{matrix} \eta_A \\ \eta_B \\ \eta_K \end{matrix} \right\}$	spanwise positions of intersection of twist segments, see Sketch 5.7		
η^*	value of $\bar{\eta}$ for wing with curved leading edge		
$\bar{\eta}_\Delta$	value of $\bar{\eta}$ for planform having required chord distribution and span, but with unswept trailing edge		
κ	taper parameter, $\int_0^1 \frac{c}{\bar{c}} \eta \, d\eta$		
λ	taper ratio, tip chord / c_r		
Λ_0	leading-edge sweep angle at $\eta = 0$	degree	degree
$\Lambda_{1/4}$	quarter-chord sweep angle at $\eta = 0$	degree	degree
$\Lambda_{1/2}$	mid-chord sweep angle at $\eta = 0$	degree	degree
Λ_1	trailing-edge sweep angle at $\eta = 0$	degree	degree
Λ_1^*	effective trailing-edge sweep angle for wing having cranked trailing edge, see Equation (4.2)	degree	degree
σ	spanwise loading due to incidence, $\frac{C_{LL} c}{\bar{C}_L \bar{c}}$		

Subscripts

A1	denotes twist of type A, with kink at η_K
A2	denotes twist of type A, with kink at η_A
B1	denotes twist of type B, with kink at η_K
B2	denotes twist of type B with kink at η_B



Sketch 1.1

The chord line is defined as the straight line connecting the leading and trailing-edge points. For a section with a finite base thickness, the trailing-edge point is taken as the mid-thickness point. The leading-edge point is defined as that unique point at which a circle centred at the trailing-edge point is tangential to the section.

2. INTRODUCTION AND SCOPE OF ITEM

2.1 Introduction

A method is presented for the rapid estimation of the theoretical spanwise loading of wings with camber and twist in wholly subsonic flow^{*}. It utilises the basic method of Derivation 2 which gives spanwise loading due to incidence for untwisted and uncambered wings and extends the procedure to take account of camber and twist. The effect of camber is treated by considering it as imparting an equivalent incremental twist to the wing. A simple artifice is used to represent the twist distribution by a combination of straight line distributions, from which values of effective local twist are derived for use in the method.

A step-by-step description of the calculation procedure to be followed for estimating the spanwise loading is given in Section 5.3.

2.2 Scope of Item

The method is applicable to a wide range of wing planforms in unseparated flow at subcritical Mach numbers. The simplifications adopted in accounting for camber and twist can involve some minor reservations on the choice of planform (see Section 6) and there are minor restrictions on the choice of leading-edge and trailing-edge shapes, see Section 4.

3. BASIS OF METHOD

3.1 Uncambered and Untwisted Wings

A rapid method for estimating the theoretical spanwise loading due to incidence was developed in Derivation 2, where a full account can be found. In subsonic compressible flow the method embodies a modified planform in which all spanwise dimensions are reduced by the factor β .

^{*} A computer program, ESDUpac A9510, described in ESDU 95010 (Reference 8), is available that calculates the spanwise loading directly from steady lifting-surface theory using the Multhopp-Richardson solution.

Derivation 2 is applicable to wings for which $\beta A \leq 8$, and the charts contained in it are reproduced in this Item. An additional chart has been generated using data obtained from Derivations 3 and 4 to extend the range of applicability to $\beta A = 12$. Details of the procedure are given in Section 4.

3.2 Cambered and Twisted Wings

The extension of Derivation 2 to cater for wings with camber and twist is based on factoring the spanwise loading due to constant unit incidence by an effective local twist angle, α_E . The angle α_E is related to the local total twist α_T that incorporates the effects of both camber and geometric twist. Sections 3.2.1 and 3.2.2 describe the calculation of α_T . The calculation of α_E is described in Section 5.

3.2.1 Representation of camber as a local twist

To cater for wings with camber, the spanwise distribution of the local zero-lift angle, α_0 , must first be obtained. Values of α_0 are estimated by the method of Derivation 1. An approximate expression for the zero-lift angle of a cambered wing section is given by

$$\alpha_0 = -2 \sum_{i=1}^{i=14} B_i \frac{z_{Ci}}{c} \quad (3.1)$$

where B_i is obtained from Table 3.1 and z_{Ci} is the camber ordinate given by

$$z_{Ci} = \frac{z_u(x_i) + z_l(x_i)}{2} \quad (3.2)$$

as shown in Sketch 1.1.

The approximate value of zero-lift angle given by Equation (3.1) is then interpreted as an incremental local twist angle, $\alpha_C = -\alpha_0$.

TABLE 3.1

i	1	2	3	4	5	6	7
$(x/c)_i$	0	0.025	0.05	0.1	0.2	0.3	0.4
B_i	1.45	2.11	1.56	2.41	2.94	2.88	3.13
i	8	9	10	11	12	13	14
$(x/c)_i$	0.5	0.6	0.7	0.8	0.9	0.95	1.00
B_i	3.67	4.69	6.72	11.75	21.72	99.85	-164.88

3.2.2 Local total twist due to camber and geometric twist

The local total twist angle, α_T , is obtained by adding the geometric local twist, α_G , and the incremental local twist, α_C , resulting from wing camber. Thus

$$\alpha_T = \alpha_C + \alpha_G; \quad (3.3)$$

for cambered, untwisted wings, $\alpha_G = 0$ and $\alpha_T = \alpha_C$; for uncambered, twisted wings, $\alpha_C = 0$ and $\alpha_T = \alpha_G$.

4. ESTIMATION OF THEORETICAL SPANWISE LOADING DUE TO INCIDENCE

4.1 Scope of Method

The method used is that of Derivation 2*, extended to $\beta A = 12$. Neither $\bar{C}_L$ nor α is required as input. Charts used in the rapid estimation of the theoretical spanwise loading due to incidence are presented as Figures 1 to 8.

The spanwise centre of pressure position, $\bar{\eta}$, is derived from Figures 1 to 5 in terms of aspect ratio, mid-chord sweep, Mach number and a taper parameter. This taper parameter caters for arbitrary leading edges, but excludes wings with inverse taper ($\lambda > 1$). The method has an upper limit of $\beta A = 12$, with mid-chord sweep angles such that $0 \leq A \tan \Lambda_{1/2} \leq 6$. The lower limit of βA is 1.5, except that for wings with straight leading and trailing edges and $A \tan \Lambda_{1/2} \leq 2$ it is known that $\bar{\eta}$ converges rapidly to the value corresponding to elliptic loading ($\bar{\eta} = 0.4244$) as $\beta A \rightarrow 0$. This may sometimes be used as an extra point on the plot required in step (iii) of Section 4.2.

The method should not be applied to planforms having cranked leading or trailing edges with discontinuities greater than about 40° .

4.2 Wings with Straight or Cranked Leading and Trailing Edges

The procedure of Derivation 2 is as follows.

- (i) Evaluate the taper parameter

$$\begin{aligned} \kappa &= \int_0^1 \frac{c}{\bar{c}} \eta d\eta \\ &= \frac{1 + 2\lambda}{3(1 + \lambda)} \text{ for a straight tapered wing.} \end{aligned} \quad (4.1)$$

- (ii) Knowing κ and $A \tan \Lambda_{1/2}$, obtain $\bar{\eta}$ from Figures 1 to 5 for $\beta A = 1.5, 3, 5, 8$ and 12, respectively.

- (iii) Plot $\bar{\eta}$ against βA to obtain $\bar{\eta}$ for the required value of M .

$$\begin{aligned} \text{(iv) Evaluate } \tan \Lambda_1^* &= \int_0^1 \frac{x_T(\eta)}{s} (12\eta - 6) d\eta \\ &= \tan \Lambda_0 - \frac{12}{A} (1 - 2\kappa) \text{ for a straight leading edge.} \end{aligned} \quad (4.2)$$

Note that $\Lambda_1^* = \Lambda_1$ for wings with straight trailing edges.

* A computer program listing for the method of Derivation 2 is contained in an Addendum to that Derivation.

- (v) Obtain $F(\eta, \bar{\eta})$, $G(\eta)$ and $H(\eta)$ from Figures 6, 7 and 8, for selected values of η corresponding to step (i) in Section 5.2.
- (vi) Evaluate the local spanwise loading due to incidence from the equation

$$\sigma = \frac{C_{LL}^c}{\bar{C}_L \bar{c}} = F(\eta, \bar{\eta}) + (A \tan \Lambda_1) G(\eta) + (\beta A - 4)(1 + 3.5\beta^{-1} \tan \Lambda_1^*) H(\eta) \quad (4.3)$$

where the last term is omitted if $\beta A \leq 4$.

4.3 Swept Wings with Curved Leading Edges

Curved leading edges are defined here as having continuous slope, apart from the root, and streamwise wing tips of zero chord. The procedures used depend on the form of the trailing edge and are only applicable when the sweep angles Λ_0 and Λ_1 are approximately equal.

- (a) Complete step (i) of Section 4.2.
- (b) Evaluate the empirical parameter A^* .

For straight trailing edges

$$A^* = -1.2A + 8.8 \left(\frac{2c_r}{s} + \tan \Lambda_1 - \tan \Lambda_0 \right)^{-1} \quad (4.4)$$

For curved or cranked trailing edges

$$A^* = -1.2A + 8.8 \left(\frac{2c_r^*}{s} + \tan \Lambda_1^* - \tan \Lambda_0 \right)^{-1} \quad (4.5)$$

$$\text{where } c_r^* = \int_0^1 x_T(\eta)(4 - 6\eta) d\eta \quad (4.6)$$

and $\tan \Lambda_1^*$ is obtained from Equation (4.2).

- (c) Step (ii) of Section 4.2 now requires the evaluation of

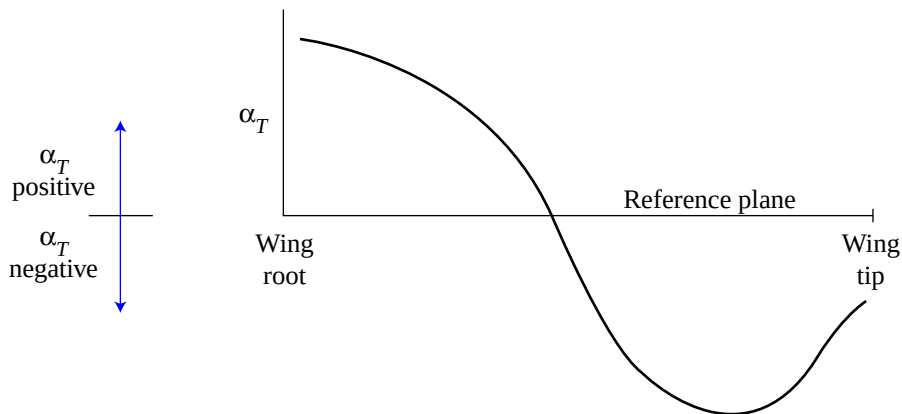
$$\bar{\eta}^* = (\bar{\eta} - \bar{\eta}_\Delta) \frac{A^*}{A} + \bar{\eta}_\Delta, \quad (4.7)$$

where $\bar{\eta}_\Delta$ is obtained from Figures 1 to 5 with the true βA and κ from Equation (4.1) but with $A \tan \Lambda_{1/2} = 6(1 - 2\kappa)$.

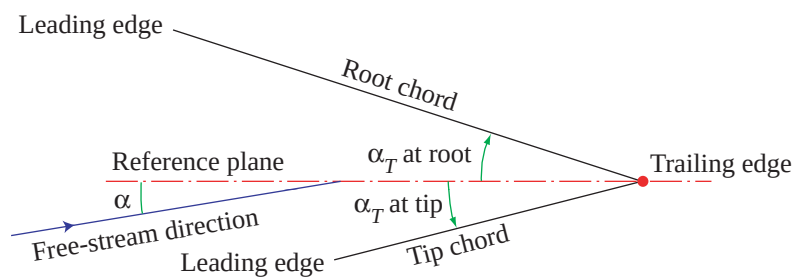
- (d) Steps (iii) to (vi) of Section 4.2 follow with $\bar{\eta}$ replaced by $\bar{\eta}^*$.

5. ESTIMATION OF THEORETICAL SPANWISE LOADING FOR CAMBERED AND TWISTED WINGS

The wing incidence, α , and the spanwise variation of α_T must be defined relative to a common reference plane (see Sketches 5.1 and 5.2).



Sketch 5.1 View in the longitudinal direction

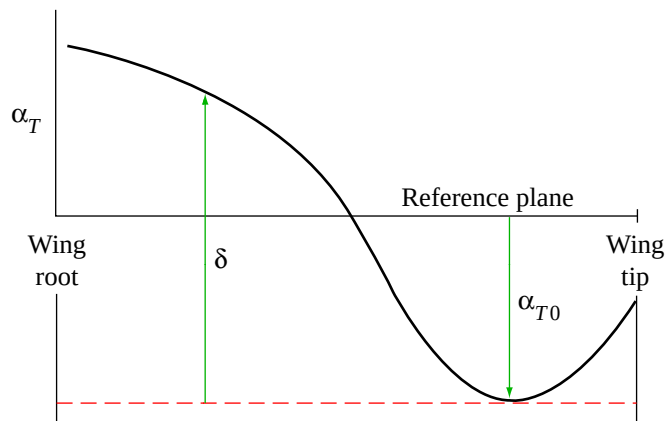


Sketch 5.2 *View in the lateral direction

It is implicit that when the reference plane lies in the stream direction the wing is at zero incidence ($\alpha = 0$).

For the purpose of this Item α_T is defined in terms of a datum value, α_{T0} , and the twist, δ , relative to this datum value (see Sketch 5.3).

* For purposes of clarity in Sketch 5.2, the root and tip chords are shown as having a common trailing edge location lying in the reference plane. In practice this will not usually be the case.

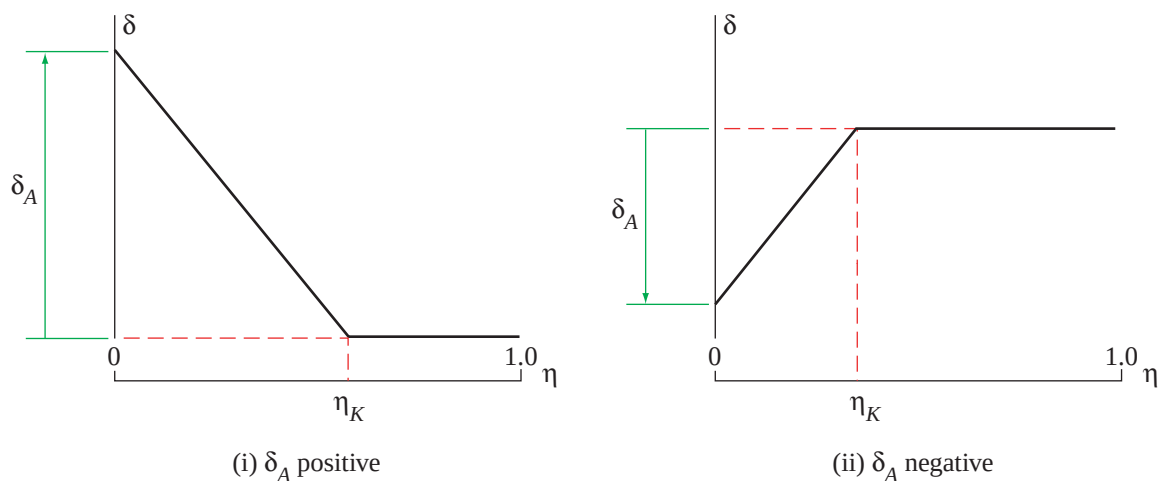


Sketch 5.3 Definition of α_T in terms of α_{T0} and δ (Note that in this sketch α_{T0} is negative)

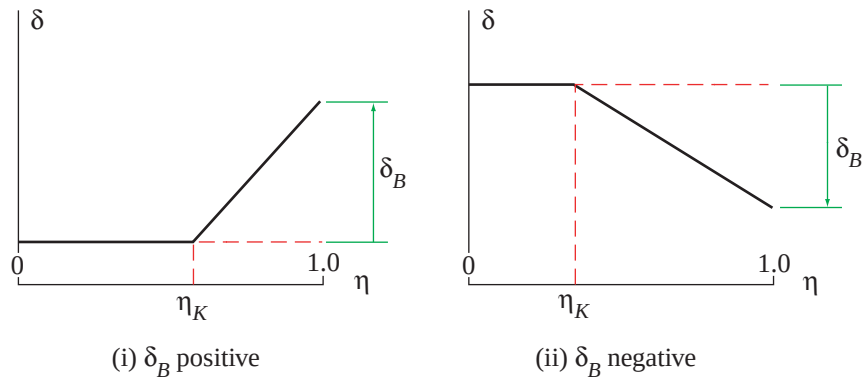
The lift distribution at $\alpha = 0$ then consists of contributions due to α_{T0} and δ . The contribution due to α_{T0} is simply an incidence loading and may be determined as such. Thus it remains to determine the spanwise loading due to the spanwise twist distribution, δ . The procedure adopted here is described in Sections 5.1 to 5.3.

5.1 Representation of Twist Distribution by Linear Segments

As stated in Section 3.2, the method requires a distribution of effective twist, α_E , to be derived from the distribution of twist, α_T . To do this, α_T must first be represented by a series of contiguous linear segments. Two basic types of linear twist distribution are used, as defined in Sketches 5.4 and 5.5 in which δ_A and δ_B respectively are the values of δ at the wing root and wing tip.



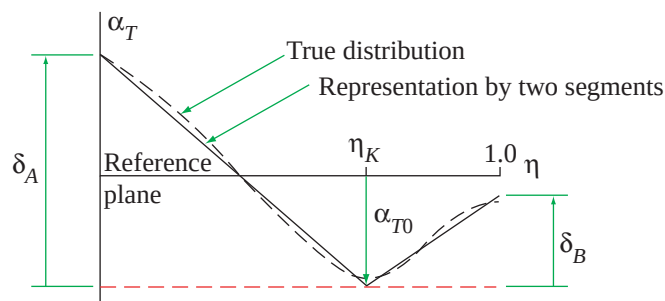
Sketch 5.4 Twist of Type A



Sketch 5.5 Twist of Type B

In principle a given twist distribution can be represented by a number of linear segments, but it is likely that most distributions encountered in practice can be adequately represented by up to four linear segments and only such cases are considered here. A two-segment representation, for example, could be used for the twist distribution illustrated in Sketch 5.6. The extension of the method for an arbitrary number of segments is considered in Appendix A. Note that the method is also used in Section 3.3.2 of ESDU 87031 (Reference 7).

In Sketch 5.6, α_{T0} is the value of α_T at $\eta = \eta_K$ derived from the two-segment representation, and is negative as drawn. For this illustration δ_A and δ_B are positive. The parameters δ_A , δ_B , η_K and α_{T0} then define the twist distribution across the span (see Example 1 in Section 8.1).



Sketch 5.6

Many practical twist distributions have marked changes in twist near the wing root and tip. For such cases four linear segments can often be used to represent the twist, as shown in Sketch 5.7.