

FLEXIBLE CIRCULAR FRAMES SUPPORTED BY A SHELL. MOMENTS, FORCES AND DISPLACEMENTS DUE TO CONCENTRATED LOADS AND COUPLES

1. NOTATION

A	cross-sectional area of longitudinal stringer supporting shell (normal to plane of frame)	m^2	in^2
C	coefficient used in moment, force and displacement equations		
d	effective frame to shell relative stiffness parameter, defined by $d = GtR^4/EIL'$ (see Data Item No. Struct. 03.06.17)		
E	modulus of elasticity of frame material	N/m^2	lbf/in^2
G	shear modulus of shell material	N/m^2	lbf/in^2
I	effective moment of inertia of cross section of frame including allowance for effect of attached skin	m^4	in^4
K	shell skin resisting force per unit tangential deflection, where $K = GtR/L'$	N/m	lbf/in
L	distance between adjacent frames, where $L = 2L_1L_2/(L_1 + L_2)$ (see Data Item No. Struct. 03.06.17)	m	in
L'	effective frame spacing (see Data Item No. Struct. 03.06.17)	m	in
L_1, L_2	distances of adjacent frames on either side of frame considered	m	in
M	bending moment in frame at section under consideration, positive if it produces tension on inside of frame	N m	lbf in
M_a	applied couple, positive if tending to rotate frame anti-clockwise	N m	lbf in
N	shear force (radial) in frame, positive if acting outward on clockwise side of section under consideration	N	lbf
N_a	applied radial load, positive if acting outward	N	lbf
n	number of longitudinal stringers supporting shell (normal to plane of frame)		
Q	shear flow from shell to frame, positive if tending to rotate frame clockwise	N/m	lbf/in
R	radius measured to centroid of frame section	m	in
T	direct force (tangential) in frame, positive if it produces tension	N	lbf

T_a	applied tangential load, positive if tending to rotate frame anti-clockwise	N	lbf
t	thickness of shell skin effective in shear	m	in
t'	equivalent thickness of shell skin effective in carrying longitudinal stress (where stringers are present, $t' = t + nA/2\pi R$, see Data Item No. Struct. 03.06.17)	m	in
δ	radial displacement, positive if measured outward	m	in
η	tangential displacement, positive if measured anti-clockwise	m	in
θ	angle between section at which load or couple is applied and section under consideration, positive if measured clockwise	degree	degree
ψ	local rotation, positive if measured clockwise	rad	rad

Suffixes

Where there are two suffixes the first refers to the dependent variable being considered: load, moment or displacement; the second refers to the applied quantity: moment or load.

First suffixes

M	bending moment
T	direct force, tangential
N	shear force, radial
Q	shear flow
δ	radial displacement
η	tangential displacement
ψ	rotational displacement

Second suffixes

M	applied couple
N	applied radial load
T	applied tangential load

Both SI and British Units are quoted but any coherent system of units may be used.

2. NOTES

Values are given for the coefficients of simple equations that enable the bending moment, direct and shear forces, shear flow and displacements to be obtained at any point round a flexible circular frame supported by a shell when subjected to concentrated loads and couples. In Table 4.1 the equations are given for the loads, shear flow and displacements, M , T , N , Q , η , δ and ψ , for applied couple M_a and applied tangential and radial loads T_a and N_a . In this table reference is also made to Data Items that present the data in graphical form.

In Tables 4.2 to 4.11 the values of the coefficients, C , appearing in the equations in Table 4.1, are given for

values of θ from 0 to 180° grouped for particular values of the frame to shell relative stiffness parameter d . The coefficients may be applied for values of $180^\circ \leq \theta \leq 360^\circ$ by using the parameter $360^\circ - \theta (= \theta)$ but since some quantities are symmetrical and some antisymmetrical about $\theta = 0^\circ$ it will be necessary to change the sign of some coefficients. This is indicated by + or – in the tables, indicating no change or change respectively. The values of the coefficients are given for single applied couples or loads but these may be superimposed for combined loading. Since usually $L' \neq L$ the value of d used for the tables should be obtained from Data Item No. Struct. 03.06.17* which considers the effect of non-uniform frame spacing and stiffness and the effect of longitudinal stringers on the structure (see example).

The couple M_a and the loads T_a and N_a are assumed to act in the plane of the frame and to be balanced entirely by tangential shear forces in the shell. These forces arise from displacement of the frame as a whole in its own plane and from deformation of the frame from its original circular shape. All displacements are assumed to be relative to an undisturbed section of the cylinder at distance L from the frame under consideration. These displacements are local and in practice likely to be small compared to the overall displacements of a structure comprising a number of frames along a cylindrical shell. Rigid connection of frames to shell is assumed, the effective value of d is reduced when flexible connections exist between shell and frame. The values for $d = 0$ are for infinitely rigid frames which are displaced as a whole with no deformation. The moments, forces and displacements for rigid frames subjected to concentrated loads are given in Data Items Nos Struct. 03.06.01, 03.06.02, 03.06.03 and 03.06.04.

3. DERIVATION

1. WIGNOT, J.E. Analysis of circular shell-supported frames. NACA tech. Note 929,
COMBS, H. December 1943.
ENSRUD, A.F.

4. EXAMPLE

It is required to find the bending moment, direct (tangential) and shear (radial) forces, the radial, tangential and angular displacements and the shear flow from shell to frame, at a point 330° (clockwise) from an offset load applied to a circular frame in a monocoque structure. The concentrated offset load can be expressed as a couple and tangential and radial direct loads so that $M_a = 25\,000$ lbf in, $T_a = 5000$ lbf and $N_a = 4000$ lbf. The frame has a radius $R = 56$ in and the effective moment of inertia $I = 1.24$ in⁴. The nearest adjacent frames on either side are at $L_1 = 42$ in and $L_2 = 60$ in and the thickness of the shell $t = 0.036$ in. There are 72 stringers between the adjacent frames spaced equally around the shell, each of cross-sectional area $A = 0.1$ in². The shear modulus and modulus of elasticity of the shell and frame material are $G = 3.8 \times 10^6$ lbf/in² and $E = 10 \times 10^6$ lbf/in², respectively.

Data Item No. Struct. 03.06.17 is first used to obtain the value of d .

* Data Item No. Struct. 03.06.17 "Flexible circular frames supporting a shell. The effect of adjacent frames and the longitudinal flexibility of the shell".

From the given data,

$$L = \frac{2 \times 42 \times 60}{(42 + 60)} = 49.4 \text{ in,}$$

$$\frac{GtR^4}{EIL} = \frac{3.8 \times 10^6 \times 0.036 \times 56^4}{10 \times 10^6 \times 1.24 \times 49.4} = 2200,$$

$$t' = 0.036 + \frac{72 \times 0.1}{2 \times \pi \times 56} = 0.056 \text{ in}$$

$$\text{and } \frac{R^6 t'}{IL^3} = \frac{56^6 \times 0.056}{1.24 \times 49.4^3} = 11\,500.$$

From Figure 1 of Item No. Struct. 03.06.17 for $GtR^4/EIL = 2200$ and $R^6 t'/IL^3 = 11\,500$,

$$d = \frac{GtR^4}{EIL'} = 1000.$$

The following additional parameter is required for use with this Item.

$$K = \frac{GtR}{L'} = \frac{dEI}{R^3} \\ = \frac{1000 \times 10 \times 10^6 \times 1.24}{56^3} = 70.6 \times 10^3 \text{ lbf/in.}$$

Tables 4.2 to 4.11 are used to obtain the coefficients for the equations given in Table 4.1. From Table 4.7, when $d = 1000$ and $360 - \theta = 30^\circ$ the coefficients obtained are as follows.

M_a	T_a	N_a
$C_{MM} = 0.04$	$C_{MT} = -0.01$	$C_{MN} = 0.025$
$C_{TM} = -1.55$	$C_{TT} = -0.05$	$C_{TN} = 0.56$
$C_{NM} = 0.525$	$C_{NT} = -0.028$	$C_{NN} = -0.05$
$C_{\delta M} = -10$	$C_{\delta T} = 1.54$	$C_{\delta N} = -0.4$
$C_{\psi M} = -29$	$C_{\psi T} = 1.0$	$C_{\psi N} = 10$
$C_{\eta M} = 1.0$	$C_{\eta T} = 0.52$	$C_{\eta N} = -1.55$
$C_{QM} = 1.0$	$C_{QT} = 0.52$	$C_{QN} = -1.55$

The equations given in Table 4.1 for the effect of each applied couple or load can be combined to give the resultant value of the particular dependent variable as shown in the following tabulation.

Dependent variable	Applied couple or load			Σ
	M_a	T_a	N_a	
M (lbf in)	$C_{MM}M_a =$ $0.04 \times 25\,000$	$C_{MT}RT_a =$ $-0.01 \times 56 \times 5000$	$C_{MN}RN_a =$ $0.025 \times 56 \times 4000$	3800 lbf in
T (lbf)	$\frac{C_{TM}M_a}{R} =$ $\frac{-1.55 \times 25\,000}{56}$	$C_{TT}T_a =$ -0.05×5000	$C_{TN}N_a =$ 0.56×4000	1298 lbf
N (lbf)	$\frac{C_{NM}M_a}{R} =$ $\frac{0.525 \times 25\,000}{56}$	$C_{NT}T_a =$ -0.028×5000	$C_{NN}N_a =$ -0.05×4000	-106 lbf
δ (in)	$\frac{C_{\delta M}M_a}{KR} =$ $\frac{-10 \times 25\,000}{70.6 \times 10^3 \times 56}$	$\frac{C_{\delta T}T_a}{K} =$ $\frac{1.54 \times 5000}{70.6 \times 10^3}$	$\frac{C_{\delta N}N_a}{K} =$ $\frac{-0.4 \times 4000}{70.6 \times 10^3}$	0.023 in
ψ (rad)	$\frac{C_{\psi M}M_a}{KR^2} =$ $\frac{-29 \times 25\,000}{70.6 \times 10^3 \times 56^2}$	$\frac{C_{\psi T}T_a}{KR} =$ $\frac{1 \times 5000}{70.6 \times 10^3 \times 56}$	$\frac{C_{\psi N}N_a}{KR} =$ $\frac{10 \times 4000}{70.6 \times 10^3 \times 56}$	0.008 rad
η (in)	$\frac{C_{\eta M}M_a}{KR} =$ $\frac{1 \times 25\,000}{70.6 \times 10^3 \times 56}$	$\frac{C_{\eta T}T_a}{K} =$ $\frac{0.52 \times 5000}{70.6 \times 10^3}$	$\frac{C_{\eta N}N_a}{K} =$ $\frac{-1.55 \times 4000}{70.6 \times 10^3}$	-0.0447 in
Q (lbf/in)	$\frac{C_{QM}M_a}{R^2} =$ $\frac{1 \times 25\,000}{56^2}$	$\frac{C_{QT}T_a}{R} =$ $\frac{0.52 \times 5000}{56}$	$\frac{C_{QN}N_a}{R} =$ $\frac{-1.55 \times 4000}{56}$	-56.3 lbf/in

TABLE 4.1 Equations for Moments, Forces, Shear Flow and Displacements
(Numbers in parenthesis refer to Data Items providing the same data in curve form.)

Dependent variable	Applied moment or load		
	M_a	T_a	N_a
M	$C_{MM}M_a$ (03.06.06)	$C_{MT}RT_a$ (03.06.07)	$C_{MN}RN_a$ (03.06.08)
T	$C_{TM}M_a/R$ (03.06.09)	$C_{TT}T_a$ (03.06.10)	$C_{TN}N_a$ (03.06.11)
N	$C_{NM}M_a/R$ (03.06.12)	$C_{NT}T_a$ (03.06.13)	$C_{NN}N_a$ (03.06.14)
δ	$C_{\delta M}M_a/KR$ (03.06.15 Fig. 1)	$C_{\delta T}T_a/K$ (03.06.15 Fig. 2)	$C_{\delta N}N_a/K$ (03.06.15 Fig. 3)
ψ	$C_{\psi M}M_a/KR^2$ (03.06.15 Fig. 4)	$C_{\psi T}T_a/KR$ (03.06.15 Fig. 5)	$C_{\psi N}N_a/KR$ (03.06.15 Fig. 6)
η	$C_{\eta M}M_a/KR$ (03.06.15 Fig. 7)	$C_{\eta T}T_a/K$ (03.06.15 Fig. 8)	$C_{\eta N}N_a/K$ (03.06.15 Fig. 9)
Q	$C_{QM}M_a/R^2$ (03.06.16 Fig. 1)	$C_{QT}T_a/R$ (03.06.16 Fig. 2)	$C_{QN}N_a/R$ (03.06.15 Fig. 3)

