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Fluid transients in pipes and tunnels Speed of propagation of pressure waves

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THE PREPARATION OF THIS DATA ITEM

This Item was developed in the Noise and Structural Dynamics Group on behalf of the Internal Flow Panel. The work of the permanent professional staff of the Engineering Sciences Data Unit on this particular Item was monitored and guided by the following Working Party:

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Dr S.J. Murray	— Atkins Research and Development
Dr D. Pollard	— GEC Power Engineering
Dr A. Spurr	— CEGB Marchwood Engineering Laboratories
Dr L. Santos	— Babcock Power
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(Continued on inside back cover)

FLUID TRANSIENTS IN PIPES AND TUNNELS SPEED OF PROPAGATION OF PRESSURE WAVES

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FLUID TRANSIENTS IN PIPES AND TUNNELS

Speed of Propagation of Pressure Waves

1. NOTATION AND UNITS

Any consistent system of units may be used in the equations in this Item

		<i>Units</i>	
		<i>SI</i>	<i>British</i>
A	internal cross-sectional area of pipe or tunnel	m ²	ft ²
a	wave speed in fluid contained in pipes or tunnels	m/s	ft/s
a_0	wave speed in large, unconfined body of fluid	m/s	ft/s
a_0'	wave speed in large, unconfined body of liquid/gas or liquid/solid mixture	m/s	ft/s
D	mean diameter of circular pipe	m	ft
$D_c, D_r,$ D_s	internal diameters of concrete lining, rock tunnel and steel lining, respectively (see Sketch 4.4)	m	ft
$D_1, D_2,$ D_3	internal diameters of lining and pipe and external diameter of pipe, respectively (see Sketch 4.5)	m	ft
d	width of flats of regular polygon (see Sketch 4.3)	m	ft
E	Young's modulus of elasticity	N/m ²	*lbf/ft ²
G	shear modulus of elasticity	N/m ²	*lbf/ft ²
K	bulk modulus of fluid	N/m ²	lbf/ft ²
p	pressure (absolute)	N/m ²	lbf/ft ²
p_{ref}	[†] reference pressure, taken as one atmosphere (101 325 N/m ² , 2116.22 lbf/ft ²)	N/m ²	lbf/ft ²
R	gas constant (typical value for air = 287.053 J/kg K, 3090 ft lbf/slug K)	J/kg K	ft lbf/ [†] slug K
T	absolute temperature	K	K
t	pipe wall thickness	m	ft
W	term defined by Equation (A1.2)	—	—
x	ratio of gas or solid mass to primary liquid mass	—	—

For footnotes see end of Notation section.

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$\alpha_{t,p}$	ratio defined in Equation (3.3)	—	—
$\beta_{t,p}$	ratio defined in Equation (3.4)	—	—
γ	ratio of specific heat capacity of a fluid at constant pressure to that at constant volume	—	—
θ	term defined by Equation (4.8)	deg	deg
ν	Poisson's ratio	—	—
Λ	area strain factor (see Section 4 and Table 7.3)	m ² /N	ft ² /lbf
λ	term given by Equation (4.13) or (4.16)	—	—
ρ	density	kg/m ³	slug/ft ³
ϕ	pipeline restraint factor (see Section 4.1)	—	—
ψ	term defined by Equation (4.14) or (4.17)	m	ft

* 1 lbf/ft² = 1/144 lbf/in².

† p_{ref} is taken as the pressure in the International Standard Atmosphere at sea level and 15°C.

‡ 1 slug = 32.174 lb (pounds mass).

Subscripts

c	refers to concrete
g	refers to gas
i	refers to lining inside pipe
l	refers to liquid
m	refers to mean value
r	refers to rock
ref	reference
s	refers to solid
st	refers to steel

2. INTRODUCTION

This Item provides data that may be used to estimate the speed of propagation of pressure waves, or wave speed, in a homogeneous body of fluid contained in a pipe or tunnel, following a disturbance to the flow. Typical causes of such disturbances are the rapid opening or closure of a valve, or the run-down of a pump pumping fluid along a pipeline. This Item is the first in a series of Data Items concerned with fluid transients in pipelines and tunnels. The magnitude of the pressure change in a system, following a disturbance to the flow, is proportional to the wave speed, and it is therefore necessary to obtain a good estimate of the wave speed for fluid transient design calculations. The magnitude of the wave speed in a fluid is dependent upon the physical properties of the fluid and the elastic properties of the conduit containing the fluid. The additional elasticity introduced by the expansion or contraction of a conduit with the passage of pressure waves may reduce the wave speed by a significant amount. For example, the wave speed in a fluid contained in very flexible hoses at low pressures* may be only of the order of 5 to 10 per cent of the wave speed in a large, unconfined body of the same fluid. Similarly, the presence of free gases in a liquid may introduce additional elasticity and lead to a reduction in the wave speed. Typically, in water supply lines at normal conditions, an accuracy better than ± 10 per cent in the wave speed estimate, and hence the estimate of the pressure change, may be expected using this Item.

The wave speed data presented in this Item are in the form of design charts and simple equations that have been shown to correspond well with experimental data. Typical data for the properties of some liquids and the elastic properties of some commonly used conduit materials are given in tabular form. In Section 3 of this Item, data are presented for the estimation of the speed of propagation of pressure waves in large unconfined bodies of some common fluids. Simple equations for the estimation of wave speeds in homogeneous, pure fluid flows in pipes and tunnels are given in Section 4. In Appendix A, simple expressions for the estimation of the wave speeds in homogeneous liquid/gas mixtures and liquid/solid mixtures are presented. Some data concerning the wave speed in steam at various temperatures and pressures are given in Appendix B. The effects of plastic deformation of the pipe walls upon the wave speed are considered in Appendix C.

* In very flexible hoses, the elastic properties of the pipe (and hence the wave speed in the fluid contained in the pipe) may be dependent upon the internal pressure and the rate of change of internal pressure. Consequently, great care must be taken in the estimation of the wave speed.

3. ESTIMATION OF THE WAVE SPEED, a_0 , IN LARGE, UNCONFINED BODIES OF FLUIDS

Data are given in this Section for the estimation of the speed of propagation of pressure waves* in large unconfined bodies of some common fluids. This parameter is commonly referred to as the speed of sound in the fluid. These data may be used to give an estimate of the upper limit of the wave speed in a particular fluid contained in a conduit. The general equation for the wave speed, or speed of sound, in a large, unconfined body of a homogeneous fluid may be expressed as

$$a_0 = \sqrt{\frac{\gamma K}{\rho}}, \quad (3.1)$$

where γ is the ratio of the specific heats of the fluid, K is the bulk modulus of elasticity of the fluid, and ρ is the density of the fluid. This equation may be used to estimate the wave speed, a_0 , for most liquids and gases in most practical applications. Great care must be taken in the estimation of the parameters for use in this equation. In particular, a reliable estimate of the bulk modulus, K , is required in order to obtain a good estimate of the wave speed. Typical values of K are given for some common liquids in Table 7.1. The application of the general equation to the estimation of this wave speed, a_0 , in liquids and gases is discussed below.

3.1 Application of the General Equation for the Wave Speed in Liquids

By assuming the liquid to be nearly incompressible, the ratio of the specific heats, γ , may be considered to be approximately unity, and Equation (3.1) may be simplified to

$$a_0 = \sqrt{\frac{K}{\rho}}, \quad (3.2)$$

for normal temperatures and pressures. If greater accuracy is required, it will be necessary to consider the effect of the ratio of the specific heats, and Equation (3.1) must be used to determine the wave speed. In general, it is unlikely that such accuracy will be required. Some data from which the wave speed, a_0 , in water at pressures from 1 to 1000 atmospheres, and over the temperature range 0 to 100°C (32 to 210°F), may be estimated are presented graphically on Figure 1. These data, derived from the experimental data presented in Derivation 1, are given as a ratio of the wave speed at the chosen temperature and pressure to the wave speed at a temperature of 15°C (59°F) and the reference pressure, p_{ref} (atmospheric). From Figure 1, a value of this ratio, $\alpha_{t,p}$, for the required temperature and pressure may be estimated, and the corresponding wave speed is calculated from the equation,

$$a_0 = \alpha_{t,p}(a_0)_{ref}, \quad (3.3)$$

where $(a_0)_{ref} = 1466.5$ m/s (4811.4 ft/s). In Figure 2, data are presented for the estimation of the bulk modulus of water over the same range of temperatures and pressures. These data were derived from the experimental results for wave speed, a_0 given in Figure 1 by using Equation (3.2). In the derivation, a value of the ratio of the specific heats of water, γ , of unity was taken, and the values of water density, ρ , were taken from Reference 8. The data are presented as a ratio, $\beta_{t,p}$, of the bulk modulus at the chosen temperature and pressure, p_{ref} . The bulk modulus may then be estimated from the equation,

$$K = \beta_{t,p}K_{ref}, \quad (3.4)$$

where $K_{ref} = 2.148$ GN/m² (4.49×10^7 lbf/ft²).

* Strictly, the theory applies to small amplitude pressure waves only, but is a suitable approximation for practical fluid transient calculations.

Some values of the wave speed, a_0 , and the bulk modulus, K , for other liquids at selected temperatures are given in Table 7.1.

3.2 Application of the General Equation for the Wave Speed in Gases

In gases at temperatures and pressures at which the perfect gas laws apply, the bulk modulus of the gas, K , is equal to the pressure of the gas, p . Hence, from Equation (3.1), for such a gas, the wave speed in a large, unconfined body of the gas is given by

$$a_0 = \sqrt{\frac{\gamma p}{\rho}} \quad (3.5)$$

or, alternatively, since $p/\rho = RT$ for an ideal gas, by

$$a_0 = \sqrt{\gamma RT}, \quad (3.6)$$

where R is the appropriate gas constant and T is the absolute temperature (in kelvin). In a gas contained in a pipe or tunnel, the effect of the elasticity of the pipe or tunnel walls upon the wave speed is usually negligible compared with that of the compressibility of the gas itself (see Section 4). In general, the wave speed, a , in a gas contained in a pipe or tunnel is approximately equal to the wave speed, a_0 , in a large, unconfined body of the gas.

Some data, from which an estimate of the wave speed, a_0 , in compressed gases can be made, are given in References 5, 6 and 7.

4. ESTIMATION OF THE WAVE SPEED, a , IN PIPES AND TUNNELS

The elasticity of the walls of pipes or tunnels containing a fluid reduces the wave speeds in the fluid by a factor dependent upon the size, cross-sectional shape and material of the pipe or tunnel. Additionally, the method of fixation of a pipeline may affect the wave speed (see Section 4.1). The wave speed is not usually dependent upon the profile of the pipeline. The general equation for the wave speed in a fluid contained within a flexible conduit is usually expressed in the form,

$$a = \left[\rho \left(\frac{1}{\gamma K} + \frac{1}{A} \frac{\partial A}{\partial p} \right) \right]^{-1/2}, \quad (4.1)$$

where the term $(1/A)(\partial A/\partial p)$ is the variation of the area strain with the internal pressure in the conduit, and may be considered to be a modification to the bulk modulus of the fluid to allow for the pipe effects. For gases contained in a pipe or tunnel, the pipe wall elasticity is usually neglected, and the wave speed, a , is given by the wave speed, a_0 , in a large, unconfined body of the gas (see Section 3.2). For liquids, a value of γ of unity is usually taken. In this Item, the area strain term is referred to as the area strain factor, and is denoted by Λ . For the pipelines and tunnels considered in this Section, the expressions for Λ are summarised in Table 7.3. In a perfectly rigid pipeline, or an infinite, unconfined body of liquid, Λ tends to zero, and Equation (4.1) takes the form given by Equation (3.2).

In this Section, methods of estimating the wave speeds in single phase, homogeneous liquids contained in circular and non-circular pipes, lined or unlined tunnels and lined, buried pipes are presented. The pipe or tunnel wall materials are assumed to be homogeneous with uniform dimensional and elastic properties. The main uncertainty in the estimated wave speeds using the methods presented in this Section concerns the specification of the elastic properties of the liquid and the pipe or tunnel wall. In particular, the value of the bulk modulus, K , of the liquid is important. The bulk moduli for most pure liquids are well defined. Some typical values for liquids commonly used in engineering applications are given in Table 7.1. Typical values of the physical properties of some common pipe and tunnel wall materials for use with the methods presented in this Section are given in Table 7.2.

If entrained bubbles of gas or lumps of solid matter are present in a liquid, the value of K may be modified significantly. Some relevant data are given in Appendix A. The wave speed data given in this Section are derived using the methods presented in Derivations 2, 3 and 4.

4.1 Wave Speed in Liquids in Circular Pipes

The wave speed, a , in a fluid contained in a thin-walled pipe of circular cross-section (see Sketch 4.1) may be estimated using the equation,

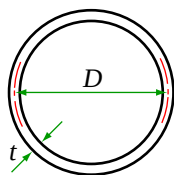
$$a = \left[\rho \left(\frac{1}{K} + \frac{D\phi}{tE} \right) \right]^{-1/2}, \quad (4.2)$$

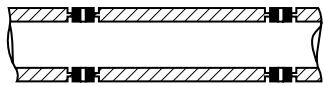
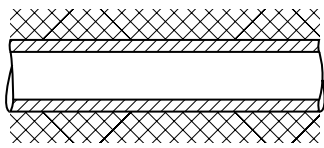
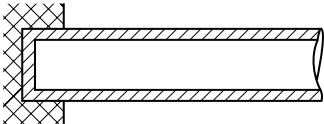
where $D\phi/tE = \Lambda$, D is the internal diameter of the pipe, t is the nominal wall thickness, E is the modulus of elasticity* of the pipe wall material, and ϕ represents the pipeline restraint. For pipeline materials with a high elastic modulus, such as steel or concrete, or for pipelines with frequent expansion joints (Sketch 4.1), a value for ϕ of unity may be taken. For pipes completely restrained against axial motion, ϕ may be approximated by

$$\phi = (1 - \nu^2), \quad (4.3)$$

where ν is the Poisson's ratio of the pipe wall material. For pipes in which axial motion occurs due to increased pressure at a closed end away from the source of pressure waves, the pipeline restraint factor is given by,

$$\phi = \left(1 - \frac{\nu}{2}\right). \quad (4.4)$$



Pipeline restraints		ϕ
Frequent expansion joints		1
Restrained against axial motion along entire length		$(1 - \nu^2)$
Restrained at closed end away from source of pressure waves		$\left(1 - \frac{\nu}{2}\right)$

Sketch 4.1 Circular pipe cross section and pipeline restraints

In practice, these idealised restraints very rarely occur. As the influence of this parameter is usually small compared with the inaccuracies of other parameters, a value of ϕ of unity is usually taken with little additional error in the wave speed estimate.

4.1.1 Graphical presentation of wave speed in circular pipes

The value of the wave speed, a , in water in thin-walled circular pipes may be estimated for a range of values of the pipe wall modulus of elasticity using Figure 3. These data, derived from Equation (4.2) assuming a unit value of the pipeline restraint, ϕ , are given for water at a temperature of 15°C (59°F) and atmospheric pressure, p_{ref} . The value of the wave speed in various common liquids in steel thin-walled pipes may be estimated using Figures 4a and 4b. All these data are given for a range of values of the pipe wall thickness to pipe diameter ratio, t/D , between 0.001 and 0.1. The data were generated using

* For orthotropic materials (i.e. those having uniform elastic properties in three mutually perpendicular directions), such as glass reinforced plastic (GRP), care should be taken in the selection of the appropriate values of E and ν for use in the equations in Section 4.1 (see the note relating to GRP pipes below Table 7.3). The problem does not arise in the case of isotropic materials (i.e. those having the same elastic properties in all directions).

Equation (4.2), which is based upon thin-walled pipe theory, and are valid for values of t/D less than approximately 0.1. The bulk modulus and density data considered in Figures 4a and 4b are given in Table 7.1.

4.2 Wave Speed in Liquids in Lined, Circular Pipes

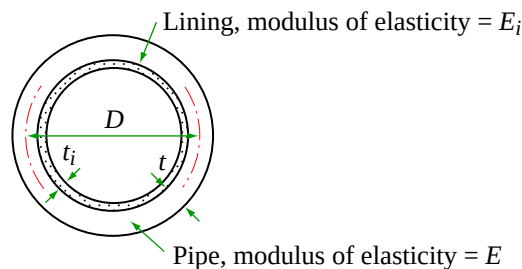
The wave speed, a , for a liquid contained in a lined pipe of circular cross section (see Sketch 4.2) may be estimated using the equation,

$$a = \left[\rho \left(\frac{1}{K} + \Lambda \right) \right]^{-1/2}, \quad (4.5)$$

where Λ is given by

$$\Lambda = \frac{(D-t)(1-\nu^2)\{(D^2+t^2)(1-\nu)+2\nu Dt\}}{EDt(D-t)(1-\nu)+E_i t_i \{(D^2+t^2)(1-\nu)+2\nu Dt\}}. \quad (4.6)$$

In Equation (4.6), E , D and t are the modulus of elasticity, and the mean diameter and thickness of the pipe, respectively, and E_i and t_i are the modulus of elasticity and thickness of the internal lining, respectively. In this expression, the same value of Poisson's ratio, ν , is assumed for both the pipe wall material and the lining material. This assumption is valid for most pairs of isotropic materials as the expression for Λ is relatively insensitive to variations in ν .



Sketch 4.2 Cross section through a lined circular pipe

4.3 Wave Speed in Liquids in Non-Circular Pipes

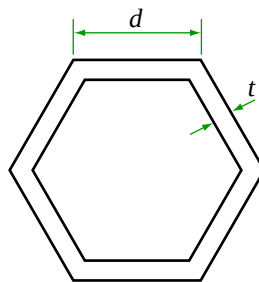
The wave speed, a , for a fluid contained in a thin-walled pipe of regular, polygonal cross section with an even number of flats (see Sketch 4.3, which illustrates such a pipe with six flats) may be estimated using the equation

$$a = \left[\rho \left(\frac{1}{K} + \frac{1}{E} \left\{ \frac{d}{t} + \frac{\tan^4 \theta}{15} \left(\frac{d}{t} \right)^3 \right\} + \frac{\tan^2 \theta}{2G} \frac{d}{t} \right) \right]^{-1/2}, \quad (4.7)$$

where d is the width of the flats, t is the pipe wall thickness, G is the shear modulus of elasticity, and θ is given by

$$\theta = \frac{180^\circ}{\text{number of flats}}. \quad (4.8)$$

Equation (4.7), for wave speed, is based upon the assumption that large deformations of the pipe do not occur. If such deformation does occur, the pipe cross section would tend towards circular, and the wave speed would increase. If plastic deformation occurs in the vicinity of the corners of the polygonal cross section, the wave speed would be altered. Some guidance on the effects of plastic deformation upon the wave speed is given in Appendix C.



Sketch 4.3 Cross section through a regular, polygonal pipe

4.3.1 Graphical presentation of wave speed in non-circular pipes

The values of the wave speed, a , for water at a temperature of 15°C (59°F) and atmospheric pressure, p_{ref} , in thin-walled, non-circular steel and reinforced concrete pipes may be estimated for several regular, polygonal cross-sectional shapes from Figure 5, using the typical values of the pipe wall elastic properties given in Table 7.2. These data are given for a range of values of the ratio of the pipe wall thickness to the width of flats ratio, t/d , between 0.001 and 0.1.

4.4 Wave Speed in Liquids in Tunnels

Data are given in this Section for the estimation of the wave speed in lined and unlined tunnels through rock. The main uncertainty in the estimation of the wave speed in a tunnel through rock lies in the accuracy with which the elastic properties of the rock are known, as they may vary along the length of the tunnel due to the non-homogeneity of the rock. Additionally, there may be trapped pockets of air or gas in the tunnel if the surface of the tunnel is not smooth and uniform.

The data for the estimation of wave speed in unlined tunnels are given for any cross-sectional shape, although it is assumed that the tunnel is completely full of liquid.