

GROWTH OF CRACKS UNDER CONSTANT AMPLITUDE FATIGUE LOADING: EXAMPLE CALCULATIONS

1. INTRODUCTION

This Item illustrates, by means of worked examples, applications of the fatigue crack propagation rate data presented in the Fatigue - Fracture Mechanics data Sub-series. The data used in the examples have been obtained from Data Item No. 83007* and the methods adopted use the principles of linear elastic fracture mechanics as discussed in Data Item No. 80036†.

The methods used in the examples are applicable for constant amplitude loading provided that the appropriate crack propagation rate data and stress intensity factor solutions are available.

2. NOTATION

A	gross cross-sectional area of stiffener	m^2	in^2
$2a$	total crack length	m	in
$2b$	total sheet width or stiffener pitch	m	in
C	coefficient in simple crack growth rate equation defined by $da/dN = C(\Delta K)^n$	$\text{m}\left(\frac{\text{m}^{3/2}}{\text{N}}\right)^n$	$\text{in}\left(\frac{\text{in}^{3/2}}{\text{lbf}}\right)^n$
E_1	modulus of elasticity of sheet material	N/m^2	lbf/in^2
E_2	modulus of elasticity of stiffener material	N/m^2	lbf/in^2
ΔK	stress intensity factor range	$\text{N/m}^{3/2}$	$\text{lbf/in}^{3/2}$
ΔK_0	stress intensity factor range for infinitely wide panel, see Example 2 in Section 4.2	$\text{N/m}^{3/2}$	$\text{lbf/in}^{3/2}$
N	number of load cycles		
n	index in simple crack growth rate equation, Equation (3.1)		
p	pitch of skin-to-stiffener fasteners	m	in
Q	coefficient defined by Equation (4.4)	$(\text{N/m}^2)^n$	$(\text{lbf/in}^2)^n$
R	$S_{\min}/S_{\max}$		
S	stress based upon gross cross-sectional area	N/m^2	lbf/in^2
t	sheet thickness	m	in

* Data Item No. 83007 "Fatigue crack propagation rates and threshold stress intensity factor ranges for aluminium alloy sheet".

† Data Item No. 80036 "Introduction to the use of linear elastic fracture mechanics in estimating fatigue crack growth rates and residual strength of components."

α	geometrical correction to stress intensity factor
μ	ratio of stiffener stiffness to total stiffness defined by $E_2 A / (E_2 A + E_1 2bt)$

Suffixes

<i>max</i> , <i>min</i>	indicates maximum and minimum values
<i>mean</i>	average value over increment of crack growth
<i>x</i>	label denoting step limits used when calculating crack growth using Method (i) (see Section 4 and Table 4.1)

2.1 Conversion Factors

Both SI and British units are quoted in the notation. The examples in this Item employ SI units throughout. Stress intensity factors may be converted into British units using the following conversion factor,

$$1.0 \text{ MN/m}^{3/2} = 0.91 \times 10^3 \text{ lbf/in}^{3/2}.$$

Note also that

$$1.0 \text{ MN/m}^{3/2} = 1.0 \text{ MPa m}^{1/2} = 31.62 \text{ N/mm}^{3/2} \text{ (that is, } 10^{3/2} \text{ N/mm}^{3/2} \text{)}.$$

3. APPLICATIONS OF CRACK PROPAGATION RATE DATA AND BASIC METHODS OF SOLUTION

In a damage tolerant aircraft structure any fatigue cracks that may develop should be detected before they reduce the strength of the structure below an acceptable level. Design in accordance with such a philosophy will generally necessitate prediction of the growth of a crack under the load history appropriate to one inspection interval (or more) from an initial crack length. The length of the initial crack is generally taken to be the maximum that might be missed by the inspection method to be used for the structural component concerned. Crack growth prediction over one inspection interval, or more, is made to check that the crack will not grow within the inspection interval to such a length that the residual strength of the structure becomes inadequate. If the performance of the component is shown to be inadequate it may be necessary to reduce the stress level in the component, or to consider shorter inspection intervals in service.

3.1 Methods of Estimating Fatigue Crack Growth

Estimates of the growth of a crack as a result of the application of a given number of load cycles may be obtained by using crack propagation rate data for the appropriate material. These are usually presented as curves of crack growth rate, da/dN , against stress intensity factor range, ΔK , as, for example, the data given in the ESDU Data Items. The use of such curves for estimation of crack growth requires a numerical integration process and the following two methods, for use under constant amplitude loading applications, are demonstrated in this Item.

Method (i). The crack growth rate curves are assumed to be straight lines between selected values of ΔK

so that the relationship

$$\frac{da}{dN} = C(\Delta K)^n \quad (3.1)$$

applies. The index n and the coefficient C may be obtained from the crack propagation rate curves appropriate to the material. The general solution for N obtained from Equation (3.1) is used to determine N over the selected increment of ΔK . Values of N are summed, for successive increments, until the number of load applications within an inspection interval is reached. The value of ΔK then applying gives the length of the crack at the end of the inspection interval.

Method (ii). The geometric mean of the crack growth rate values at two selected crack lengths is calculated and is assumed to be the crack growth rate that prevails between these crack lengths. This is then used to calculate the number of cycles required to propagate the crack between the two lengths. As with Method (i) values of N are summed until the number of load applications within an inspection interval is reached. The value of ΔK then applying gives the length of crack at the end of the inspection interval.

3.1.1 Solution of stress intensity factor range, ΔK

The general solution of ΔK is given by Equation (4.2) of Data Item No. 80036 so that,

$$\Delta K = (S_{max} - S_{min})(\pi a)^{1/2} \alpha. \quad (3.2)$$

Both Methods (i) and (ii) use Equation (3.2) to calculate ΔK and assume that the value of α is constant over the increment of crack growth considered.

The accuracy of the numerical integration procedure used is dependent upon the size of increment of ΔK used. Account must be taken of the local shape of the da/dN versus ΔK curve to ensure that the estimate of N , using either Method (i) or Method (ii), is sensible. In addition, since the value of ΔK is dependent upon α , which is held constant over the increments of crack length considered in both Methods (i) and (ii), variations in the value of α with crack length will also influence the accuracy of the calculation. Therefore, variations in the value of α must be taken into account when determining the size of the increments of crack length to be used in the numerical integration.

The user should first determine whether or not the da/dN versus ΔK curves include corrections for the effect of plasticity at the crack tip. Where such corrections are included the same method of allowing for plasticity at the crack tip in the crack under investigation should be employed throughout the calculation. The examples given in this Item do not contain corrections for crack tip plasticity because the analysis of the test data for production of the $da/dN - \Delta K$ curves, on which these examples rely, also do not include corrections for crack tip plasticity effects (see Data Item No. 83007). Notes on crack tip plasticity are given in Section 4.1.2 of Data Item No. 80036.

4. EXAMPLES

Worked examples are provided in Sections 4.1 and 4.2. In each example flat sheets or panels, manufactured from 2024-T3 aluminium alloy sheet material, each containing a central through-thickness crack and subjected to constant amplitude tensile fatigue loading are considered. In each example an estimate is made of the number of load cycles, N , that would cause the existing crack to grow from an initial length to a predetermined final length. The appropriate fatigue crack propagation rate data for 2043-T3 aluminium alloy sheet is given on Figure 2 of Data Item No. 83007.

Example 1 (Section 4.1) considers the simple case of a thin sheet, without edge stiffeners or restraint. In Section 4.1.1 the value of N is obtained by solving Equation (3.1), using Method (i), and in Section 4.1.2 the value of N is obtained using Method (ii).

Example 2 (Section 4.2) considers the more practical case of a typical aircraft structure consisting of a sheet reinforced by equally spaced stiffeners and uses Method (ii) to calculate the value of N . In addition, the effect of changing the stiffness of the stiffeners is illustrated.

4.1 Example 1 – Sheet with a Central Crack (No Edge Restraint)

A 2 mm thick sheet with unsupported edges is 440 mm wide and contains an initial crack 44 mm in length at its centre. The sheet is subjected to a constant amplitude tensile gross stress cycle of 0 to 80 MN/m² acting normal to the direction of the crack length. The growth of the crack will be investigated up to a crack length of 200 mm and the number of load cycles to reach this length determined using both Methods (i) and (ii) together with the appropriate da/dN versus ΔK data of Data Item No. 83007.

The loading cycle is $S_{max} = 80$ MN/m², $S_{min} = 0$, so that $R = 0$ and from the given data $b = 220$ mm and initial $a = 22$ mm.

4.1.1 Estimation of number of load cycles using Method (i)

Substituting Equation (3.2) into Equation (3.1), the crack propagation rate may be expressed as

$$\frac{da}{dN} = C[(S_{max} - S_{min})(\pi a)^{1/2} \alpha]^n \quad (4.1)$$

The solution for α for a flat sheet with no edge restraint and containing a central crack is given by Equation (3.2) of Data Item No. 83007 as

$$\alpha = \left[1 - 0.5 \frac{a}{b} + 0.326 \left(\frac{a}{b} \right)^2 \right] \left[1 - \frac{a}{b} \right]^{-1/2} \quad (4.2)$$

Although α is a function of crack half length, a , it will be treated as a fixed value over each single increment of crack growth. In this particular example it is taken as the mean value over the increment of crack half length considered. This allows Equation (4.1) to be rewritten as

$$\frac{da}{dN} = CQa^{n/2} \quad (4.3)$$

where

$$Q = [(S_{max} - S_{min})(\pi)^{1/2} \alpha_{mean}]^n \quad (4.4)$$

Equation (4.3) may be expressed as an integral with limits a_{x+1} , a_x and N_{x+1} , N_x . Thus,

$$\frac{1}{CQ} \int_{a_x}^{a_{x+1}} a^{-n/2} da = \int_{N_x}^{N_{x+1}} dN \quad (4.5)$$

Integrating Equation (4.5) yields

$$\frac{1}{CQ} \left[\frac{a^{[1 - (n/2)]}}{1 - (n/2)} \right]_{a_x}^{a_{x+1}} = \left[N \right]_{N_x}^{N_{x+1}} \quad (4.6)$$

and therefore, inserting the limits, Equation (4.6) becomes

$$\frac{1}{CQ} \left\{ \frac{1}{1 - (n/2)} \left[a_{x+1}^{[1 - (n/2)]} - a_x^{[1 - (n/2)]} \right] \right\} = (N_{x+1} - N_x). \quad (4.7)$$

Table 4.1 sets out the solution to Equation (4.7) to obtain the total number of loading cycles using the $R = 0$ to 0.11 mean curve of Figure 2 of Data Item No. 83007.

In order to represent the da/dN versus ΔK curve over the range $a = 22$ mm to 100 mm by Equation (3.1) it is necessary to select increments over which the da/dN versus ΔK curve is sensibly straight. The initial and final crack half lengths correspond to ΔK values of $21.1 \text{ MN/m}^{3/2}$ and $51 \text{ MN/m}^{3/2}$. Inspection of the da/dN versus ΔK curve shows that selection of intermediate values of ΔK of about $25 \text{ MN/m}^{3/2}$ and $40 \text{ MN/m}^{3/2}$ will separate the curve into 3 portions over each of which the curve may be represented by a straight line. Values of a corresponding to these values of ΔK are 30 mm and 70 mm and these values have been taken to define the ends of the crack growth increments. Thus, Equation (4.7) can be solved for three values of n and four values of a . The total number of load cycles required to reach a crack half length of 100 mm, based upon the mean crack growth curve, is 25 680.

4.1.2 Estimation of number of load cycles using Method (ii)

Table 4.2 sets out Method (ii) for estimating the crack growth, as described in Section 3.1. In this example the crack length is increased by approximately 10 per cent up to a crack half length of 100 mm. The geometric mean value of crack growth rate is calculated from the values of da/dN at the ends of each increment of crack half length, and is taken to apply over the whole increment. Two sets of values of da/dN , and subsequently dN , are presented in Table 4.2. The first set relates to the mean da/dN versus ΔK curves and the second set relates to the 95 per cent limit da/dN versus ΔK curves, for $R = 0$ to 0.11 of Figure 2 of Data Item No. 83007. The summation of dN , the load cycles within each of the prescribed intervals, gives the total number of load cycles for the crack to grow to a crack half length of 100 mm. It shows that using the mean crack growth curve about 29 200 cycles will be required for the crack to grow up to 100 mm. The 95 per cent limit curve indicates that the crack will become unstable before 100 mm is reached because the maximum value of ΔK on the upper limit curve is $45 \text{ MN/m}^{3/2}$, which corresponds to a crack half length of 84.6 mm. At this crack half length the values given in Table 4.2 are enclosed in brackets. The number of load cycles that can be applied to reach a crack half length of 84.6 mm is 5500. The table also shows a maximum crack growth rate, based upon the mean curves, of 0.1 mm/cycle is reached at 100 mm and that using the 95 per cent limit curve the maximum growth rate, at 84.6 mm, is as high as 0.2 mm/cycle.