



Estimation of sideforce, yawing moment and rolling moment derivatives due to rate of yaw for complete aircraft at subsonic speeds

Endorsed by
The Royal Aeronautical Society

ESDU DATA ITEMS

Data Items provide validated information in engineering design and analysis for use by, or under the supervision of, professionally qualified engineers. The data are founded on an evaluation of all the relevant information, both published and unpublished, and are invariably supported by original work of ESDU staff engineers or consultants. The whole process is subject to independent review for which crucial support is provided by industrial companies, government research laboratories, universities and others from around the world through the participation of some of their leading experts on ESDU Technical Committees. This process ensures that the results of much valuable work (theoretical, experimental and operational), which may not be widely available or in a readily usable form, can be communicated concisely and accurately to the engineering community.

We are constantly striving to develop new work and review data already issued. Any comments arising out of your use of our data, or any suggestions for new topics or information that might lead to improvements, will help us to provide a better service.

THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item was monitored and guided by the Aerodynamics Committee. This Committee first met in 1942 and now has the following membership:

Chairman	
Mr H.C. Garner	— Independent
Vice-Chairman	
Mr P.K. Jones	— British Aerospace, Aircraft Group, Manchester
Members	
Mr D. Bonenfant	— Aérospatiale, Toulouse, France
Mr E.A. Boyd	— Cranfield Institute of Technology
Mr K. Burgin	— Southampton University
Mr E.C. Carter	— Aircraft Research Association
Mr J.R.J. Dovey	— British Aerospace, Aircraft Group, Warton
Dr J.W. Flower	— Bristol University
Mr A. Hipp	— British Aerospace, Dynamics Group, Stevenage
Mr J. Kloos*	— Saab-Scania, Linköping, Sweden
Mr J.R.C. Pedersen	— Independent
Mr I.H. Rettie*	— Boeing Aerospace Company, Seattle, Wash., USA
Mr A.E. Sewell*	— Northrop Corporation, Hawthorne, Calif., USA
Mr F.W. Stanhope	— Rolls-Royce Ltd, Derby
Mr H. Vogel	— British Aerospace, Aircraft Group, Weybridge
Mr J. Weir	— Salford University.

* Corresponding Member

The construction and subsequent development of the Data Item was undertaken by

Mr R.W. Gilbey — Senior Engineer.

ESTIMATION OF SIDEFORCE, YAWING MOMENT AND ROLLING MOMENT DERIVATIVES DUE TO RATE OF YAW FOR COMPLETE AIRCRAFT AT SUBSONIC SPEEDS

CONTENTS

	Page
1. NOTATION AND UNITS	1
2. INTRODUCTION	5
3. COMPARISONS WITH EXPERIMENTAL DATA	6
3.1 Discussion of Data	6
3.2 Accuracy of Prediction for $\alpha \approx 0$	7
3.3 Variation with α	10
3.3.1 Sideforce and Yawing Moment Derivatives	10
3.3.2 Rolling Moment Derivative	10
3.4 High-lift Devices	11
3.4.1 Sideforce and Yawing Moment Derivatives	11
3.4.2 Rolling Moment Derivative	12
3.5 Variation with Mach Number	12
3.6 Comparisons with Flight-test Data	12
4. DERIVATION AND REFERENCES	19
4.1 Derivation	19
4.2 References	22
5. EXAMPLE	24
5.1 Calculation of Wing Planform Parameters	26
5.2 Calculation of Wing and Flap Lift and Zero-lift Profile Drag Coefficients	27
5.2.1 Wing Coefficients C_L , C_{D0}	27
5.2.2 Flap Coefficients ΔC_{Lf} , ΔC_{D0f}	29
5.2.3 Total Lift Coefficient	29
5.3 Calculation of Wing and Flap Contributions $(N_r)_W$ and $(N_r)_f$	30
5.3.1 Wing Yawing Moment Contribution	30
5.3.2 Flap Yawing Moment Contribution	30
5.4 Calculation of Wing and Flap Contributions $(L_r)_W$ and $(L_r)_f$	32
5.4.1 Wing Rolling Moment Contribution	32
5.4.2 Flap Rolling Moment Contribution	32
5.4.3 Calculation of $(\Delta L_r)_{corr}$	33
5.5 Calculation of Body Contributions $(Y_r)_B$ and $(N_r)_B$	38
5.6 Calculation of Fin Contributions $(Y_r)_F$, $(N_r)_F$ and $(L_r)_F$	39
5.7 Summary of Results and Total Values	41

APPENDIX A SIMPLIFIED METHOD	46
A1. INTRODUCTION	46
A2. METHOD	46
A3. ACCURACY	47
FIGURE A1	48

ESTIMATION OF SIDEFORCE, YAWING MOMENT AND ROLLING MOMENT DERIVATIVES DUE TO RATE OF YAW FOR COMPLETE AIRCRAFT AT SUBSONIC SPEEDS

1. NOTATION AND UNITS (see Sketch 1.1)

The derivative notation used is that proposed in ARC R&M 3562 (Hopkin, 1970) and described in Item No. 86021. Coefficients and aeronormalised derivatives are evaluated in aerodynamic body axes with origin at the aircraft centre of gravity and with the wing span as the characteristic length. The derivatives Y_r , N_r and L_r are often written as C_{Yr} , C_{nr} and C_{lr} in other systems of notation, but attention must be paid to the reference dimensions used. In particular, in forming C_{Yr} , C_{nr} and C_{lr} differentiation may be carried out with respect to $rb/2V$ not rb/V as implied in the Hopkin system. It is also to be noted that a constant datum value of V is employed by Hopkin.

This Item makes use of several other Items which have been produced at different times over a period of many years. Although the nomenclature in these Items is consistent for the important parameters such as stability derivatives, it involves some variation and duplication for the less significant parameters. Because of this, and to avoid repetition, the Notation given here is limited to the major quantities appearing in the main text of this Item and to quantities not appearing in other Items. When referred to the method in another Item the user should consult the Notation at the front of that particular Item before carrying out any calculations.

		SI	British
A	aspect ratio, b^2/S		
b	wing span	m	ft
C_{D0}	zero-lift profile drag coefficient		
ΔC_{D0f}	increment in C_{D0} due to trailing-edge flap deflection		
C_L	lift coefficient, $L/1/2\rho V^2 S$		
ΔC_{Lf}	increment in C_L due to trailing-edge flap deflection		
C_l	rolling moment coefficient, $\mathcal{L}/1/2\rho V^2 S b$		
C_n	yawing moment coefficient, $\mathcal{N}/1/2\rho V^2 S b$		
C_Y	sideforce coefficient, $Y/1/2\rho V^2 S$		
L	lift	N	lbf
$\mathcal{L}$	rolling moment	N m	lbf ft
L_v	rolling moment derivative due to sideslip; $L_v = (\partial \mathcal{L} / \partial v) / 1/2\rho V S b$		
L_{v0}	value of L_v when wing $C_L = 0$		

Issued April 1984 - 48 pages

$L_{\dot{v}}$	rolling moment derivative due to rate of change of sideslip; $L_{\dot{v}} = (\partial \mathcal{L} / \partial \dot{v}) / \frac{1}{2} \rho S b^2$		
L_r	rolling moment derivative due to rate of yaw; $L_r = (\partial \mathcal{L} / \partial r) / \frac{1}{2} \rho V S b^2$		
$(\Delta L_r)_{corr}$	empirical correction added to allow for partially separated flow on wing (see Section 3.3.2)		
$\bar{l}'_F$	yawing moment tail arm; distance from moment reference centre to centre of pressure position of fin sideforce, measured parallel to longitudinal body axis	m	ft
M	Mach number		
$\mathcal{N}$	yawing moment	N m	lbf ft
N_v	yawing moment derivative due to sideslip; $N_v = (\partial \mathcal{N} / \partial v) / \frac{1}{2} \rho V S b$		
$N_{\dot{v}}$	yawing moment derivative due to rate of change of sideslip; $N_{\dot{v}} = (\partial \mathcal{N} / \partial \dot{v}) / \frac{1}{2} \rho S b^2$		
N_r	yawing moment derivative due to rate of yaw; $N_r = (\partial \mathcal{N} / \partial r) / \frac{1}{2} \rho V S b^2$		
r	rate of yaw	rad/s	rad/s
S	wing planform area	m ²	ft ²
S_F	fin side area	m ²	ft ²
V	velocity of aircraft relative to air	m/s	ft/s
v	sideslip velocity	m/s	ft/s
$\dot{v}$	rate of change of sideslip velocity	m/s ²	ft/s ²
Y	sideforce	N	lbf
Y_v	sideforce derivative due to sideslip; $Y_v = (\partial Y / \partial v) / \frac{1}{2} \rho V S$		
$Y_{\dot{v}}$	sideforce derivative due to rate of change of sideslip; $Y_{\dot{v}} = (\partial Y / \partial \dot{v}) / \frac{1}{2} \rho S b$		
Y_r	sideforce derivative due to rate of yaw; $Y_r = (\partial Y / \partial r) / \frac{1}{2} \rho V S b$		
$\bar{z}'_F$	rolling moment tail arm; distance from moment reference centre to centre of pressure position of fin sideforce, measured normal to longitudinal body axis	m	ft
α	angle of attack of longitudinal body axis	degree	degree

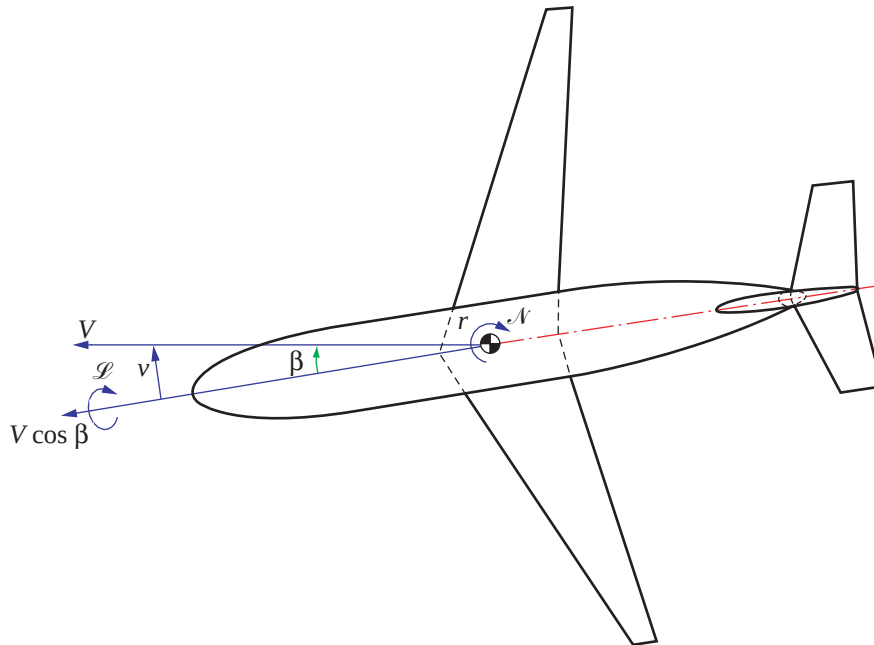
α_w	angle between wing zero-lift line and longitudinal body axis, so that angle of attack of wing zero-lift line is $\alpha + \alpha_w$	degree	degree
β	angle of sideslip	rad	rad
$\dot{\beta}$	rate of change of angle of sideslip	rad/s	rad/s
Λ_0	wing leading-edge sweep angle	degree	degree
$\Lambda_{1/4}$	wing quarter-chord sweep angle	degree	degree
$\Lambda_{1/2}$	wing half-chord sweep angle	degree	degree
λ	ratio of tip chord to centre-line chord (taper ratio)		
ρ	density of air	kg/m ³	slug/ft ³

Subscripts

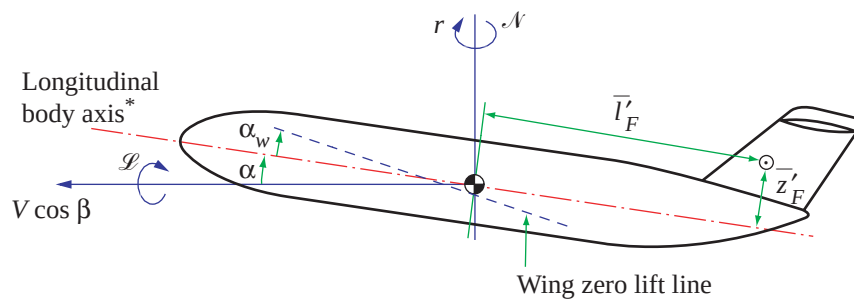
<i>exp</i>	denotes experimental value
<i>pred</i>	denotes predicted value

Additional symbols

$()_B$	denotes component due to body
$()_F$	denotes component due to fin
$()_f$	denotes component due to trailing-edge flap deployment at constant C_L
$()_p$	denotes component due to wing planform in fully-attached flow
$()_W$	denotes total component due to wing
$()_\Gamma$	denotes component due to wing dihedral
$()_\epsilon$	denotes component due to wing twist
'	as in L'_r and L'_v denotes values in fully-attached flow



● Centre of gravity position



○ Centre of pressure position of fin sideforce

Sketch 1.1 Sign conventions

* The longitudinal body axis is a reference axis, fixed in the body in the plane of symmetry and passing through the centre of gravity position. The exact direction of the axis in the plane of symmetry is conventionally determined by considerations of mid-body geometry.

2. INTRODUCTION

An aircraft's sideforce, yawing moment and rolling moment derivatives due to rate of yaw, Y_r , N_r and L_r , are customarily estimated by calculating the effects of the major components of the aircraft individually and adding together the part derivatives so obtained. For aircraft at subsonic speeds, separate Items on the various part derivatives have been issued over a number of years and each of the major components is covered. This Item demonstrates how the methods in those separate Items may be combined to provide an estimate of the derivatives of a complete aircraft, and illustrates the overall accuracy of prediction by means of comparisons with wind-tunnel and flight-test data.

Tables 2.1 and 2.2 list the major components of Y_r , N_r and L_r and the Items from which they may be estimated. The total values of the derivatives are obtained by evaluating each component at the same angle of attack and summing the results. The Items that deal specifically with Y_r , N_r and L_r often require further information, such as wing lift coefficient or the lift and drag coefficient increments due to trailing-edge flap deflection. Such information may be obtained from other Items and these are also listed in the Tables. If static stability wind-tunnel data are available for the aircraft of interest then these can sometimes be used as an alternative source for the additional information required.

Comparisons between experimental and predicted values of Y_r , N_r and L_r are discussed in Section 3 in terms of magnitude and variation with angle of attack. The Derivation and References are given in Section 4.

Section 5 contains a detailed worked example that demonstrates the calculation of the component parts of the derivatives for a particular aircraft and their subsequent combination. The choice of aircraft is the same as that used to demonstrate the calculation of derivatives due to sideslip in Item Nos 81032 and 82011 (Derivations 57 and 59). The example devotes a separate subsection to the calculation of each component and describes at which stages in the calculation use is made of the additional Items listed in Tables 2.1 and 2.2 and when experimental data, if available, may be useful. As the example provides guidance and information on points that are not covered in other Items it is advisable to refer to the appropriate subsection of the example to obtain the best use of each Item dealing with the separate components of Y_r , N_r and L_r .

A simplified method for estimating complete aircraft values rapidly is given in Appendix A.

TABLE 2.1 SIDEFORCE AND YAWING MOMENT DERIVATIVES

Component	Due to	Calculated from Item No.	Possible additional Item Nos
$(N_r)_w$	Wing*	71017 ⁴⁶	Aero W.02.04.01³⁴, 02³⁵ and 03³⁸ 76003⁵⁴, 70011⁴⁵ 75013⁵³, 74009⁴⁹, 74011⁵¹, 74012⁵² Aero F.01.01.08³⁷ and 09³⁶ Aero W.01.01.01³⁹ for wing zero-lift drag coefficient for wing lift coefficient for lift coefficient increments due to trailing-edge flap deflection
$(N_r)_f$	Trailing-edge flap deployment (at constant C_L)	71017 ⁴⁶	75013⁵³, 74010⁵⁰ Aero F.02.01.06³² and 07³¹, 87024⁶⁶ for increment in wing zero-lift drag coefficient due to trailing-edge flap deflection
$(Y_r)_B$, $(N_r)_B$	Body	83026 ⁶¹	
$(Y_r)_F$, $(N_r)_F$	Fin	82017 ⁶⁰	82010⁵⁸, 70011⁴⁵ Aero C.01.01.01⁴⁰ for fin sideforce derivative and centre of pressure position in sideslip
Item No. Aero A.07.01.00 ³³ gives a brief introduction to the various components of the yawing-moment stability derivatives and their related Data Items. (Derivation numbers are given as indices.)			

* This component depends on the wing lift coefficient. If trailing-edge flaps are deployed the lift coefficient of the wing with flap deflected is used. It therefore includes the effect of the lift increment produced by the flap. (See Sections 5.3 and 5.4.)

TABLE 2.2 ROLLING MOMENT DERIVATIVE

Component	Due to	Calculated from Item No.	Possible additional Item Nos
$(L_r)_W \left\{ \begin{array}{l} (L_r)_p \\ (L_r)_\Gamma \\ (L_r)_\epsilon \end{array} \right.$	Wing $\left\{ \begin{array}{l} \text{planform}^* \\ \text{dihedral} \\ \text{twist} \end{array} \right.$	72021 ⁴⁷	$\left. \begin{array}{l} 76003^{54}, 70011^{45} \\ 75013^{53}, 74009^{49}, 74011^{51}, 74012^{52} \\ \text{Aero F.01.01.08}^{37} \text{ and } 09^{36} \\ \text{Aero W.01.01.01}^{39} \end{array} \right\}$ for wing lift coefficient for lift coefficient increment due to trailing-edge flap deflection
$(L_r)_f$	Trailing-edge flap deployment (at constant C_L)	72021 ⁴⁷	$\left. \begin{array}{l} \text{Aero C.01.01.03}^{43}, 74009^{49} \\ \text{Aero F.01.01.08}^{37} \text{ and } 09^{36} \end{array} \right\}$ for slope of lift increment curve with trailing-edge flap deflection
$(\Delta L_r)_{corr}$	Partial separation of wing flow	This Item: Equations (3.1) and (3.2)	$\left. \begin{array}{l} 81032^{57} \\ \text{Aero A.06.01.03}^{41} \text{ and } 09^{42}, \text{ Aero C.01.01.01}^{40} \\ 73006^{48}, 80033^{55}, 80034^{56}, 82010^{58} \end{array} \right\}$ for predicted value of L_v
$(L_r)_F$	Fin	82017 ⁶⁰	$\left. \begin{array}{l} 82010^{58} \\ 70011^{45} \\ \text{Aero C.01.01.01}^{40} \end{array} \right\}$ for fin sideforce derivative and centre of pressure position in sideslip

Item No. Aero A.06.01.00⁴⁴ gives a brief introduction to the various components of the rolling-moment stability derivatives and their related Data Items. (Derivation numbers are given as indices.)

* This component depends on the wing lift coefficient. If trailing-edge flaps are deployed the lift coefficient of the wing with flap deflected is used. It therefore includes the effect of the lift increment produced by the flap. (See Sections 5.3 and 5.4.)

3. COMPARISONS WITH EXPERIMENTAL DATA

3.1 Discussion of Data

The accuracy with which the methods in the Items listed in Tables 2.1 and 2.2 predict Y_r , N_r and L_r for complete aircraft has been assessed by making comparisons with the values measured in the wind-tunnel tests reported in Derivations 1 to 20 and the values extracted from flight-test results that are reported in Derivations 21 to 30. Data have been studied for a wide variety of aircraft types representing civil transport aircraft, high performance fighter aircraft and light and general aviation aircraft, together with results for a number of simpler wind-tunnel models of the type employed to study the effect of systematic variations of geometric parameters. The majority of these data have been taken from low-speed tests for clean-wing configurations with no high-lift devices deployed. Experimental data for other conditions are less abundant, but the analysis included a limited number of results from tests at high subsonic Mach numbers ($M \approx 0.8$) and a few data from low-speed tests on configurations with wing leading-edge slats or trailing-edge flaps deployed.

It should be noted that wind-tunnel data from two types of test have been used. In one type a curved flow represents the yaw rate of the aircraft and this enables the direct measurement of the rate of yaw derivatives. In the second type of test the aircraft is mounted on a forced-oscillation rig and the rate of yaw derivatives are measured only in combination with a derivative due to the rate of change of sideslip, *i.e.* these tests give $Y_r - Y_v \cos \alpha$, $N_r - N_v \cos \alpha$ and $L_r - L_v \cos \alpha$. The $\dot{v}$ derivatives arise from the body, wing and the fin and become increasingly important as the angle of attack increases, see References 62 and 63. Reference 62 discusses the causes and magnitudes of the wing and fin contributions to the derivatives N_v and L_v in detail and concludes that they can not be estimated satisfactorily by simple means. In the case of the derivative Y_v the body also contributes significantly at low α (see Item No. 83026). In order to provide an indication of the practical importance of the rate of change of sideslip, data from oscillatory tests have been included in the present study and have been compared directly with the predicted rate of yaw derivatives, except in the case of Y_v when the slender-body theory estimate discussed in Item No. 83026

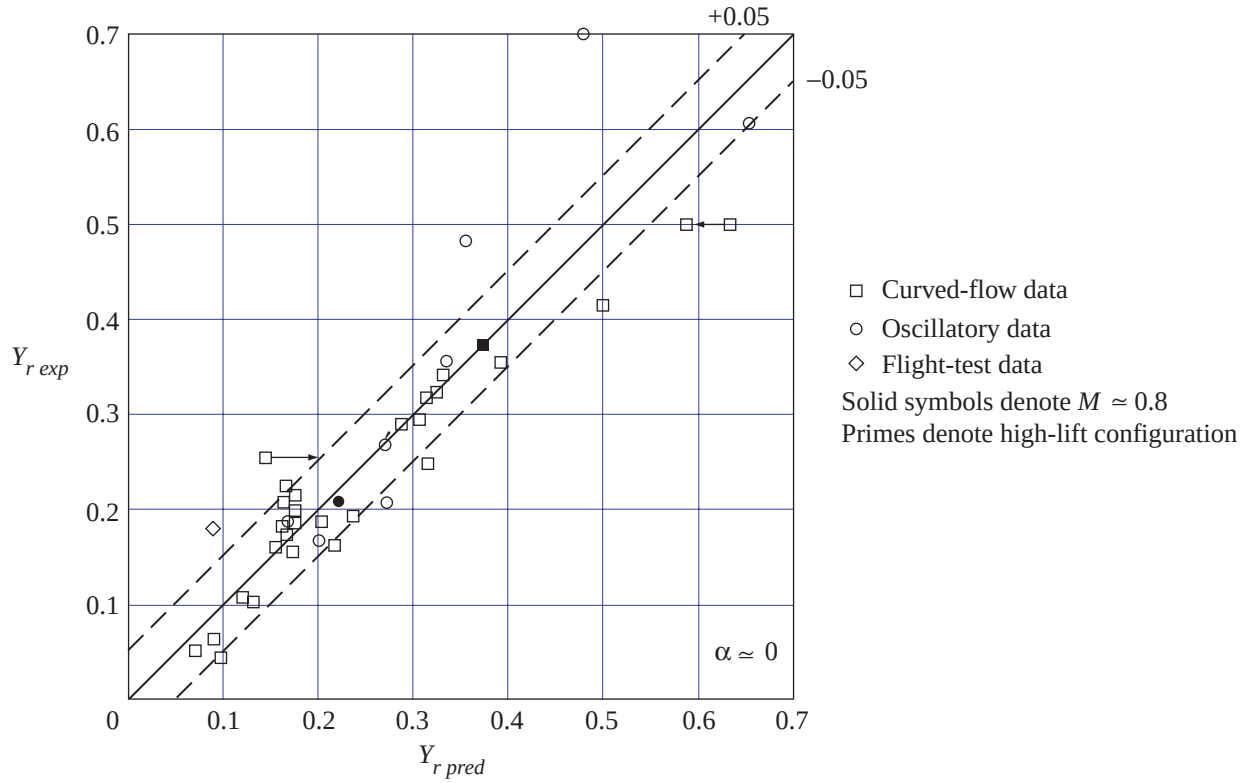
has been used. These comparisons have been confined to low values of angle of attack ($\alpha < 10^\circ$) where the derivatives $N_{\dot{v}}$ and $L_{\dot{v}}$ should not be too large. Errors between predicted and experimental values of this second type can, however, be expected to be higher than those for curved-flow wind-tunnel data.

The flight-test data that have been used come from tests in which the dynamic response of an aircraft to various control inputs is measured and then a complete set of aerodynamic stability derivatives is deduced which corresponds to that response. The accuracy with which the various derivatives can be identified by this process depends on the sensitivity of the response of the aircraft to each derivative. Usually, satisfactory estimates of N_r and L_r can be obtained, although L_r is sometimes subject to a fairly high uncertainty. The response of an aircraft is too insensitive to Y_r for an accurate value of this derivative to be necessary and so Y_r is frequently omitted from such analyses, a point which should be remembered when examining the accuracy of prediction of Y_r in general. In the extraction of flight-test results the $\dot{v}$ derivatives are usually ignored. Reference 63 shows that they can have a significant effect on the estimated rate of yaw derivatives at high angles of attack. However, at low angles of attack ($\alpha < 10^\circ$) they are considered to be sufficiently small to be ignored in direct comparisons between the flight-test data and the predicted values of N_r and L_r . Because of the errors involved in isolating particular derivatives from the flight-test data and the omission of the $\dot{v}$ derivatives, the errors between predicted and experimental flight-test values can be expected to be higher than for curved-flow wind-tunnel data.

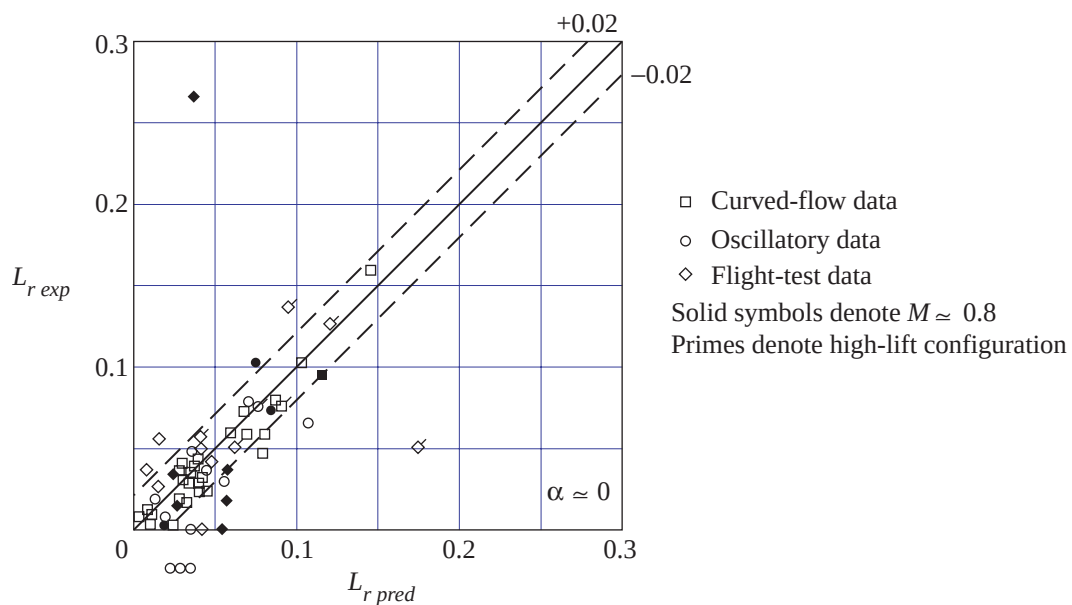
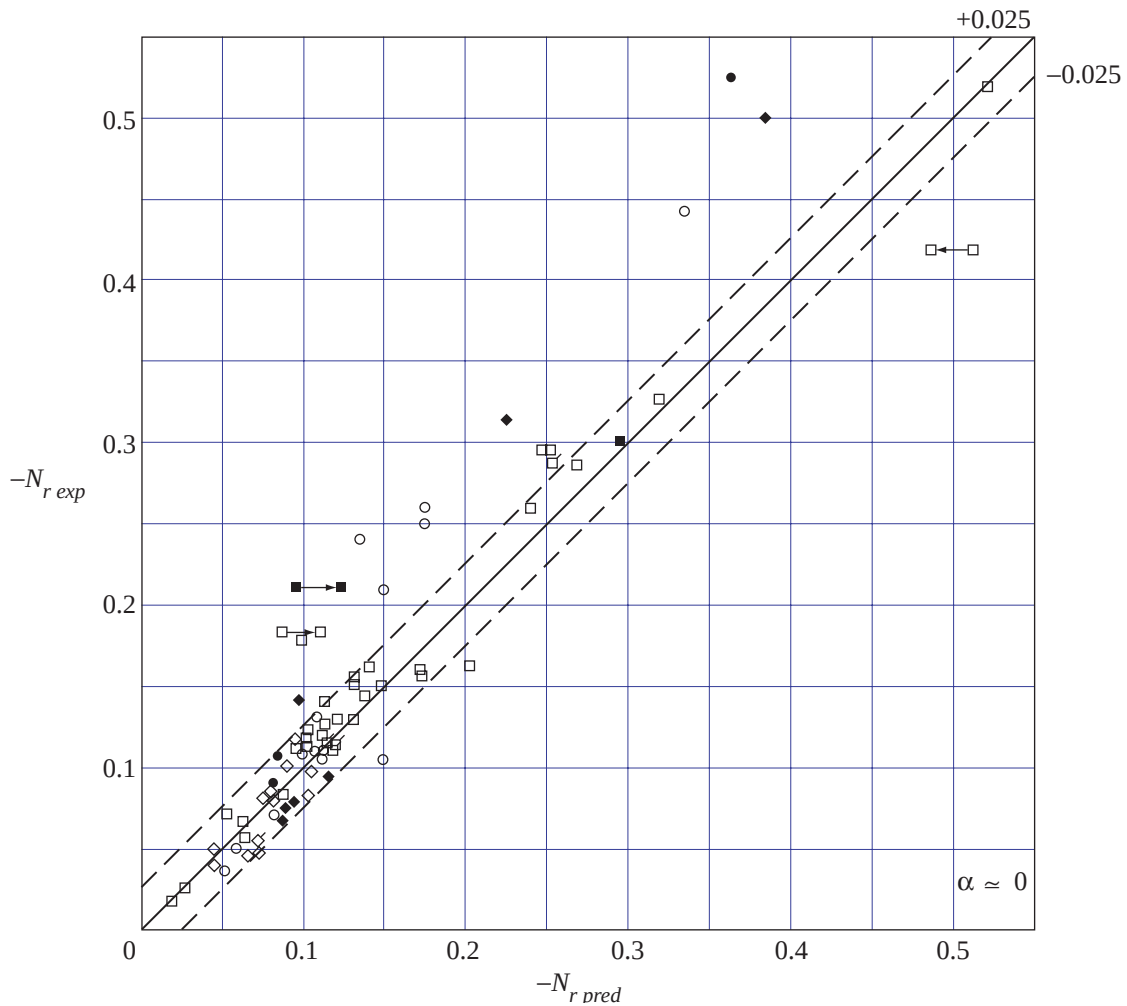
3.2 Accuracy of Prediction for $\alpha \approx 0$

The predicted and experimental values of Y_r , N_r and L_r at low angles of attack are compared in Sketches 3.1, 3.2 and 3.3. Sketch 3.1 shows that the experimental values of Y_r are generally predicted to within ± 0.05 or, at high values, to within ± 15 per cent. The prediction for the oscillatory rig data have $-(Y_{\dot{v}})_B$ added (see Section 3.1). Sketch 3.2 shows that the experimental values of N_r are generally predicted to within ± 0.025 or, at high values, to within ± 15 per cent, with poorer agreement being shown for some oscillatory rig and flight-test data. Sketch 3.3 shows that L_r is generally predicted to within ± 0.02 , although there are some larger errors mainly in cases of flight-test data. Possible causes of the discrepancies in the oscillatory rig and flight-test data have been discussed in the preceding section. Overall the accuracy of prediction is consistent with the accuracy figures quoted in individual Items, approximating to the square root of the sum of the squares of the individual errors.

For those cases in Sketches 3.1 and 3.2 where curved-flow data show a large prediction error, it has been found that this can be attributed, at least in part, to errors in the fin contributions predicted by Item No. 82017. Such inaccuracies can arise if the aircraft configuration departs significantly from the simple geometries that are considered in Item Nos 82010 and Aero C.01.01.01, when those Items are used to provide estimates of the fin sideforce derivative in steady sideslip, $(Y_v)_F$, and the yawing and rolling moment tail arms $\bar{l}'_F$ and $\bar{z}'_F$ that form part of the method in Item No. 82017. However, if values of $(Y_v)_F$, $(N_v)_F$ and $(L_v)_F$ are available from wind-tunnel tests these can be compared with the predicted values to determine empirical corrections, see Section 5.6. Examples of the improvements that can be introduced by such corrections are illustrated by arrows in Sketches 3.1 and 3.2.



Sketch 3.1 Comparison of predicted and experimental Y_r



Sketch 3.3 Comparison of predicted and experimental L_r

3.3 Variation with α

A typical set of experimental data is used for illustration purposes throughout Sections 3.3, 3.4 and 3.5. These data have been taken from Derivations 3 and 7 which report on a number of curved-flow wind-tunnel tests of the aircraft model shown in Sketch 3.4. The data are presented as a whole in Sketches 3.5 to 3.7 and cover both the cruise (clean) configuration and the landing configuration with leading-edge slats, trailing-edge flaps and undercarriage deployed. Reference to particular features of the sketches is made where appropriate. Section 3.3 considers the effect of α without the deployment of high-lift devices, Section 3.4 the changes due to such devices and Section 3.5 the effect of Mach number. The behaviour of the derivatives is a good representation for a wide variety of aircraft configurations.

3.3.1 Sideforce and Yawing Moment Derivatives

The predicted values of Y_r and N_r vary only slightly as the angle of attack and wing lift coefficient increase. This is because the fin provides the major contributions to Y_r and N_r and these are affected only by the small variation in the effective yawing moment tail arm (see Sections 5.6 and 5.7). There is also an increase in the magnitude of the lift-dependent part of the wing contribution to N_r but this remains small compared to other components (see Sections 5.3 and 5.7). The experimental values of Y_r and N_r vary only slightly as the angle of attack increases, in agreement with the predicted values, until large departures from the low angle of attack values accompany the appearance of flow separation effects. Significant variations with angle of attack are not usually apparent until α exceeds 10° , and for highly swept wings the values of Y_r and N_r remain essentially constant up to 15° or 20° . The predicted values therefore model the experimental behaviour quite well over the initial angle of attack range. In general the experimental values of Y_r show more variation than N_r , but as Y_r is comparatively unimportant in stability calculations such variation can be neglected. The cruise-configuration data in Sketch 3.5 illustrate these points.

3.3.2 Rolling Moment Derivative

The wing planform and the fin provide the major contributions to L_r . The value predicted in Item No. 72021 for the wing planform contribution in fully-attached flow, $(L_r)_p$, is proportional to the wing C_L and varies linearly with α (see Section 5.4). The predicted contribution of the fin varies non-linearly with α because of the changes in the effective rolling moment tail arm but this is less significant (see Section 5.6). Consequently the value predicted for a complete aircraft under the assumption of fully-attached flow, L'_r , increases almost linearly with α (see Section 5.7)*. Experimental values of L'_r depart quickly and severely from that prediction, falling well below L'_r from low values of α onwards, as demonstrated by the cruise-configuration data in Sketch 3.6a. This is due to the fact that the magnitude of the planform contribution is dramatically reduced by the occurrence of small areas of separated flow. For the same reason, the experimental planform contribution to $-L_v$ shows a very similar departure from its value in attached flow, that is also proportional to the wing C_L . Reference 64 gives a method that makes use of this similarity to produce an empirical correction term $(\Delta L_r)_{corr}$ that is added to $(L'_r)_w$ to allow for the effect of partial flow separation. The method requires appropriate experimental values of L_v , and Item No. 72021 recommends that it be employed whenever such data are available.

The correction term is equal to half the difference between the value predicted for the planform contribution to L_v in attached flow and the experimental value in partially-separated flow. For an isolated plain wing this can be written

$$(\Delta L_r)_{corr} = 0.5[(L_v)_{p\ pred} - (L_v)_{w\ exp}]. \quad (3.1)$$

* Primes as in L'_r and L'_v denote values in attached flow. Note that the planform contributions denoted by $(L_r)_p$ and $(L_v)_p$ are only defined as values in attached flow and therefore they do not carry primes.

Reference 64 presents a large number of comparisons with experimental data for a wide range of wing planforms that demonstrate that a correction of this magnitude leads to much improved estimates. It should be noted that the factor 0.5, which is empirical, must be adjusted if the parameters that are used to make L_v and L_r dimensionless are different from those in the Notation.

In most cases the experimental values of L_v that are available do not allow a direct comparison of predicted and experimental planform contributions because the wing has camber, dihedral and twist and is tested only in combination with a body or as part of a complete configuration. To allow for this Equation (3.1) is rewritten in a more general form that permits direct substitution of those data,

$$(\Delta L_r)_{corr} = 0.5[(L'_{v\ pred} - L'_{v0\ pred}) - (L_{v\ exp} - L_{v0\ exp})]. \quad (3.2)$$

The total wing contribution to L_r is

$$(L_r)_W = (L'_r)_W + (\Delta L_r)_{corr}. \quad (3.3)$$

The values of L_v at zero wing C_L , L_{v0} , are included to ensure that $(\Delta L_r)_{corr}$ is zero when the planform contribution is zero. The values of L_v may be for an isolated wing, a wing-body or a complete aircraft, provided the same configuration is taken throughout. Equation (3.2) reduces precisely to Equation (3.1) only if all components of L_v , apart from the wing planform contribution, are predicted exactly. The correction obtained is therefore likely to be better when deduced from data for the simpler configurations, although a qualitative improvement will always be obtained.

Section 5.4.3 demonstrates the application of Equations (3.2) and (3.3). Estimates of the various component parts of L_v for an aircraft can be evaluated by using Item Nos Aero A.06.01.03 and 09, Aero C.01.01.01, 73006, 80033, 80034 and 82010 as described in Item No. 81032.

Sketches 3.6a and 3.6b illustrate how a major improvement in the prediction of L_r is introduced through $(\Delta L_r)_{corr}$. Sketch 3.6a shows the predicted values L_r and L'_r together with the experimental data. Sketch 3.6b compares the experimental variation of L_v with the predicted values of L'_v for the wing-body configuration, and those data have been substituted in Equation (3.2) to evaluate $(\Delta L_r)_{corr}$. The close similarity in the experimental behaviour of L_r and L_v should be noted. It is this that leads to the excellent prediction of L_r . For other aircraft this similarity is sometimes less marked, with the result that the corrected values of L_r do not always follow the experimental data quite as well as in Sketch 3.6a.

3.4 High-lift Devices

3.4.1 Sideforce and Yawing Moment Derivatives

The deployment of leading-edge slats or trailing-edge flaps has little effect on the experimental values of Y_r and N_r . Their main effect is to delay to higher angles of attack any tendency to depart from the essentially constant values maintained at low α and otherwise they cause little change in magnitude. Theoretically, (see Section 5.3.2), a small change in N_r is predicted as a result of the deployment of trailing-edge flaps and this is in reasonable agreement with the experimental data. Trailing-edge flap deployment is assumed to have no effect on the predicted values of Y_r . Sketches 3.5a and 3.5b compare some cruise and landing-configuration data to show the effect of slat and flap deployment. Overall the agreement between prediction and experiment is comparable to that achieved for aircraft without high-lift devices although the qualitative dependence on α is only partially predicted.

3.4.2 Rolling Moment Derivative

The deployment of high lift-devices has a much greater effect on L_r than on Y_r and N_r . Such devices influence the attached-flow value of the wing planform contribution by changing the wing C_L , and for trailing-edge flaps there is also a small contribution that is independent of the wing C_L (see Section 5.4.2). Furthermore, because they change the flow separation characteristics of the wing a separate calculation of $(\Delta L_r)_{corr}$ has to be made by substituting into Equation (3.2) values of L_v appropriate to the high-lift configuration (see Section 5.4.3).

Comparisons with experimental data have shown that for high-lift configurations the general accuracy of prediction of L_r at low values of α is comparable to that achieved for clean-wing configurations. As α increases the predicted values of L_r do not follow the experimental variation so closely because the corrections made with $(\Delta L_r)_{corr}$ are too small, although they still introduce a substantial improvement at high values of α .

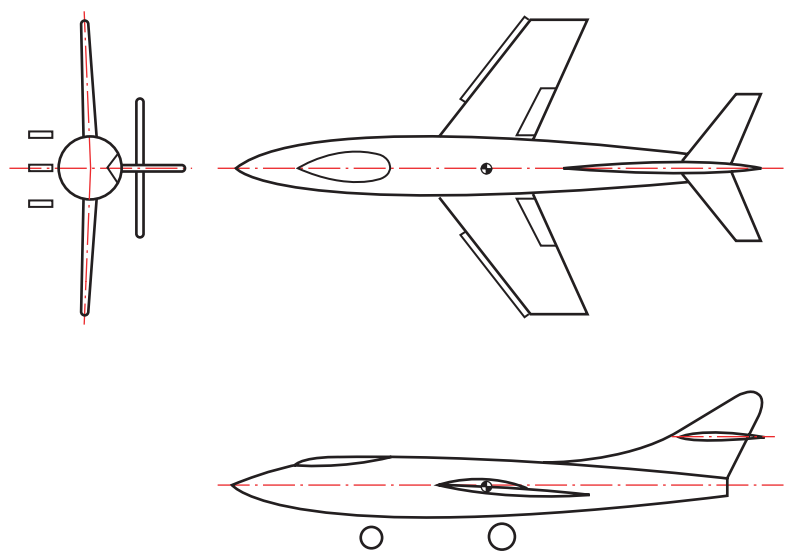
Sketches 3.6c and 3.6d show landing-configuration data that may be compared with the cruise-configuration data in Sketches 3.6a and 3.6b. The variation of the predicted values of L'_v and the experimental values of L_v are shown for the fin-off configuration in Sketch 3.6d. These have been substituted in Equation (3.2). It can be seen that, compared with the clean-wing data, the high-lift devices moderate the departure of the experimental data from L'_r and L'_v at high values of α , but that $(\Delta L_r)_{corr}$ underestimates the departure from L'_r in Sketch 3.6c.

3.5 Variation with Mach Number

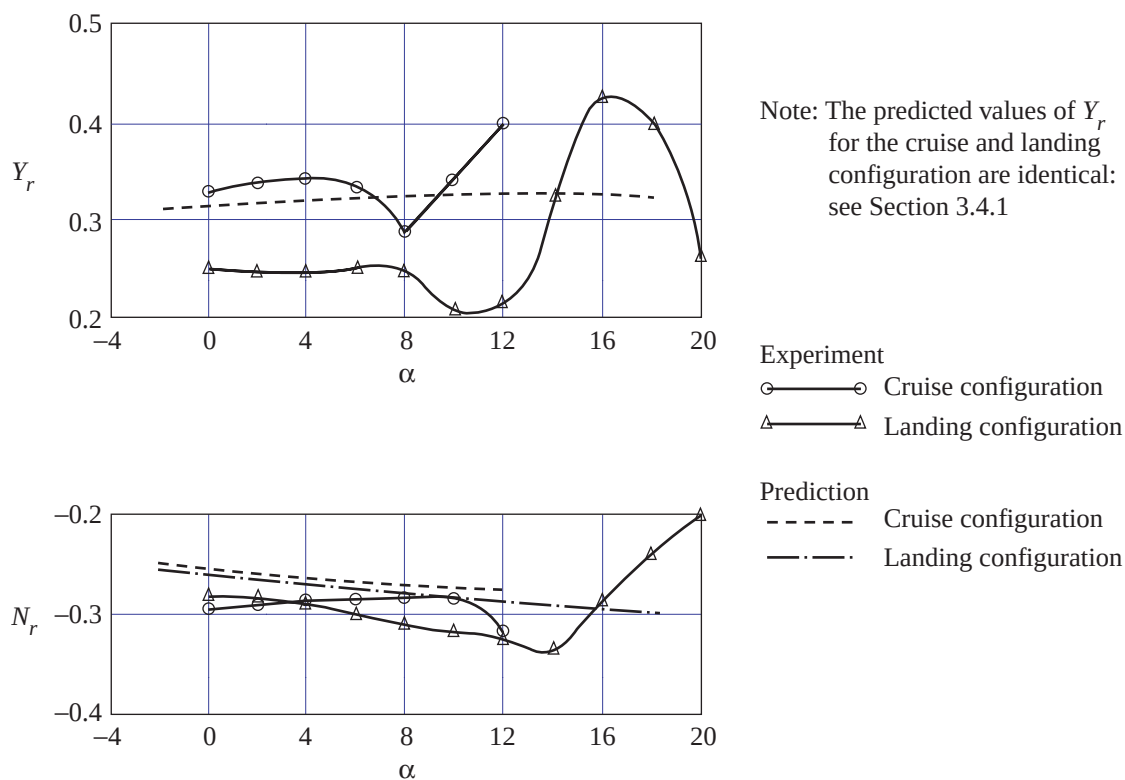
The effect of Mach number on the derivatives Y_r , N_r and L_r is moderate for Mach numbers up to about 0.8, the derivatives increasing slowly in magnitude. Comparisons with experimental data suggest that the theoretical methods model the effect of Mach number quite well, until the appearance of shock waves at high Mach numbers causes the experimental derivatives to depart rapidly from their low speed values. Sketch 3.7 gives an example of the experimental and predicted variations with Mach number for each of the derivatives.

3.6 Comparisons with Flight-test Data

Sketches 3.8a to 3.8d show four aircraft configurations for which predicted values of N_r and L_r have been compared with flight-test data. These indicate the range of aircraft geometries to which the Data Item methods can be applied. The magnitudes of the predicted components of Y_r , N_r and L_r are illustrated for cruise conditions. The general agreement with flight test data is quite good, as demonstrated in Sketch 3.9 for an aircraft with variable wing sweep from $\Lambda_0 = 26^\circ$ to 58° .



Sketch 3.4 Aircraft model tested in Derivations 3 and 7



Sketch 3.5 Illustration of predicted and experimental values of Y_r and N_r