

ELASTIC BUCKLING OF THIN-WALLED UNSTIFFENED CIRCULAR CYLINDERS UNDER BENDING, AXIAL COMPRESSION AND INTERNAL PRESSURE

1. NOTATION

E	modulus of elasticity of cylinder material	N/m ²	lbf/in ²
F	axial compressive force acting on cylinder	N	lbf
f	cylinder extreme fibre compressive stress defined by $f = \frac{M}{\pi R^2 t} + \frac{F}{2\pi R t} - \frac{pR}{2t}$	N/m ²	lbf/in ²
f_b	value of f at which buckling occurs	N/m ²	lbf/in ²
f_{bm}	value of f_b under bending alone ($F = p = 0$)	N/m ²	lbf/in ²
f_{bt}	theoretical elastic local buckling stress for geometrically perfect cylinder loaded in axial compression ignoring end effects $f_{bt} = \frac{E}{[3(1 - \nu^2)]^{1/2}} \left(\frac{t}{R} \right)$	N/m ²	lbf/in ²
f_{Br}	extreme fibre stress associated with M_{Br} , based upon undeformed cylinder section (see Section 4)	N/m ²	lbf/in ²
L	length of cylinder	m	in
M	bending moment applied to cylinder	N m	lbf in
M_{Br}	limiting value of M for long cylinder owing to ovalisation of section (see Brazier effect in Section 4)	N m	lbf in
p	internal pressure	N/m ²	lbf/in ²
R	radius of cylinder	m	in
t	thickness of cylinder	m	in
ν	Poisson's ratio for cylinder material		

Both SI and British units are quoted but any coherent system of units may be employed.

2. GENERAL NOTES

This Item provides data for estimating the extreme fibre stress at which elastic buckling occurs in thin-walled unstiffened circular cylinders subject to bending or to bending combined with axial compression. Data are also provided on the influence of internal pressure on buckling under the action of a bending moment alone. The data relate to cylinders of nominally uniform wall thickness manufactured

Issued May 1997

from isotropic materials. In addition data are provided on the collapse of cylinders subject to bending as a result of the Brazier effect.

3. UNPRESSURISED CYLINDERS

3.1 Bending Loading Alone

Figure 1 presents the results of 54 tests on cylinders subjected to bending alone plotted on a grid of f_{bm}/f_{bt} against RL/t^2 . The choice of f_{bm}/f_{bt} as a correlating parameter is not ideal since f_{bt} does not account for the fact that only part of the cylinder is subjected to f nor does it account for the influence of cylinder ends in shorter cylinders. Consequently the buckling mode developed does not conform to that assumed for f_{bt} (see Item No. 83034*). Nevertheless, this parameter accounts satisfactorily for many of the governing factors and is in accord with that used in other Data Items on cylinders.

The solid line on Figure 1 is a least squares fit to the test results. The equation for the line is

$$\frac{f_{bm}}{f_{bt}} = 5.5 \left(\frac{RL}{t^2} \right)^{-0.18} \quad (3.1)$$

The test results are log-normally distributed about the solid line with a standard deviation in $\log_{10}(f_{bm}/f_{bt})$ of 0.10. The dashed lines shown on Figure 1 are derived from the log-normal distribution fitted to the sample and show the ± 95 per cent probability lines that are a factor of 1.46 different from the solid line. Thus, for the dashed lines the coefficient of $(RL/t^2)^{-0.18}$ in Equation (3.1) is 3.76 for the lower line and 8.03 for the upper line.

The low values of buckling stress found in cylinders subjected to axial compression alone are attributed to the effect of initial inward irregularities (see Item No. 83034). The probability of such irregularities occurring in the region of the maximum compressive stress, f , is likely to increase with cylinder surface area and decrease with thickness. The reduction in f_{bm}/f_{bt} with increasing RL/t^2 may be attributed to this effect. Furthermore, at any value of RL/t^2 , the scatter may be due in part to the bending moment being applied to the test cylinders in a predetermined plane which may, or may not, associate the maximum compressive stress, f , with the largest inward initial irregularity. For many cylinders the service bending loading can be applied in any plane and in some cases, such as rotating cylinders, the entire area can be subjected to the maximum compressive stress at some time. Thus, for such cylinders the lower curve on Figure 1 is more likely to apply.

At the lower values of RL/t^2 some of the tests gave values of f_{bm}/f_{bt} greater than one. As Figure 1 shows, all the tests for which $f_{bm}/f_{bt} \geq 0.9$ are for short cylinders for which $11 \leq L^2/(Rt) \leq 29$. Three results are also recorded for $44 \leq L^2/(Rt) \leq 47$ and these occur at $f_{bm}/f_{bt} \geq 0.62$. For all other results on Figure 1 $L^2/(Rt)$ exceeds 73. Short cylinders will have resistance to buckling from both the end restraints and their curvature. It follows that for very short cylinders the buckling stress must be greater than that for a wide plate with free edges so that, for simply-supported ends,

$$f_{bm} > \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{L} \right)^2 \quad [\text{which is } \geq f_{bt} \text{ when } L^2/(Rt) \leq 1.5] \quad (3.2)$$

* Data Item No. 83034 "Elastic local buckling stresses of thin-walled unstiffened circular cylinders under combined axial compression and internal pressure".

and, for clamped ends,

$$f_{bm} > \frac{\pi^2 E}{3(1-\nu^2)} \left(\frac{t}{L}\right)^2 \quad [\text{which is } \geq f_{bt} \text{ when } L^2/(Rt) \leq 6.0]. \quad (3.3)$$

Note that Equations (3.2) and (3.3) are for small deflection thin plate theory and begin to become invalid if either L or R is less than $20t$.

Figure 1 may be applied to ring stiffened cylinders where the rings provide the end conditions of at least simple support. In such cases L may be taken to be the distance between rings.

3.2 Combined Bending and Axial Loading

Figure 2 presents the results of 207 tests on cylinders subject to combined bending and axial loading. In addition the lines shown on Figure 1 are superimposed together with the mean line for axial loading alone obtained from the data used in Item No. 83034. It would be expected that results for combined bending and compression would tend to fall between the results for bending alone and axial loading alone. This is not confirmed by the very wide scatter shown on Figure 2 in which some results are comparable with the lower line of Figure 1 and others are above the upper line. There is no consistent variation in f_b/f_{bt} with the proportion of f_b arising from bending loading. All the cylinders are relatively long, having $1360 \leq L^2/(Rt) \leq 5307$. The data tend to confirm the lower line of Figure 1 and indicate that, for large values of RL/t , values of f_b/f_{bt} similar to those obtained in cylinders under compression alone may apply.

4. NON-LINEAR EFFECTS - UNPRESSURISED CYLINDERS UNDER BENDING

Under the action of a bending moment the cross-sectional form of a long slender thin walled circular cylinder will become oval thereby reducing its second moment of area. Increasing the bending moment causes the second moment of area to decrease progressively until the section is incapable of supporting the applied moment and collapse occurs. This effect is non-linear with increasing applied bending moment and is often termed "Brazier's effect". The maximum bending moment at collapse (when $F = p = 0$) is given by

$$M_{Br} = \frac{2\sqrt{2}}{9} \frac{E\pi R t^2}{(1-\nu^2)^{1/2}}. \quad (4.1)$$

The maximum bending moment may be used to calculate an equivalent extreme fibre stress based upon the original undeformed circular cross section of the cylinder. Thus,

$$M_{Br} = \frac{2\sqrt{2}}{9} \frac{E}{(1-\nu^2)^{1/2}} \left(\frac{t}{R}\right). \quad (4.2)$$

Ovalisation of the section reduces the curvature (increases R) in the region of maximum compressive stress and this can reduce the local buckling stress to a value below that given by Equation (4.2). Approximate analysis indicates that, when $\nu = 0.3$, f_{Br} may fall to about 90 per cent of the value given in Equation (4.2). This value, $0.9f_{Br}/f_{bt}$, is indicated on the right-hand side of Figure 1. A few of the test results shown on Figure 1 fall below this value. This may be a result of initial irregularities which may serve to reduce f_{bm} below f_{Br} even in a very long cylinder.