

# BUCKLING OF THIN-WALLED UNSTIFFENED CIRCULAR CYLINDERS UNDER COMBINED TORSION AND PRESSURE

## 1. NOTATION

|                 |                                                                                           |                  |                     |
|-----------------|-------------------------------------------------------------------------------------------|------------------|---------------------|
| $E$             | modulus of elasticity of cylinder material                                                | N/m <sup>2</sup> | lbf/in <sup>2</sup> |
| $E_T$           | tangent modulus of cylinder material                                                      | N/m <sup>2</sup> | lbf/in <sup>2</sup> |
| $f_n$           | stress at which $E_T = \frac{1}{2}E$                                                      | N/m <sup>2</sup> | lbf/in <sup>2</sup> |
| $K$             | elastic buckling coefficient defined by Equation (2.1)                                    |                  |                     |
| $L$             | length of cylinder                                                                        | m                | in                  |
| $m$             | material characteristic for cylinder material (see Item No. 76016*)                       |                  |                     |
| $n$             | number of complete waves into which cylinder buckles around circumference                 |                  |                     |
| $p$             | pressure loading, external pressure positive                                              | N/m <sup>2</sup> | lbf/in <sup>2</sup> |
| $p_b$           | elastic critical external (positive) pressure (see Section 3 and Data Item No. 04.09.01†) | N/m <sup>2</sup> | lbf/in <sup>2</sup> |
| $q_b$           | shear buckling stress under torsion alone ( $p = 0$ )                                     | N/m <sup>2</sup> | lbf/in <sup>2</sup> |
| $q_{be}$        | elastic shear buckling stress under torsion alone ( $p = 0$ )                             | N/m <sup>2</sup> | lbf/in <sup>2</sup> |
| $q_{bep}$       | elastic shear buckling stress under torsion and pressure $p$                              | N/m <sup>2</sup> | lbf/in <sup>2</sup> |
| $R$             | radius of cylinder                                                                        | m                | in                  |
| $t$             | thickness of cylinder wall                                                                | m                | in                  |
| $\nu$           | elastic value of Poisson's ratio for cylinder material                                    |                  |                     |
| $\alpha, \beta$ | indices, see Equation (2.1)                                                               |                  |                     |
| $\gamma$        | index in Equation (3.1), see Section 3                                                    |                  |                     |
| $\eta$          | material plasticity reduction factor defined by $q_b = \eta q_{be}$                       |                  |                     |

Both SI and British Units are quoted but any coherent system of units may be employed.

\* Item No. 76016 "Generalisation of smooth continuous stress-strain curves for metallic materials".

† Item No. Struct 04.09.01. "Collapse coefficients for unstiffened circular cylinders under uniform external pressure".

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## 2. BUCKLING UNDER TORSION ALONE

This Item provides the buckling stress of nominally uniform thickness thin-walled circular cylinders manufactured from isotropic material, for which  $\nu = 0.3$ , subject to torsion alone. The theoretical value of the buckling stress for a perfectly round, straight tube of uniform section under pure torsion may always be expressed in the form

$$(q_{be}/E) = K(t/R)^\alpha (R/L)^\beta, \quad (2.1)$$

where the values of the factor  $K$  and of the indices  $\alpha$  and  $\beta$  are functions of  $n$ , the number of complete waves round the circumference of the tube in its buckled form. When  $L/R$  is large,  $n$  is restricted to its least possible value, namely 2, and for that case  $K = 0.292$ ,  $\alpha = 3/2$  and  $\beta = 0$ , so that

$$(q_{be}/E) = 0.292(t/R)^{3/2}. \quad (2.2)$$

When  $L/R$  is moderate,  $n$  may be treated as a continuous variable ranging from 2 upwards and for that case the well attested value of  $\alpha$  for the perfect tube is  $5/4$ , with  $K$  in the range 0.70 to 0.75 and  $n$  between 2 and 6. This case is therefore given approximately by

$$(q_{be}/E) = 0.73(t/R)^{5/4} (R/L)^{1/2}. \quad (2.3)$$

When  $L/R$  is small,  $n$  becomes very large so that  $\alpha = \beta = 2$  and for this case

$$(q_{be}/E) = K(t/L)^2 \quad (2.4)$$

with  $K = 4.83$  for ends simply-supported or 8.14 for ends clamped\*.

In practice the majority of tubes have  $L/R$  in the moderate range, and of the 189 experimental results available, see Table 2.1, nearly all relate to this range. Experimental values of  $q_{be}/E$  necessarily include the effect of the initial departures of the tubes from the perfectly cylindrical form and have the added complication of the difficulty of determining the precise point at which buckling occurs. The best fit to the experimental results is given when  $\alpha = 6/5$ ,  $\beta = 2/5$  and  $K = 0.480$ , that is by

$$(q_{be}/E) = 0.480(t/R)^{6/5} (R/L)^{2/5}. \quad (2.5)$$

Figure 1 is based on Equations (2.5) and (2.4) with the upper parts of the lines calculated from Equation (2.5) at high values of  $L/R$  combined with low values of  $R/t$  cut off by the line calculated from Equation (2.2).

A statistical analysis of the 144 results for which  $L^2/(Rt) \geq 100$ , and which relate to Equation (2.5), shows the data to be consistent with the assumption of a normal distribution with a standard deviation of  $0.117q_{be}$ . It follows that for probabilities of survival of 90 or 99 per cent  $q_{be}$  should be factored by 0.85 or 0.76†, respectively.

\* These values are obtained from Data Item No. 71005 "Buckling of flat plates in shear".

† These factors both relate to a confidence level of 95 per cent and are derived for a sample of 144.

The data may be applied to ring stiffened cylinders by taking  $L$  to be the ring spacing. Note that the rings must be sufficiently stiff to maintain the cylinder ends circular for Equation (2.5) to apply and must also provide the appropriate conditions of support for Equation (2.4) to apply.

**TABLE 2.1 Range of values and geometry ratios covered by tests**

|             |       |                   |                   |                     |                    |         |
|-------------|-------|-------------------|-------------------|---------------------|--------------------|---------|
| $R$ minimum | 12.7  | mm                | 0.5               | in                  | $R/t$ minimum      | 12.33   |
| maximum     | 381   | mm                | 15                | in                  | maximum            | 1396    |
| $t$ minimum | 0.152 | mm                | 0.006             | in                  | $L/R$ minimum      | 0.2     |
| maximum     | 0.813 | mm                | 0.032             | in                  | maximum            | 160     |
| $L$ minimum | 22.9  | mm                | 0.9               | in                  | $L^2/(Rt)$ minimum | 20.2    |
| maximum     | 2835  | mm                | 111.6             | in                  | maximum            | 710 000 |
| $E$ minimum | 2.89  | GN/m <sup>2</sup> | $4.2 \times 10^5$ | lbf/in <sup>2</sup> |                    |         |
| maximum     | 200   | GN/m <sup>2</sup> | $29 \times 10^6$  | lbf/in <sup>2</sup> |                    |         |

### 3. TORSION PLUS PRESSURE LOADING

Figure 2 provides an interaction curve relating  $q_{bep}/q_{be}$  and  $p/p_b$  through a range of values of  $p$  from buckling under external pressure ( $p = p_b$ ) to high internal pressure ( $p$  negative). The figure relates strictly to elastic stresses in cylinders where pressure is applied to the sides of the cylinder only and not to the ends. Values of  $p_b$  may be obtained from Data Item No. Struct 04.09.01, for the appropriate end conditions, and may be taken to relate to pressure loading on the cylinder sides only (see Section 2 of that Item).

The solid line on Figure 2 represents the interaction equation

$$(q_{bep}q_{be})^\gamma + (p/p_b) = 1, \quad (3.1)$$

where  $\gamma$  is taken to be 2. In the external pressure region of Figure 2 limited experimental results (30 tests only) show wide scatter when  $q_{be}$  and  $p_b$  are based upon this Item and Data Item No. Struct 04.09.01, respectively. Ten per cent of results are between the lines given by  $\gamma = 0.7$  and  $\gamma = 1$  in Equation (3.1) (shown dashed on Figure 2) and 70 per cent are above a  $\gamma = 2$  line. In the internal pressure region the curve is close to the experimental results (36 tests) over the range  $-1.5 \leq p/p_b \leq 0$ . Beyond  $p/p_b = -1.5$  the experimental results are just above the  $\gamma = 2$  line of Figure 2.

### 4. MATERIAL EFFECTS

#### 4.1 Torsion Alone

Figure 1 can be applied to materials having values of Poisson's ratio other than 0.3 by factoring  $q_{be}/E$  by  $0.945/(1-\nu^2)^{3/5}$ . If the value of  $q_{be}$  obtained from Figure 1 is such that the material has begun to yield the effective value of  $E$  will be less than that originally assumed in the calculation of  $q_{be}$ . Figure 3 is provided to account for this effect directly by giving the material plasticity correction factor,  $\eta$ , for use in the relationship

$$q_b = \eta q_{be}. \quad (4.1)$$

Figure 3 provides curves of  $\eta$  plotted against  $q_{be}/f_n$  for a range of values of  $m$  and is based upon the secant

modulus, as given by Equation (3.4) of Item No. 76016. Item No. 76016 shows how values of  $f_n$  and  $m$  may be determined from a stress-strain curve for the cylinder material or estimated from two or more proof stress values for the material.

#### 4.2 Torsion with Internal Pressure

Figure 2 shows that the effect of internal pressure is to raise the buckling stress. A proven method of accounting for the effects of material plasticity in such cases is not available. However, practical internal pressures usually produce hoop tension stresses that are negligibly small in relation to the shear buckling stress and in such cases an estimate of the effect of material plasticity on  $q_{bep}$  may be made by multiplying it by  $\eta$  obtained from Figure 3 with  $q_{bep}/f_n$  in place of  $q_{be}/f_n$  in the abscissa.

### 5. DERIVATION

This Section lists selected sources that have assisted in the preparation of this Data Item.

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