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Cylinder groups: mean forces on pairs of long circular cylinders

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THE PREPARATION OF THIS DATA ITEM

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CYLINDER GROUPS: MEAN FORCES ON PAIRS OF LONG CIRCULAR CYLINDERS

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APPENDIX A CALCULATION OF EFFECTIVE REYNOLDS NUMBER **29**

A1. CALCULATION PROCEDURE **29**

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CYLINDER GROUPS: mean forces on pairs of long circular cylinders

1. NOTATION AND UNITS

		Units	
		SI	*US customary*
C_D	mean drag coefficient: (drag force per unit length)/($\frac{1}{2}\rho V^2 D$)		
C_{Dd}, C_{Du}	mean drag coefficients for downstream and upstream cylinders respectively		
C_{D0}	reference mean drag coefficient for an isolated cylinder		
C_L	mean lift or side-force coefficient: (lift or side force per unit length)/($\frac{1}{2}\rho V^2 D$)		
C_{Ld}, C_{Lu}	mean lift coefficients for downstream and upstream cylinders respectively		
D	cylinder diameter	m	ft
H	cylinder length	m	ft
I_u	intensity of u-component of turbulence		
r_{L_u}	integral scale, normal to cylinder axis, of u-component of turbulence (see Appendix A)	m	ft
Re	Reynolds number: VD/υ		
Re_e	effective Reynolds number; $\lambda_R\ \lambda_T\ Re$ (see Section 4.3 and Appendix A)		
s	cylinder pitch: spacing between centres (see Sketch 1.1)	m	ft
s_l	cylinder spacing along flow: $s\ \cos\alpha$ (see Sketch 1.1)	m	ft
s_t	cylinder spacing across flow: $s\ \sin\alpha$ (see Sketch 1.1)	m	ft
V	mean flow velocity	m/s	ft/s
α	flow incidence: angle between flow direction and line joining cylinder centres (see Sketch 1.1)	degree	degree
ε	effective roughness height of cylinder surface (see Appendix A)	m	ft

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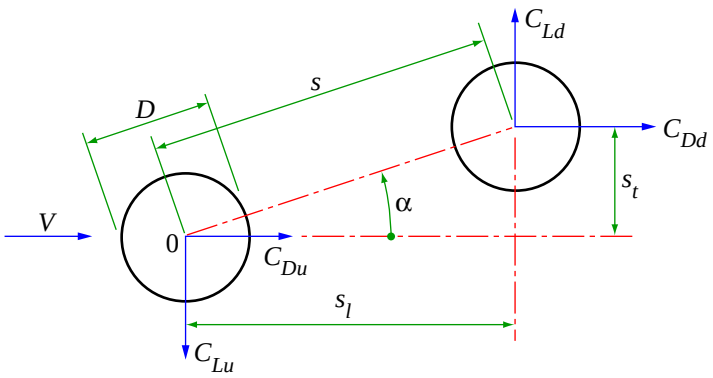
υ	kinematic viscosity	m^2/s	ft^2/s
λ_R ,λ_T , λ_{Tcrit}	parameters defining effects of surface roughness and turbulence on Re_e (see Section 4.3 and Appendix A)		
ρ	fluid density	kg/m^3	lb/ft^3



Note: C_D and C_L are positive as shown

Sketch 1.1

2. INTRODUCTION

When two (or more) cylinders are close together in a fluid flow, both the mean and the fluctuating* forces on each cylinder differ from those on an isolated cylinder†, particularly in the possible occurrence of mean lift or across-flow forces. This Data Item, which supersedes ESDU 83047, presents information on the mean drag and lift forces on pairs of long circular cylinders.

2.1 Scope

The data in this Item cover pairs of equal-diameter‡, circular cylinders normal to a flow at Reynolds numbers from approximately 10^4 upward for the upstream cylinder. Data are provided for C_D and C_L on both cylinders for spacings in the range 1 to 50 diameters and for flow incidences in the range 0° to 90°.

The data presented and the location of the relevant Figures are summarised in Table 4.1 on page 9 and the flow mechanisms which give rise to the forces are discussed in Section 3. Section 4 discusses the effects of turbulence and surface roughness and the background to the data analysis. Section 4 also includes some notes on the derivation and application of the Figures.

The data in this Item are presented in terms of an effective Reynolds number, Re_e, (see Section 4.3) and equivalent-roughness ratio, ε/D. Information for calculating Re_e and typical values of ε are given in Appendix A, taken from Reference 3. In low turbulence flows over smooth cylinders, and for $Re \geq 2 \times 10^6$, $Re_e \approx Re$.

* Information on the fluctuating forces on, and response of, small groups of cylinders is discussed in Reference 1.
† Information on the mean forces on isolated circular cylinders is given in References 3 and 4.
‡ Some information on the mean forces on, and flow around, pairs of unequal-diameter cylinders in tandem may be found in References 2 and 5 in Derivation 12.

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2.2 Uncertainty

The quoted values of uncertainty are based on a comparison between experimental data and predictions from this Item.

The uncertainty generally increases for $Re_e \le 2 \times 10^4$ because of experimental uncertainty associated with measurement of small forces and for $3 \times 10^5 \le Re_e \le 2 \times 10^6$ (the critical range) because of rapid changes in flow conditions with Re_e.

There are relatively few data for forces on pairs of cylinders in supercritical flow, $Re_e \ge 4.5 \times 10^5$, especially at Reynolds numbers typical of full-scale applications. Considerable recourse must therefore be made to appropriate extrapolation from subcritical flow data and the uncertainty is therefore large.

2.3 Applicability

The data in this Item apply directly to pairs of long circular cylinders normal to the flow, typical of structural elements such as pipes, cables, masts and cylindrical members of frames.

In the absence of better information it is suggested that the data can also be applied to structures such as stacks and towers in the natural wind by application of the appropriate factors for finite length and shear flow effects from Item No. 81017[4], provided that H/D is greater than about 4.

The data do not apply for yawed cylinders (where the cylinder axis is inclined to the flow). Application of 'cross-flow' theory (see Reference 3) may give misleading results except at small angles of yaw and is not recommended.

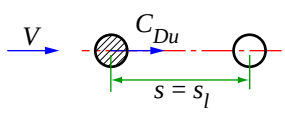
3. DESCRIPTION OF THE FLOW MECHANISMS

The flow mechanisms giving rise to the variation of both the mean and the fluctuating forces with spacing and incidence are complex. They are associated with interaction between the downstream cylinder and the upstream wake except for incidences near 90° where interaction between the two wakes predominates.

The descriptions which follow are generally applicable to subcritical flow, $Re_e \le 4.5 \times 10^5$, except that for $Re_e \le 10^4$ the effect of the upstream wake turbulence on the effective Reynolds number of the flow around the downstream cylinder is insignificant. In supercritical flow the flow mechanisms are broadly similar but the spacings and incidences at which changes in flow pattern occur are somewhat reduced because of the smaller extent of the upstream wake. For $Re_e \ge 10^6$ the upstream wake turbulence again exerts little influence on the flow around the downstream cylinder.

3.1 Cylinders in Tandem Arrangements (α = 0°)

3.1.1 Upstream cylinder



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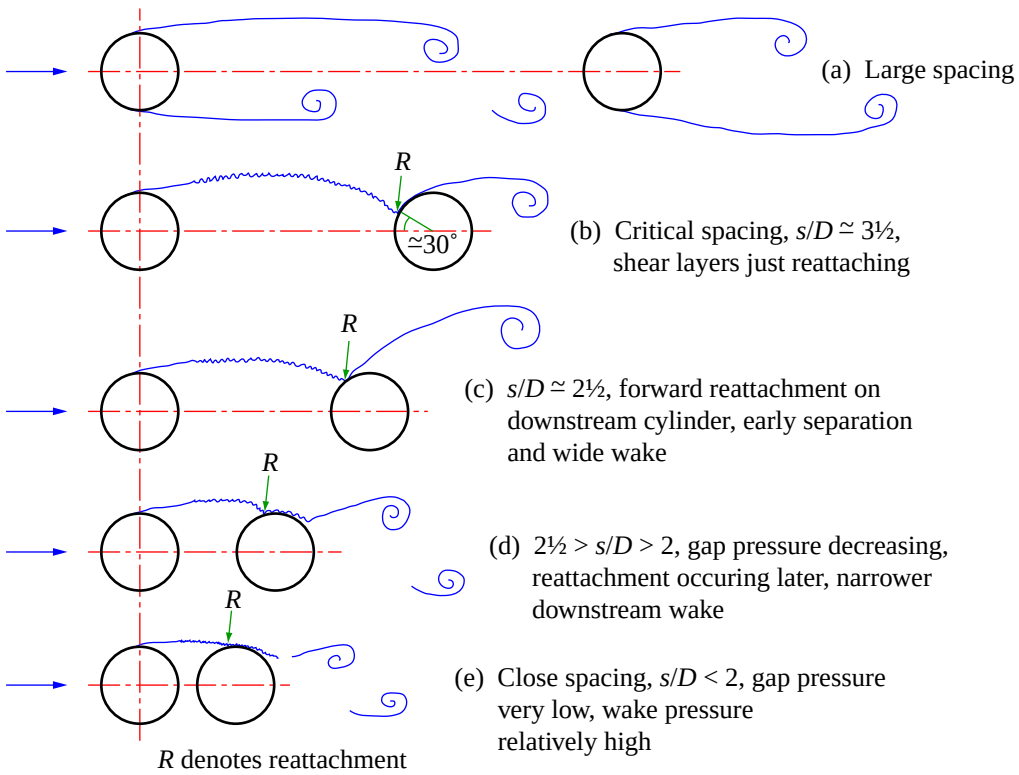
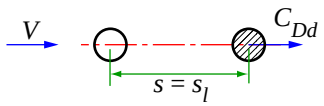
For $s_l/D \geq 7$ the upstream cylinder is not significantly affected by the presence of the downstream cylinder and $C_{Du} = C_{D0}$. For smaller spacings however, the natural wake development from the upstream cylinder is disturbed and C_{Du} is reduced. At a spacing of about s_l/D = 3½ the separated shear layers from the upstream cylinder just impinge on the downstream cylinder (Sketch 3.1). This inhibits vortex shedding from the upstream cylinder and its drag is suddenly reduced even further. As the downstream cylinder enters the separation bubble region of the upstream wake (Sketch 3.1c), C_{Du} continues to decrease to a minimum at around s_l/D = 3. For smaller spacings (Sketch 3.1d and 3.1e) the gap pressure reduces again and C_{Du} increases although it is never greater than C_{D0}. The data do not indicate the occurrence of two different flow patterns as occurs for the downstream cylinder but this may simply be lost in the overall scatter.

Because of symmetry of the system, the mean lift coefficient is zero, but for spacings greater than about 3½D there will be fluctuating lift forces arising from vortex shedding.



Sketch 3.1 Flow around tandem pairs

3.1.2 Downstream cylinder



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The spacing at which significant interference effects disappear for the downstream cylinder is very large, in excess of 50 to 60 diameters. For spacings down to about 4*D*, two effects predominate. Firstly, the mean velocities and static pressures in the upstream cylinder wake are lower than in the free stream, which reduces the drag. This effect is predominantly a function of spacing. Secondly, the turbulence generated in the wake of the upstream cylinder can result in transition to turbulent flow in the surface boundary layer of the downstream cylinder at much lower Reynolds numbers than for an isolated cylinder, *i.e.* its effective Reynolds number is increased. This will result in delayed flow separation from the cylinder surface, a narrower wake, a higher base pressure and hence a reduced drag. The magnitude of this effect depends on both cylinder spacing and Reynolds number.

For $Re \le 10^4$ and in supercritical flow at $Re \ge 10^6$, turbulence has little effect on the effective Reynolds number of the flow and the drag reduction is largely due to the first effect, the "shielding" by the upstream cylinder.

For values of Re_e greater than about 10^5, continuance of the large reduction in drag to very large spacings (40*D* to 50*D*) is probably due largely to the increased turbulence in the upstream wake promoting an effectively "supercritical flow" around the downstream cylinder. It is possible that C_{Dd} will ultimately return rapidly to C_{D0}, after some larger value of spacing $s_l/D \ge 50$, once the upstream wake turbulence has decayed sufficiently, in the same way as the drag of an isolated cylinder increases as the effective Reynolds number decreases through the critical value (see Item No. 80025[3]).

As the spacing is reduced the wake effects become more pronounced and the drag coefficient of the downstream cylinder falls. At a spacing of around $s/D \approx 3½$, the separated shear layers from the upstream cylinder just reattach to the downstream cylinder and it enters the low pressure "separation bubble" in the wake of the upstream cylinder (as described in Section 3.1.1 and Sketch 3.1b). The drag on the downstream cylinder drops sharply as the pressure on its front face falls. As the downstream cylinder moves closer, the pressure on its front face decreases and the area affected increases (Sketch 3.1c) leading to a further decrease in drag on the downstream cylinder. Further decrease in spacing causes the pressure on its front face to decrease further but the reattachment and final separation of the shear layers emanating from the upstream cylinder occur further round the downstream cylinder, increasing its base pressure. The balance between the negative pressures on the front and rear of the downstream cylinder leads ultimately to negative drag coefficients as shown in Figure 2a. The variation is a function of Reynolds number, the peaks in drag (Figure 2a) at small spacings disappearing at $Re_e \le 3 \times 10^4$. Sketch 3.1 shows a suggested sequence of flow patterns.

The foregoing description applies to the interference effects leading to the lower curves on Figure 2a. However, the data suggest that an alternative flow pattern may exist for spacings between about 2½*D* and 3½*D* as shown by the upper curves. This may be associated with reattachment further forward in the downstream cylinder. The flow patterns previously described have been associated with reattachment moving rearward from about 30° around the cylinder from the leading edge (Sketch 3.1b), exposing a significant area of the front face to the low pressure region. Derivation 8, however, reports reattachment much further forward which would correspond to higher C_{Dd}. Alternatively these higher C_{Dd} values may be associated with reattachment on one side only, a pattern noted in Derivation 25 and implied by asymmetries in the flow noted in Derivation 22.

Many sources [13, 22, 25, 27, 29] give both drag coefficients or pressure distributions and flow patterns around a pair of cylinders at small spacing, but not from simultaneous measurements. Because of the sensitivity of these flows to small disturbances, direct correlation between the measurements cannot be assumed.

For supercritical flow, $Re_e \ge 10^6$, data from Derivation 23 suggest that the critical spacing remains at about 3½*D*. However, because the upstream cylinder drag is smaller in supercritical flow, its wake pressure is higher and, although the interference effects are similar to those for subcritical flow, the reduction in drag is rather smaller as shown by Figure 2b.

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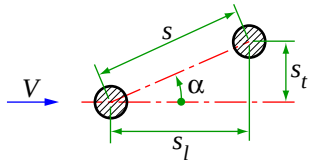
As in the case of the upstream cylinder, the mean lift coefficient, C_{Ld}, is zero (except possibly in the asymmetric flow noted above) but fluctuating lift forces may occur.

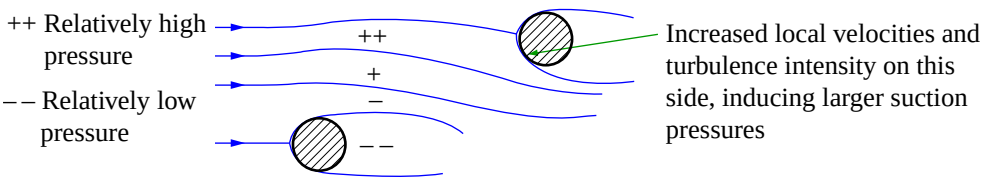
3.2 Cylinders in Staggered Arrangements, $0° \le \alpha \le 90°$



For staggered arrangements, the downstream cylinder experiences an asymmetrical flow field which becomes more pronounced as the gap between the two cylinders decreases. The drag on the downstream cylinder increases compared with the tandem case and the asymmetry induces a net side force on the downstream cylinder that is directed towards the centre line of the upstream wake. The origins of this force are explained in Sections 3.2.1 and 3.2.2. For lateral spacings, s_t/D, greater than about 4 or 5 (increasing with increasing s_l/D) interference is insignificant and both cylinders behave as isolated cylinders.

3.2.1 Large spacing, $s_l/D \ge 3$

For lateral spacings greater than about $s_t/D = 1.1$, and longitudinal spacings in the range $3 \le s_l/D \le 20$, the lateral pressure gradient that exists in the region across the gap between the cylinders causes the flow approaching the downstream cylinder to be deflected towards the upstream wake as fluid is entrained in the relatively low pressure wake region (Sketch 3.2). This flow deflection produces a resultant side force on the downstream cylinder directed towards the upstream wake axis. In addition, the higher intensities of turbulence and higher local flow velocities (see Sketch 3.2) act to move the separation point rearward on the side nearer the upstream wake axis, thus allowing lower pressures to be sustained on that side compared with the outer side and giving a net force towards the upstream wake axis.



Sketch 3.2 Flow field for larger gaps between cylinders

These effects produce the significant side force acting toward the upstream wake centre line and increasing with decreasing s_t/D as shown by Figure 6. The increased velocities initially cause a small increase in C_{Dd} above the isolated cylinder value but, as s_t/D decreases, the increasing base pressure caused by the rearward movement of the inner separation point becomes dominant and C_{Dd} decreases.

For s_l/D greater than about 20, and for lateral spacings less than about $s_t/D = 1.1$ for $s_l/D \le 20$, the deflection of the upstream wake does not occur and the downstream cylinder is enveloped by the upstream wake. The gradients of pressure, velocity and turbulence across the upstream wake again lead to differences in separation location on the inner and outer sides of the downstream cylinder such that a lift force acting towards the upstream wake axis is developed. The lift force decreases as the differences between conditions on the two sides of the cylinder decrease with decreasing lateral spacing. The drag is reduced for reasons similar to those for the tandem configuration.

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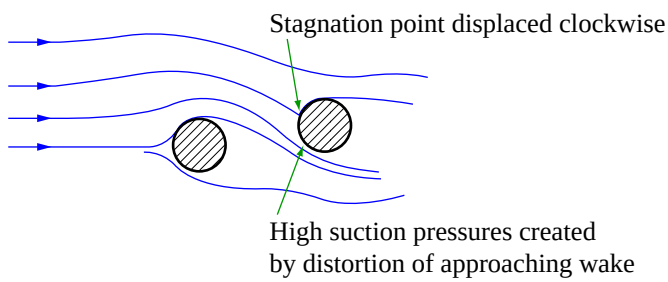
The preceding discussion of the flow mechanisms applies for effective Reynolds number, Re_e, less than about 3×10^5. When Re_e is greater than this value (due to increased flow velocity, increased free-stream turbulence or greater surface roughness) then as a result of the onset of the transcritical flow regime the upstream wake is narrower and the base pressure in the upstream wake increases, reducing the pressure gradient across the wake and thereby reducing C_{Ld} compared with the subcritical flow values. For lateral spacings at which the wake of the upstream cylinder is deflected a similar increase in Re_e is likely to reduce C_{Dd} through the promotion of transcritical flow, while at small lateral spacings the shielding effect on the downstream cylinder is reduced and C_{Dd} increases. Thus it can be anticipated that there will be less variation in C_{Dd} with lateral spacing across the approaching wake at higher effective Reynolds numbers.

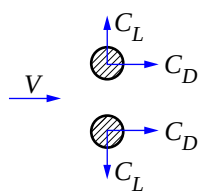
3.2.2 Small spacings; $s_l/D \leq 3$

For small cylinder spacings, at lateral spacings greater than about $s_t/D = 0.25$, the main feature of the flow is the distortion and 'squeezing' of the upstream wake by the downstream cylinder[22] as illustrated in Sketch 3.3. This effect accelerates the flow in the gap between the cylinders and induces high suction pressures on the downstream cylinder surface nearest the upstream wake axis. This in turn results in the large values of lift acting towards the upstream wake that are featured in Figure 4b. As the lateral spacing decreases, there comes a point when the flow in the gap ceases abruptly and the upstream wake envelops the downstream cylinder. The magnitude of C_{Ld} falls dramatically, the flow pattern then becomes similar to that described in Section 3.1.2 for the tandem arrangement and negative values of C_{Dd} can occur.



Sketch 3.3 Flow field for $\frac{s_l}{D} < 3$ and $\frac{s_t}{D} < 0.5$

3.3 Cylinders in Side-by-side Arrangements ($\alpha = 90°$)



For spacings greater than about $s_t/D = 5$ the two cylinders behave independently, $C_L = 0$ and the drag coefficients, C_D, are equal. At spacings between about 5 and 3 diameters, although two wakes are formed they interfere along their inner edges (Sketch 3.4a). The flow remains nearly symmetrical, C_L is small and the drag coefficients on the two cylinders are nearly equal.

For smaller spacings, $3 \geq s_t/D \geq 1.1$ one narrow wake and one wide wake is formed with a corresponding low drag on one cylinder and high drag on the other. At the same time the lack of symmetry and the bias in the flow direction through the gap causes large lift forces tending to separate the cylinders (Sketch 3.4b).

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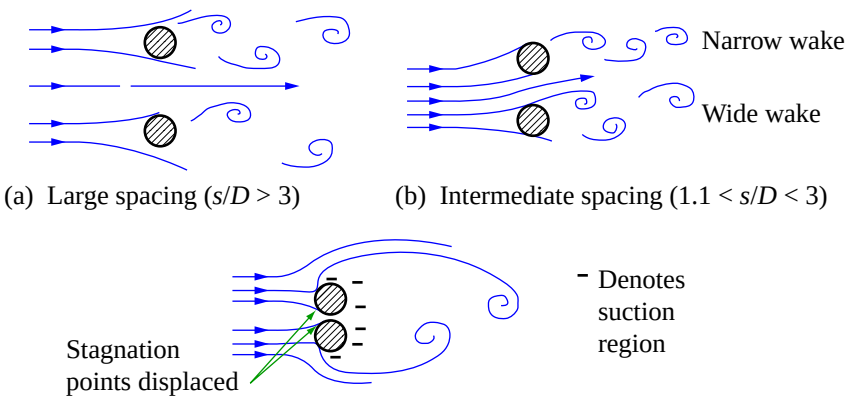
The forces usually take the maximum values (on Figures 1d, 1h and 3b) on one cylinder and the minimum values on the other, rarely taking intermediate values. The wide and narrow wakes may switch unpredictably between the two cylinders leading to instability problems if the forces are large enough. Further discussion of the complex flows at these spacings is given in Derivation 14.

For still smaller spacings, $s_t/D \leq 1.1$, the cylinder pair acts as a single body, generating a single wake with alternate vortex shedding from the outer cylinder sides (Sketch 3.4c). As the gap between the cylinders decreases, reducing flow into the base region and increasing the base pressure (Sketch 3.4c), the drag coefficients, C_D, increase rapidly, and become approximately equal. At the same time the stagnation point on each cylinder moves round towards the gap and shear layer separation on the outside of the cylinders occurs further forward leading to large suction regions over the outward-facing surfaces. This generates lift forces which rise to a maximum at zero gap ($s/D = 1$). The lift forces in this case always act to separate the cylinders, in contrast to the attractive forces generated between closely spaced long bodies such as ships and road vehicles.



(a) Large spacing ($s/D > 3$) (b) Intermediate spacing ($1.1 < s/D < 3$)

(c) Very small spacing ($s/D < 1.1$)

Sketch 3.4

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