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Contact phenomena. II: stress fields and failure criteria in concentrated elastic contacts under combined normal and tangential loading

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(Continued on inside back cover)

CONTACT PHENOMENA II: STRESS FIELDS AND FAILURE CRITERIA IN CONCENTRATED ELASTIC CONTACTS UNDER COMBINED NORMAL AND TANGENTIAL LOADING

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CONTACT PHENOMENA II: STRESS FIELDS AND FAILURE CRITERIA IN CONCENTRATED ELASTIC CONTACTS UNDER COMBINED NORMAL AND TANGENTIAL LOADING

1. NOTATION

Most of the results are given in non-dimensional form, stresses being normalised by dividing them by the maximum compressive stress, p_0 , and distances by dividing them by the minor semi-axis, b , of the ellipse of contact (Figures 16a and 16b are an exception, in that the depth z is divided by the major semi-axis, a , of the ellipse). They are therefore valid in any self-consistent set of units. For the purposes of illustration, however, units are given in this notation for both SI and a familiar British system. The hardness, H , is the only exception.

		SI	British
A	body geometry parameter, one half minor principal relative curvature	m^{-1}	in^{-1}
a	major semi-axis of contact ellipse	m	in
B	body geometry parameter, one half major principal relative curvature	m^{-1}	in^{-1}
b	minor semi-axis of contact ellipse	m	in
f	direct stress within solid body	Pa	lbf in^{-2}
H	hardness (Brinell or Vickers)		kgf mm^{-2}
h	gap between surfaces of two bodies	m	in
J_2'	second invariant of stress deviator tensor (see Equation (4.1))	Pa^2	(lbf in^{-2}) ²
k	yield stress in simple shear	Pa	lbf in^{-2}
n	$= (1 - \beta^2)^{1/2}$	—	—
P	total normal applied load	N	lbf
p	pressure on contact surface	Pa	lbf in^{-2}
p_0	maximum compressive stress in contact	Pa	lbf in^{-2}
Q	total applied tangential force	N	lbf
q	shear stress within solid body	Pa	lbf in^{-2}
q_e	equivalent or effective shear stress, $J_2'^{1/2}$	Pa	lbf in^{-2}
q_{orth}	orthogonal shear stress	Pa	lbf in^{-2}

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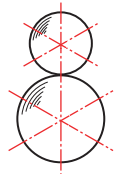
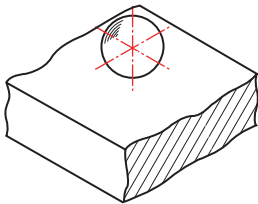
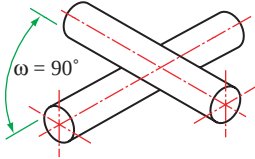
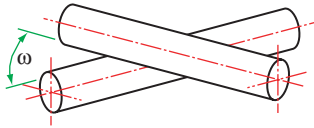
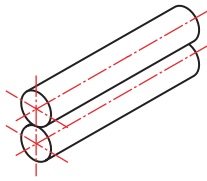
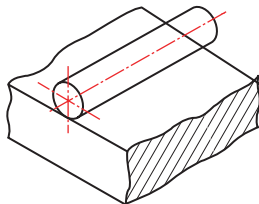
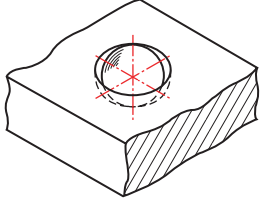
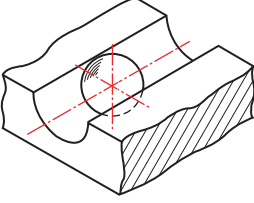
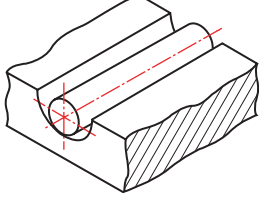
R	minimum radius of relative curvature of solid bodies	m	in
T	tensile stress	Pa	lbf in ⁻²
x	distance measured from centre of ellipse, parallel to major axis (see Sketch 3.1)	m	in
Y	yield stress in uniaxial tension or compression	Pa	lbf in ⁻²
y	distance measured from centre of ellipse parallel to minor axis (see Sketch 3.1)	m	in
z	distance measured from centre of ellipse, normal to surface, towards the interior of the body (see Sketch 3.1)	m	in
β	= $b/a \leq 1$, axis ratio of ellipse of contact	—	—
γ	angle between force of friction and minor axis of ellipse of contact	—	—
Δ	range, maximum value less minimum value	—	—
μ	coefficient of friction for gross sliding	—	—
σ	Poisson's ratio	—	—

Subscripts

$crit$	critical value
e	equivalent or effective
max	maximum value
$orth$	orthogonal
x, y, z	denote directions in which direct stresses act
xz, yz, zy	double subscripts used in conjunction with q , in which first subscript gives direction normal to plane in which stress acts and second subscript gives direction of the stress
0	pertaining to contact centre at surface

2. INTRODUCTION

When two bodies, one or both of whose surfaces are curved, are brought into contact and subjected to a normal load, a nominal “point contact” becomes an elliptical area as a result of elastic deflections within the bodies. Some typical examples of such contacts are shown in Sketch 2.1. Two limiting cases may be identified. In the first, the contact area is circular, for example, sphere on sphere, (a) in Sketch 2.1, sphere on plane, (b), identical cylinders crossed at right-angles, (c). In the second limiting case, a nominal “line contact” becomes a strip, for example, cylinder on cylinder with parallel axes, (e), cylinder on plane, (f), cylinder in groove with parallel axes, (i).

 (a) Sphere on sphere (non-conformal) Contact shape is circular	 (b) Sphere on plate (non-conformal) Contact shape is circular	 (c) Cylinder on cylinder (non-conformal) Contact shape is circular when $\omega = 90^\circ$ and cylinders are of equal diameter.
 (d) Cylinder on cylinder (non-conformal) Contact shape is elliptical when $0^\circ < \omega < 90^\circ$ and/or when the principal radii of curvature are not equal	 (e) Cylinder on cylinder (non-conformal) Contact shape is an ellipse of infinite length (i.e. line contact) when $\omega = 0^\circ$	 (f) Cylinder on plate (non-conformal) Contact shape is an ellipse of infinite length (i.e. line contact)
 (g) Sphere in socket (conformal) Contact shape is circular	 (h) Sphere in groove (conformal) Contact shape is elliptical	 (i) Cylinder in groove (conformal) Contact shape is an ellipse of infinite length (i.e. line contact)

Sketch 2.1

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The elastic deflections imply the presence of stress fields within and on the surface of the bodies. These stress fields are characterised by the values at each point of the six components of the stress tensor, defined in Section 3.

ESDU 78035 (Reference 1) deals with the calculation of stresses and contact dimensions for normally loaded contacts. The maximum values of all the stress components then occur along the line of symmetry, normal to the surface and passing through the centre of the ellipse, and much of the information of interest could be presented by a plot of the values of the stress components, and of quantities derived from them, against position along this line.

The present Item, the second in a series of which ESDU 78035 is the first, considers the effect on the stress fields of the addition of tangential shear stresses over the surface of the ellipse, the ratio of the shear stress to the normal stress having everywhere a constant value, equal to the coefficient of friction for gross slip. The inclusion of these shear stresses destroys the symmetry of the field, and the maximum values no longer necessarily occur on the axis of symmetry. This complicates the presentation of the results of calculations of the stress field, and the coverage of the effect of different combinations of the various factors, such as coefficient of friction, Poisson's ratio, axis ratio of ellipse of contact, *etc.*, is less comprehensive than in ESDU 78035. Some linear interpolation will often be required in the application to practical cases of the results presented in this Item, and worked examples are provided. Further results, not included here, will be available to all holders of this Item in a forthcoming ESDU Memorandum (Reference 2).

In the application of the information in the present Data Item to the solution of practical problems, it will be assumed that the essential quantities, the maximum compressive stress and the lengths of the axes of the ellipse of contact, have been calculated from the information given in ESDU 78035. It will also be assumed that the values of these quantities are unaffected by the imposition of the tangential stresses. This latter assumption is not strictly valid if the elastic properties of the two bodies are different, but in general the effects of the difference are small. This point is discussed further in Section 8.3.

Shear stresses of the form assumed arise in the case of gross sliding between dry (unlubricated) bodies. The distribution of shear stresses is more complicated if the tangential forces are insufficient to cause gross sliding, but the assumption of gross sliding will result in higher values of the stresses than will occur in other circumstances, *i.e.* the situation that has been assumed is the worst that could arise. The stress distribution may differ from that assumed if the surfaces are lubricated, but the results given in this Data Item are valid for boundary lubrication, and are approximately correct for the limiting case of very thin films in elastohydrodynamic lubrication, *i.e.* with low surface velocities and low lubricant viscosities. It is planned to cover the case of elastohydrodynamically lubricated bodies in a further Item in this series.

Analytical expressions for the stress fields in the two limiting cases of circular and line contact under combined normal and tangential loading have been available in the literature for some time and are well known (References 3 to 6 inclusive and Derivations 21 and 22), but solutions for the general elliptical contact are more complicated, involve elliptic integrals, and have been published only recently (Reference 7, Derivations 23 to 25 inclusive).

A surprising feature of the results of these treatments is that the stress fields vary very little with what would appear to be important factors, such as the axis ratio of the ellipse, the direction of the force of friction, and Poisson's ratio. Indeed, in many cases, the values of the various stress criteria, discussed later, vary by not more than 5 per cent over the range of circular contact, line contact with friction parallel to the short axis, and line contact with friction parallel to the long axis. Results for elliptical contacts with any general direction of the force of friction may be expected to fall also within this range. The depths at which the maxima occur are, however, more sensitive to the axis ratio. The value of the coefficient of friction can have a major influence on the stress field, but only if it is above $0.1 \rightarrow 0.2$. This means that many contacts of practical interest, *e.g.* those lubricated by a boundary or elastohydrodynamic mechanism, may be treated as normally loaded, *i.e.* without tangential stresses.

Many users of this Item will be concerned with the failure of materials. There are many modes of failure, and the behaviour of materials approaching failure may be very complicated. For accurate and reliable prediction of failure, a detailed model of the behaviour of the material is required together with a detailed knowledge of the variation of the individual stress components with space and with time. It is planned to issue a further Item covering the calculations of the individual stress components, including the general elliptical contact with friction acting in any direction.

In the present Item, attention is concentrated on three simplified models of failure:

plastic yielding of ductile materials	(Section 4)
cracking of brittle materials	(Section 5)
fatigue failure under repeated stressing	(Section 6)

For each failure mode, an appropriate failure criterion is defined according to the current state of knowledge, and charts are provided from which the value of this criterion may be read. It is emphasised that this treatment is over-simplified, and that very few materials behave exactly in the manner assumed, but this simplification is acceptable in most cases.

It is necessary to distinguish between materials with uniform properties, *i.e.*, those in which the critical value of the failure criterion is independent of the position and those with non-uniform properties. The best known example of a material with non-uniform properties is case-hardened steel, in which the critical value of a failure criterion will depend on depth below the surface. For uniform materials, only the overall maximum value of the failure criterion is of interest, its position being of less consequence, but for non-uniform material it is necessary to take into account the variation of the failure criterion with depth below the surface. Results covering these cases are presented separately in the various sections.

Another vital distinction is between static stress fields, in which the stress experienced by a given element of material remains constant in time, and cyclic stress fields, such as those in rolling contacts, in which a given element of material is subjected to a range of stresses as it traverses the contact zone. The stresses often fall to zero as the material leaves the contact zone, only to be re-applied at the next approach. In some cases the materials of one body may be stressed statically, and that of the other cyclically.

This distinction between static and cyclic loading is important mainly in connection with Section 4, plastic yielding of ductile materials, while Section 6, fatigue failure, is concerned wholly with cyclic stressing. The various calculation procedures appropriate in each of these individual cases are summarised for easy reference in Table 7.1. A list of the figures, showing the ranges of the variables covered in each, is given in Table 7.2.

3. REFERENCE AXES AND STRESS COMPONENTS

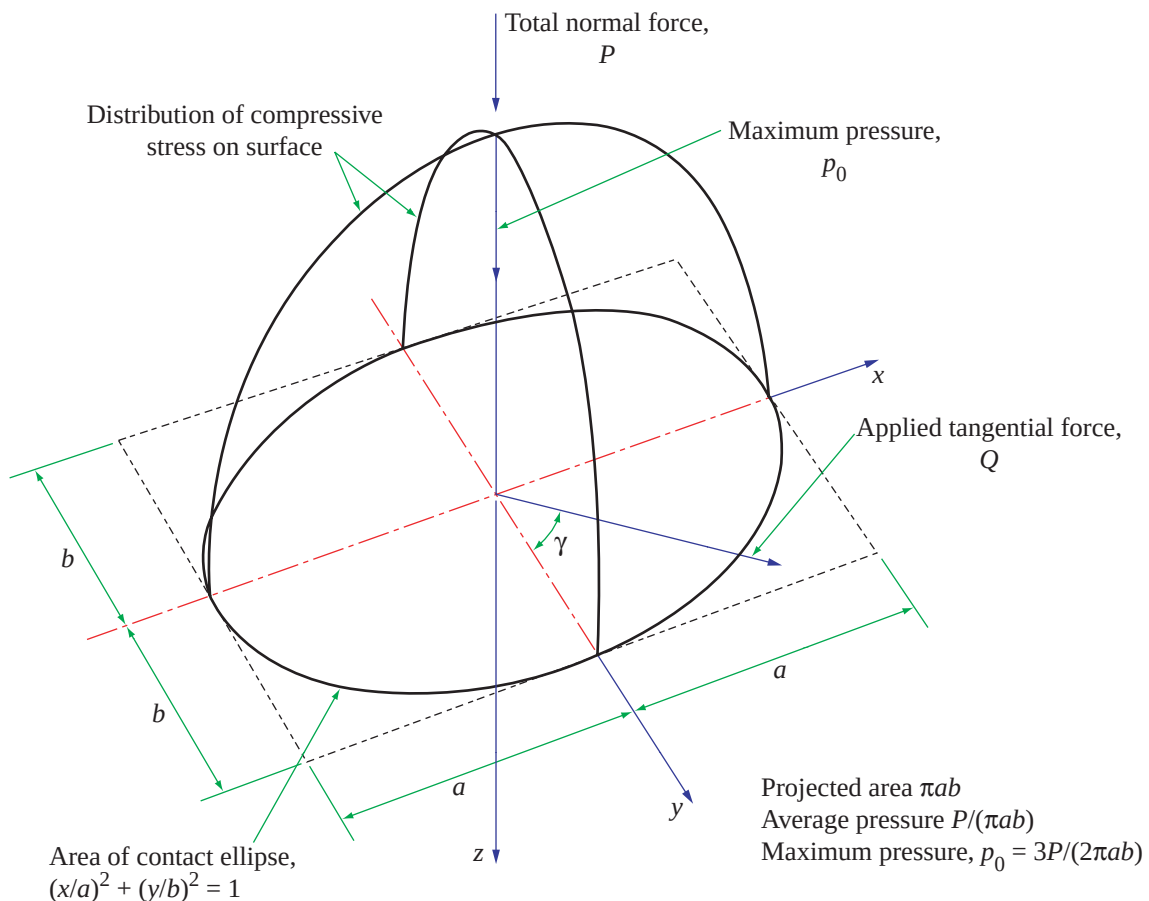
Sketch 3.1 represents the area of contact between two solid elastic bodies as the ellipse,

$$(x/a)^2 + (y/b)^2 = 1, \quad a \geq b.$$

This area supports the total load P imposed on the contact by a distribution of compressive stress p over the ellipse:

$$p = p_0 \left(1 - (x/a)^2 - (y/b)^2 \right)^{1/2}, \quad (x/a)^2 + (y/b)^2 \leq 1.$$

An applied tangential force, Q , acts at a general angle, γ , to the y -axis, and it is assumed that the resultant shear stress, q , on the surface is everywhere proportional to p .



Sketch 3.1 Area of contact, forces and stresses

The stress field within and on the surface of each body is characterised by the values of the six components of the stress tensor. These are the three direct stresses:

$$f_x, f_y, f_z$$

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acting in directions parallel to, and over planes perpendicular to, the x -, y - and z -axes respectively, and the three shear stresses:

$$q_{xy} = q_{yx}, \quad q_{yz} = q_{zy}, \quad q_{zx} = q_{xz}.$$

The z -axis is normal to the tangent plane at the centre of the ellipse and directed toward the interior of the body. The usual convention that positive direct stresses are tensile will be adopted here. In the symbol for the general shear stress, q_{xy} , the first subscript gives the direction of the normal to the plane in which the stress acts, and the second subscript gives the direction of the stress.

On the surface, and within the ellipse of contact, the values of f_z , q_{zx} and q_{zy} are prescribed by the boundary conditions:

$$\left. \begin{aligned} f_z &= -p_0 \left(1 - (x/a)^2 - (y/b)^2 \right)^{1/2}, \\ q_{zx} &= -\mu p_0 \left(1 - (x/a)^2 - (y/b)^2 \right)^{1/2} \sin \gamma, \\ \text{and} \quad q_{zy} &= -\mu p_0 \left(1 - (x/a)^2 - (y/b)^2 \right)^{1/2} \cos \gamma. \end{aligned} \right\} \quad (3.1)$$

where μ is the coefficient of sliding friction and γ is the inclination of the direction of the force of friction to the y -axis, the minor axis of the ellipse.

It is assumed that p_0 , a and b have been calculated from the information given in ESDU 78035, and that μ and γ are known. The problem is: firstly, to calculate the stress fields throughout the body; secondly, to construct from these fields values of an appropriate criterion of failure; and thirdly, to obtain either the overall maximum value of this criterion, or the variation of the maximum value with depth below the surface. Three different criteria, corresponding to three different modes of failure, will be considered in the following sections.

4. DUCTILE MATERIALS FAILING BY PLASTIC YIELDING

4.1 Failure Criterion

Various criteria have been proposed for these materials, but the one with the most experimental and theoretical support is that of von Mises, the distortional strain energy per unit volume. This is obtained by subtracting from the total strain energy that component resulting from the change in volume. For a material with given elastic properties, the distortional strain energy per unit volume is proportional to the second invariant of the stress deviator tensor:

$$J_2' = q_{xy}^2 + q_{yz}^2 + q_{zx}^2 + \frac{1}{6} \left((f_x - f_y)^2 + (f_y - f_z)^2 + (f_z - f_x)^2 \right). \quad (4.1)$$

The equivalent or effective shear stress, q_e , is defined as $J_2'^{1/2}$.

The most direct information on the critical value of this criterion for most materials is obtained from tests in uniaxial tension or compression. Here the critical value of J_2' from Equation (4.1) is

$$(J_2')_{crit} = \left((q_e)_{crit} \right)^2 = \frac{1}{3} Y^2 = k^2, \quad (4.2)$$

where Y is the yield stress in uniaxial tension or compression and k is the yield stress in simple shear.

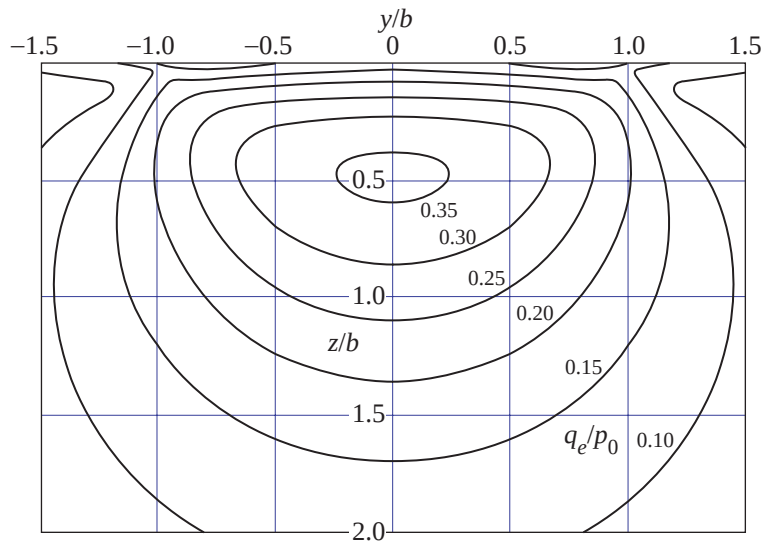
If Y is not known, an appropriate estimate may be obtained by taking 1/3 of the hardness, measured either by the Brinell or by the Vickers method, but care must be taken in the conversion to the appropriate units if necessary. The hardness is usually given in units of kgf mm^{-2} , while the results of the calculations will be given as the ratio q_e/p_0 where p_0 may be expressed in different units. Thus

$$\left. \begin{array}{lll} Y/\text{kgf mm}^{-2} & \approx & 1/3 H/\text{kgf mm}^{-2}; \quad k/\text{kgf mm}^{-2} \approx 0.192 H/\text{kgf mm}^{-2} \\ Y/\text{lbf in}^{-2} & \approx & 474 H/\text{kgf mm}^{-2}; \quad k/\text{lbf in}^{-2} \approx 274 H/\text{kgf mm}^{-2} \\ Y/\text{MPa} & \approx & 3.27 H/\text{kgf mm}^{-2}; \quad k/\text{MPa} \approx 1.89 H/\text{kgf mm}^{-2}. \end{array} \right\} \quad (4.3)$$

The relationships quoted in this section apply to materials obeying the von Mises criterion of yield, which implies strict isotropy. Materials used in practice may not behave exactly in this way, but deviations will be small for most ductile materials. The criterion is for the onset of plastic deformation. If the critical value is exceeded in cyclic loading, plastic deformation will occur in the first cycle, but not necessarily in subsequent cycles. Work hardening may have a similar effect in suppressing plastic deformation. These points are discussed further in Section 8.5.

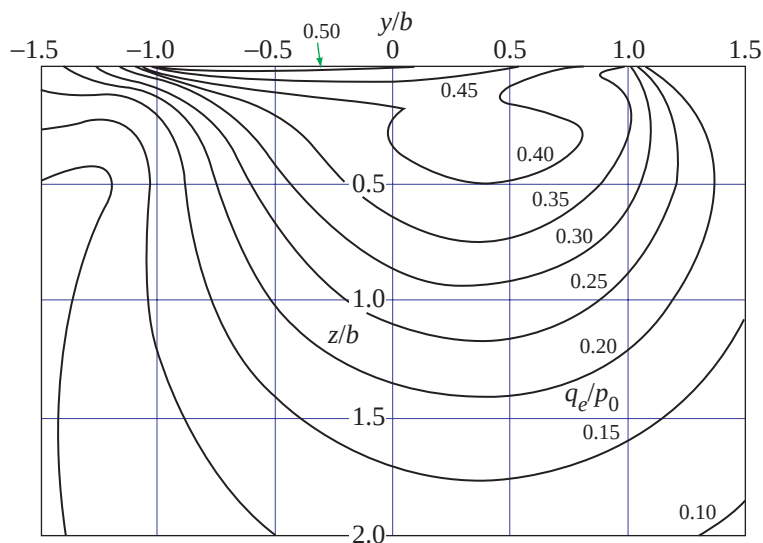
4.2 Contours

Sketches 4.1 and 4.2, taken from Derivation 21, show contours of constant values of q_e/p_0 as a function of y/b and z/b in the plane $x = 0$ beneath a circular contact on the surface of a material with Poisson's ratio equal to 0.3.



Sketch 4.1 Contours of q_e/p_0 under circular contact with no friction

In Sketch 4.1 the loading is normal and there are no tangential forces at the surface. The maximum value of q_e occurs below the surface, at a depth equal to approximately half the radius of the circle of contact.

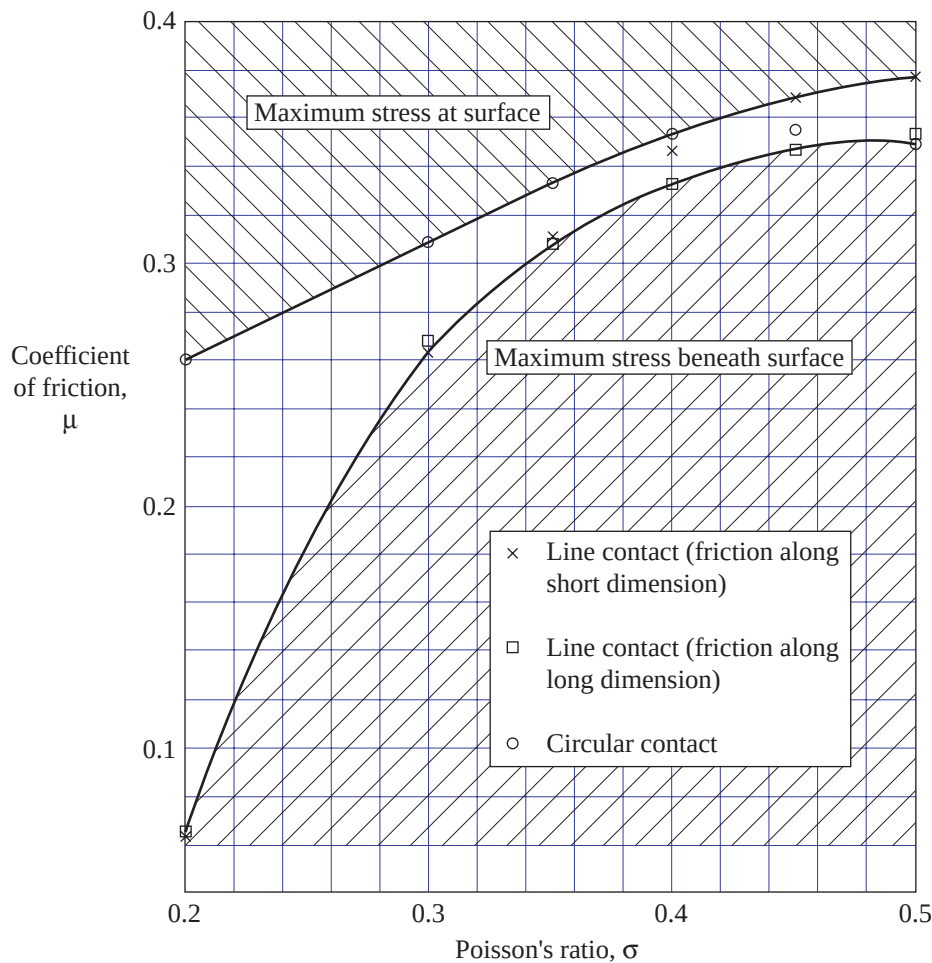


Sketch 4.2 Contours of q_e/p_0 under circular contact with coefficient of friction 0.5

In Sketch 4.2, however, an additional tangential loading produced by a coefficient of friction of 0.5 acting in the positive y direction has been introduced. The value of the sub-surface maximum of q_e has been increased but it is now nearer the surface, while a larger maximum has appeared on the surface, at the trailing edge of the contact. The essential point is that the maximum value of q_e occurs below the surface at low coefficients of friction but on the surface at high coefficients of friction.

The change-over point depends primarily on Poisson's ratio and contact geometry and is illustrated in Sketch 4.3. The effect of contact geometry is relatively small for Poisson's ratios of 0.3 or higher: the change-over occurs in the un-shaded area of the sketch, whatever the geometry.

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Sketch 4.3 Location of maximum value of q_e

4.3 Overall Maximum

If the material is uniform, then only the overall maximum value of q_e is of interest, the position of the maximum being immaterial. This overall maximum value depends mainly on the coefficient of friction, Poisson's ratio being important only at low coefficients of friction. The axis ratio of the ellipse and the direction of the frictional force have surprisingly little effect.

The overall maximum value of q_e/p_0 is presented approximately in Figure 1. The effects of variations in contact geometry are contained within the shaded area, to an accuracy of within 10%.

Figure 5 shows maxima and minima in $(q_e)_{max}/p_0$ when plotted against the axis ratio of the ellipse, but deviations from the values shown in Figure 1 are unlikely to exceed 5%.

More precise values of $(q_e)_{max}$, and particularly those of the lower values of Poisson's ratio, may be obtained from Figures 2, 3 and 4, which cover the three configurations just described. The solid lines give the higher of the sub-surface and surface maxima, and they are therefore the ones to be used in the prediction of failure. The dotted lines give the lower of the two maxima, for information only. Figure 5 shows the effects of the axis ratio of the ellipse of contact and of the direction of the friction for a range of values of

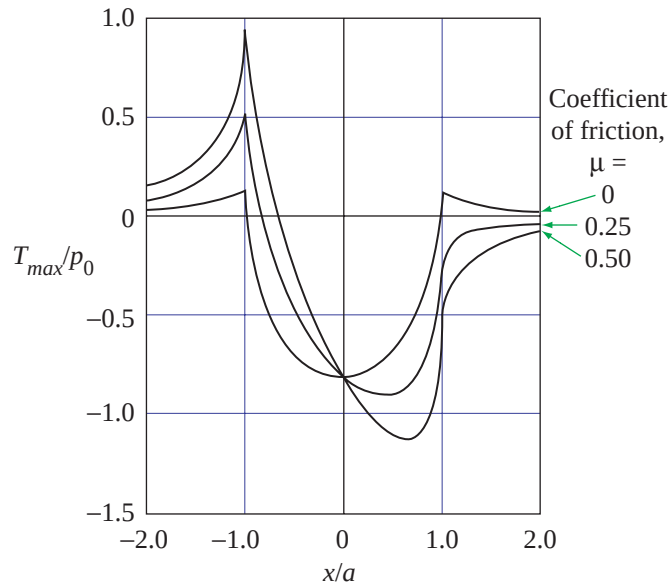
the coefficient of friction, but for a single value, 0.3, of Poisson's ratio. The effect of axis ratio on the depth of the sub-surface maximum is shown in Figure 6. Figures 5 and 6 are taken from Derivation 24. The depth of the maximum is more sensitive to the axis ratio than is the actual value of the maximum.

4.4 Variation with Depth

In some cases the properties of the material, and in particular the value of k , will vary with position. The materials will usually be ferrous, such as case-hardened steel and chilled cast iron, for which the hardness varies with depth below the surface. For these materials a plot of $(q_e)_{max}$ against depth is required, the (x,y) plane at each depth, z , being searched for the maximum value of q_e . Such plots are shown in Figures 7, 8, 9, 10 and 11, in which each curve relates to a different value of μ . Poisson's ratio has been taken to be 0.3 throughout. Interpolation, as in Example 2, will probably be necessary in the application of the results of Figures 7 to 11 to practical cases.

5. BRITTLE MATERIALS FAILING BY CRACKING

Here the best criterion is the maximum tensile stress, T_{max} (Reference 8). Sketch 5.1, from Derivation 21, shows the largest tensile stress along the centre-line at the surface of a circular contact, due to a friction force acting in the x direction for various values of the coefficient of friction. The system of tensile stresses is very sharply localised, not only in a radial direction, as is evident from Sketch 5.1, but also in depth; it decays very rapidly at small depths below the surface.



Sketch 5.1 Maximum tensile stress for circular contact

There are very simple relationships for the limiting configurations:

$$\left. \begin{aligned} T_{max}/p_0 &= \frac{1}{3}(1 - 2\sigma) + \frac{1}{8}(4 + \sigma)\pi\mu, \text{ for circular contact (Derivation 22)} \\ T_{max}/p_0 &= 2\mu, \text{ for line contact with friction along the short dimension (Reference 5)} \\ T_{max}/p_0 &= \mu, \text{ for line contact with friction along the long dimension.} \end{aligned} \right\} \quad (5.1)$$

The relationships for the three limiting configurations are plotted in Figure 12, and it is seen that over a large part of the region of interest there is little difference in the results for the first two configurations, which are the most common. Most elliptical contacts with friction approximately along the short dimension would be expected to fall in this region, common to the first two limiting configurations.

For line contact with a general direction of friction,

$$T_{max}/p_0 = \mu \left((1 + \sigma) \cos \gamma + \left((1 - \sigma)^2 \cos^2 \gamma + \sin^2 \gamma \right)^{1/2} \right). \quad (5.2)$$