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Estimation of damping in laminated and fibre-reinforced plates

Associated software: ESDUpac A8512

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ESTIMATION OF DAMPING IN LAMINATED AND FIBRE-REINFORCED PLATES

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ESTIMATION OF DAMPING IN LAMINATED AND FIBRE-REINFORCED PLATES

1. NOTATION

a	length of plate in x-direction (see Sketch 1.1)	m	in
b	length of plate in y-direction (see Sketch 1.1)	m	in
c_{mn}	coefficient in Equation (2.2)	m	in
E_α, E_β	lamina moduli of elasticity in α, β-directions, respectively	N/m^2	lbf/in^2
h	thickness of laminated plate	m	in
$G_{\alpha\beta}$	shear stiffness of lamina in (α, β) directions	N/m^2	lbf/in^2
m	mode number in x-direction		
N	number of layers in plate		
n	mode number in y-direction		
Q	magnification factor		
U_{max}	maximum strain energy	J	erg
V_f	fibre volume fraction (see Section 2.2)		
w_{mn}	deflection of plate normal to xy-plane in mode (m, n)	m	in
x, y	coordinate system (see Sketch 1.1)		
α, β	lamina coordinates aligned with principal axes of orthotropy of lamina (see Sketch 1.2)		
γ_m, γ_n	coefficients in Equations (2.5) and (2.6)		
Δ	logarithmic decrement		
δ	viscous damping ratio		
η	loss factor		
$\theta_n(y)$	displacement function in y-direction [see Equations (2.4), (2.6) and (2.10)]		
θ	angle of principal axis of orthotropy, α, of lamina measured from plate principal axis, x (see Sketch 1.2)	degree	degree

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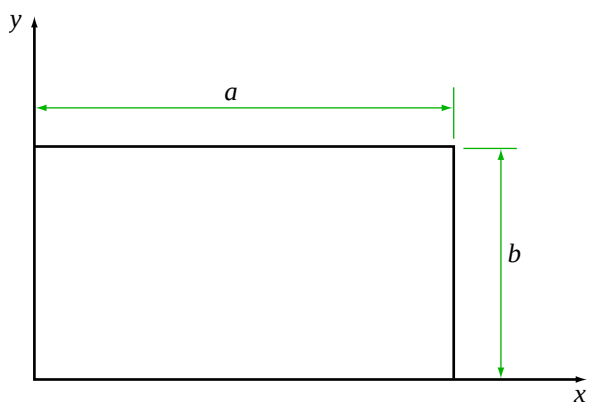
λ_m, λ_n	parameters in Equations (2.5) and (2.6)		
$\nu_{\alpha\beta}$	Poisson's ratio* for lamina resulting from load applied in α-direction		
$\nu_{\beta\alpha}$	Poisson's ratio* for lamina resulting from load applied in β-direction		
ρ	density of layer	kg/m^3	†
$\bar{\rho}$	average density of laminated plate	kg/m^3	†
$\phi_m(x)$	displacement function in x-direction [see Equations (2.3), (2.5) and (2.9)]		
ψ	plate specific damping capacity		
ψ_f	fibre specific damping capacity in longitudinal‡ flexure [see Equation (2.1)]		
ψ_L	lamina specific damping capacity in longitudinal‡ flexure**		
ψ_{mF}, ψ_{mt}	lamina matrix specific damping capacity in flexure and torsion respectively		
ψ_{LT}	lamina specific damping capacity in longitudinal‡ shear**		
ψ_T	lamina specific damping capacity in transverse‡ flexure**		

* In some Items in this series, Poisson's ratio has been designated σ.

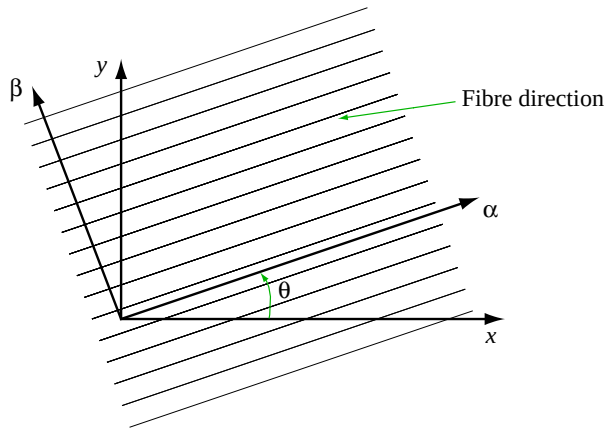
† A density value expressed in British units as pounds per cubic inch has to be divided by 386.4 before being used in this Item. (A force of 1 lbf acting on a mass of 1 lbf produces an acceleration of 386.4 in/s^2.) For example, if the density is 0.097 lbf/in^3, the value input into the computer program should be 2.51×10^{-4} lbs^2/in^4.

‡ The terms 'longitudinal' and 'transverse' refer here to the direction of the fibres.

** Note that the pairs of subscripts L and α, T and β and LT and $\alpha\beta$ are synonymous. The subscripts for specific damping capacities most commonly used in the literature are used in this Item.



Sketch 1.1 Plate dimensions



Sketch 1.2 Relationship between plate and layer axes

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2. NOTES

2.1 General

This Data Item provides a means of estimating the damping in a rectangular, multilayered laminate that is vibrating in one of its lower frequency natural modes. The data are particularly applicable to fibre-reinforced composite laminated plates. The estimation procedure is applicable only to plates in which laminae having the same material properties, and orientations of their principal axes of orthotropy, are symmetrically arranged about the plate mid-plane (*i.e.* balanced symmetric laminates). This Item may be used to estimate the damping and the natural frequencies of laminated plates having all edges simply-supported, all edges clamped or all edges free. The last case can be used for comparing laboratory measurements of damping with values predicted by this Data Item.

The practical combinations of lay-up configurations are too numerous for graphical presentation of damping data; therefore, a computer program contained in ESDUpac A8512 is provided for the estimation of damping. Example input and output files are also provided. Inputs for this program are the individual layer material elastic properties and damping properties. Damping properties are discussed in Section 2.2 of this Item and the estimation of lamina elastic properties is discussed in ESDU 83035*.

Unlike conventional metallic built-up structures, fibre-reinforced laminated structures are often constructed using adhesive joints with which there are fewer additional sources of energy dissipation associated than, for example, with bolted or riveted joints. Therefore, for these fibre-reinforced structures, material damping significantly contributes to the overall damping in any mode of vibration. The reduction of the structural component of damping in fibre-reinforced materials may be partly redressed by the suitable choice of matrix and fibre constituents to give optimum material damping. This Item provides a means of evaluating the material damping for various component materials and laminate lay-ups to enable the designer to investigate the practical combinations of materials and laminate arrangements for a particular application.

2.2 Damping in Composites

For composite materials, damping is taken to be any process within the body of the material in which energy is dissipated. The principal processes are associated with hysteresis losses within each of the constituent materials and interfacial effects at the fibre-matrix interfaces. Damping is dependent on the cyclic stress amplitude but it has been shown (see for example Derivation 4) that for small vibration amplitudes the variation of damping with stress amplitude is negligible.

When considering fibre-reinforced laminates it is convenient to express damping in terms of specific damping capacity, ψ. Specific damping capacity is the ratio of energy dissipated in a unit volume per stress cycle to the maximum strain energy per unit volume. It should be noted that in Data Items for estimating the response of structures to acoustic loading, damping is required in terms of viscous damping ratio, δ. The output from the computer program provides damping both in terms of specific damping capacity and viscous damping ratio.

The relationships between a number of the most common measures of damping are given in ESDU 73011† and more details are given in Reference 13. The relationships between various damping measures for harmonic vibration are summarised in the table on the following page. The tabulated values are equivalent (*e.g.* $\psi = 4\pi\delta$) for small values of damping. The relationships are sufficiently accurate for use in most engineering applications.

* Estimation of the stiffnesses and apparent elastic properties of laminated flat plates.
† Damping in acoustically excited structures.

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Viscous damping ratio	Specific damping capacity	Magnification factor	Logarithmic decrement	Loss factor
δ	$\frac{\psi}{4\pi}$	$\frac{1}{2Q}$	$\frac{\Delta}{2\pi}$	$\frac{\eta}{2}$

In this Item, the specific damping capacity for an individual lamina is considered in terms of damping components in longitudinal flexure, ψ_L, transverse flexure, ψ_T, and longitudinal shear, ψ_{LT}. For a particular fibre-reinforced lamina it is preferable to obtain values of the three damping components by direct measurement. Typical test procedures are described in Reference 12. If measured values of damping for a particular lamina are not available, the curves described in the following sub-sections may be used to obtain approximate values of the damping components. Component damping values are given in terms of the lamina fibre volume fraction, V_f, where V_f is the volume proportion of fibre in a representative volume of the lamina material. The relationship between the fibre volume fraction and the fibre mass fraction (the equivalent mass ratio) is given in ESDU 83035. In the curves, damping components are normalised relative to a damping component of the pure matrix material. Typical matrix damping values, for commonly used matrix materials, are given in Table 8.1. The curves in this Item are particularly applicable for estimating the specific damping capacity at a specific fibre volume fraction from a known value at some other fibre volume fraction. The use of the curves is illustrated in the example in Section 7.

2.2.1 Specific damping capacity in longitudinal* flexure, ψ_L

In Figure 1, ψ_L/ψ_{mF} is plotted against V_f, where ψ_{mF} is the specific damping capacity of the unreinforced matrix in flexure. This curve has been validated using measured data obtained from tests on unidirectional fibre specimens with either carbon† or glass fibres. The curve should be used with caution for other fibre materials.

According to the law of mixtures for tensile or compressive loading in the fibre direction, the values of ψ_L (for a lamina without voids) is given by

$$\psi_L = \psi_f V_f + \psi_{mF}(1 - V_f) \ , \tag{2.1}$$

where ψ_f is the specific damping capacity of the fibre in longitudinal flexure. Since $\psi_{mF} \geq \psi_f$, from Equation (2.1) ψ_L should decrease with increasing V_f. However, measured damping values suggest that ψ_L remains unchanged as fibre volume fraction increases beyond about 0.4 (but see Section 3.1).

2.2.2 Specific damping capacity in transverse* flexure, ψ_T

The empirical relationship between ψ_T/ψ_{mF} and V_f presented in Figure 2 has been deduced (Derivation 8) from a limited number of flexural tests on specimens made from glass fibre in an epoxy resin matrix. The curve has been shown to correlate well with values of ψ_T measured on carbon fibre specimens at a fibre volume fraction of 0.5.

* The terms 'longitudinal' and 'transverse' refer here to the direction of the fibres.
† In reports published in the USA, carbon fibre is usually referred to as graphite fiber.

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2.2.3 Specific damping capacity in longitudinal* shear, ψ_{LT}

In Figure 3, a theoretical curve (Derivation 5) is given for the variation of ψ_{LT}/ψ_{mt} with fibre volume fraction. On this Figure, ψ_{mt} is the torsional specific damping capacity of the unreinforced matrix. The correlation of measured and theoretical values of ψ_{LT} is poor, but the variation of ψ_{LT} with V_f within individual measured data sets follows that given by the theory. Figure 3 may therefore be reliably used to obtain a value of ψ_{LT} at a particular value of fibre volume fraction from a measured value at some other known fibre volume fraction.

2.3 Method of Calculation of Damping

The method of analysis for estimating damping is based on that given in Derivation 3. In the computer program it has been assumed that plates are symmetric (*i.e.* layers having the same material properties, and orientations of their principal axes of orthotropy, are symmetrically arranged about the plate mid-plane) so that the coupled in-plane flexural stiffness sub-matrix* for the plate is a null matrix; this greatly simplifies the computation of damping. It is also assumed that the amplitude of vibration of the plate is small so that damping is independent of cyclic stress amplitude and it is further assumed that the effect of in-plane forces on bending is negligible.

To evaluate the bending moments within the computer program, the deflected shapes in the modes under consideration are needed. The mode shapes are based on characteristic beam functions and are assumed to be in the form

$$w_{mn} = c_{mn}\phi_m(x)\theta_n(y) \tag{2.2}$$

where, for all plate edges simply-supported,

$$\phi_m(x) = \sin\frac{m\pi x}{a}, \tag{2.3}$$

and

$$\theta_n(y) = \sin\frac{n\pi y}{b}, \tag{2.4}$$

and, for all plate edges clamped,

$$\phi_m(x) = \cosh\lambda_m x - \cos\lambda_m x - \gamma_m(\sinh\lambda_m x - \sin\lambda_m x), \tag{2.5}$$

and

$$\theta_n(y) = \cosh\lambda_n y - \cos\lambda_n y - \gamma_n(\sinh\lambda_n y - \sin\lambda_n y). \tag{2.6}$$

For all edges free, $\phi_1(x) = \theta_1(y) = 1$ and

$$\phi_2(x) = 1 - \frac{2x}{a}, \tag{2.7}$$

and

$$\theta_2(y) = 1 - \frac{2y}{b}, \tag{2.8}$$

* This sub-matrix occurs in the in-plane and flexural load deformation relationships and is usually designated the **B** matrix. See, for example, ESDU 83035.

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