

EXAMPLE A – DESIGN OF A DISC CAM WITH A TRANSLATING ROLLER FOLLOWER**SYNOPSIS**

This example deals with the design of a cam mechanism for a moderate-speed, lightly loaded application. The main requirement is to meet specified intermediate displacements and velocities to synchronise associated parts of the machine. This is met by blending portions of different profiles to form the lift segments.

In this embodiment a disc cam co-operates with a translating roller follower to operate the feeder mechanism of a packaging machine. Compression springs keep the follower in contact with the cam. The Modified Sine Acceleration cam law is used. Profile blending is needed, together with considerations of contact stress and lubrication to satisfy the design requirements.

A1. NOTATION AND UNITS

In this notation, units are given for both SI and a familiar British system. All angles are measured positively in an anti-clockwise sense, and the cam is considered to rotate in an anti-clockwise direction.

		<i>SI</i>	<i>British</i>
A	calculation parameter, given by $180H/(\pi\phi)$	m/rad	in/rad
a	distance from loading pivot to plane of spring seating for a spring with free ends	m	in
b_1, b_3	distance between cam centre of rotation and instantaneous centre of rotation of cam corresponding to maximum follower velocity for rise and return segments respectively	m	in
C	end condition factor of spring		
c	spring index factor, D/d		
D	mean coil spring diameter	m	in
d	spring wire diameter	m	in
E	Young's modulus	N/m ²	lbf/in ²
E'	effective Young's modulus = $\frac{2 E_c E_f}{E_c(1 - \nu_f^2) + E_f(1 - \nu_c^2)}$	N/m ²	lbf/in ²
F_{all}	allowable compressive contact stress between cam and follower	N/m ²	lbf/in ²
F_{max}	maximum compressive contact stress between cam and follower	N/m ²	lbf/in ²
f	natural frequency of spring	Hz	Hz
G	modulus of rigidity	N/m ²	lbf/in ²

Issued June 1985

With Amendment A, December 1985

1



Documents

Document Sharing Hub

H	total follower displacement	m	in
H_i	total follower displacement for a complete motion segment, i	m	in
H_{ij}	total follower displacement for a portion, j , which forms part of a complete motion segment, i	m	in
h	thickness of lubricant film	m	in
I	instantaneous centre of rotation of cam with respect to follower		
l	spring free length	m	in
M	mass of follower assembly	kg	slug
n	number of working coils in spring		
O	cam centre of rotation coincident with origin of co-ordinate axis system		
P	total normal force at point of contact between cam and follower	N	lbf
q	maximum spring shear stress	N/m ²	lbf/in ²
R	spring rate	N/m	lbf/in
R_a	prime circle radius of cam, given by $R_r + R_b$	m	in
R_b	reference circle radius of cam	m	in
R_e	effective radius of curvature at contact, given by $[1/\rho_P + 1/R_r]^{-1}$	m	in
R_G	barrelling radius of follower	m	in
R_{qc}, R_{qf}	RMS (root mean square) roughness of cam and follower, respectively	m	micro in
R_{qt}	combined RMS roughness of cam and follower	m	micro in
R_r	radius of roller follower	m	in
S_f	spring force	N	lbf
u_e	lubricant entrainment velocity	m/s	in/s
u_i	normalised input displacement (cam rotation), for a motion segment, i , given by $(\phi - \phi_0) / \Phi_i$		
u_{ij}	normalised input displacement (cam rotation), given by $(\phi - \phi_0) / \Phi_{ij}$, for a portion, j , of a motion segment, i		

v	track velocity	m/s	in/s
w	normalised output displacement of follower		
w_1'	normalised follower velocity corresponding to position of maximum follower load		
X_c, Y_c	axes fixed in cam having origin at O		
X_m, Y_m	axes fixed in machine having origin at O		
x_p, y_p	co-ordinates of roller centre in X_c, Y_c axis system	m	in
Y	difference between free length and installed length of a spring	m	in
y_m	distance in Y_m -direction of centre of roller follower from X_m -axis	m	in
y_{m0}	value of y_m at start of motion segment	m	in
α	pressure angle ($-90^\circ \leq \alpha \leq +90^\circ$)	degree	degree
η	lubricant dynamic viscosity	poise	poise
λ	film thickness ratio		
ν	Poisson's ratio		
ρ	radius of curvature of pitch curve	m	in
ρ_p	radius of curvature of cam profile	m	in
ρ_s	density of spring material	kg/m ³	slug/in ³
Φ_i	cam angle of rotation for a motion segment, i , producing follower displacement, H_i	degree	degree
Φ_{ij}	cam angle of rotation for a portion, j , which forms part of a motion segment, i	degree	degree
ϕ	cam angle of rotation measured from X_m -axis to X_c -axis in anti-clockwise sense	degree	degree
ϕ_0	cam angle at start of either a motion segment or a portion of a motion segment	degree	degree
ω	angular velocity of cam	rad/s	rad/s
ω_n	natural frequency of follower system	Hz	Hz

Subscripts

c, f cam and follower, respectively

max indicates a maximum

min indicates a minimum

One, two or three dots above a variable indicates the first, second or third derivative, respectively, of that variable with respect to time.

One, two or three superscript dashes alongside a variable indicates the first, second or third derivative, respectively, of that variable with respect to normalised input displacement.

A2. INTRODUCTION

This example is concerned with the design of a cam mechanism for a moderate-speed, lightly loaded application. The principal requirement is to meet specified intermediate displacements and velocities to synchronise associated parts of the machine. These requirements are met by blending portions of different profiles to form the lift segments.

This example considers the design of a disc cam driving a translating roller follower which operates a feeder mechanism of a packaging machine. Compression springs are used to keep the follower in contact with the cam. The design uses the Modified Sine Acceleration cam law and profile blending, together with considerations of contact stress and lubrication, to satisfy the requirements of the mechanism.

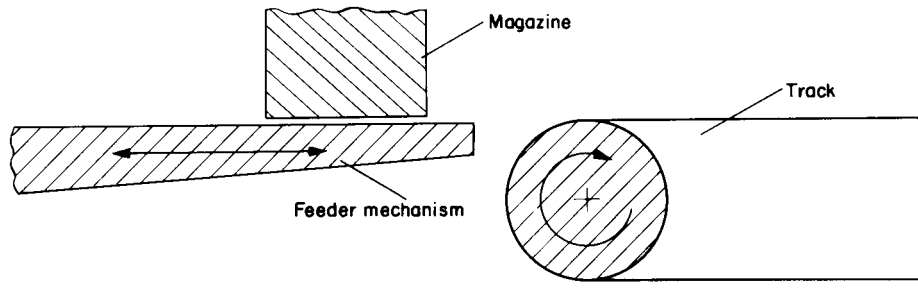
The design of the compression springs is considered in this design together with the effect they have on the performance of the mechanism.

It is assumed that the user has the preceding Items in the Mechanisms series on cams listed in the references of Section 1.

A3. MACHINE SPECIFICATION

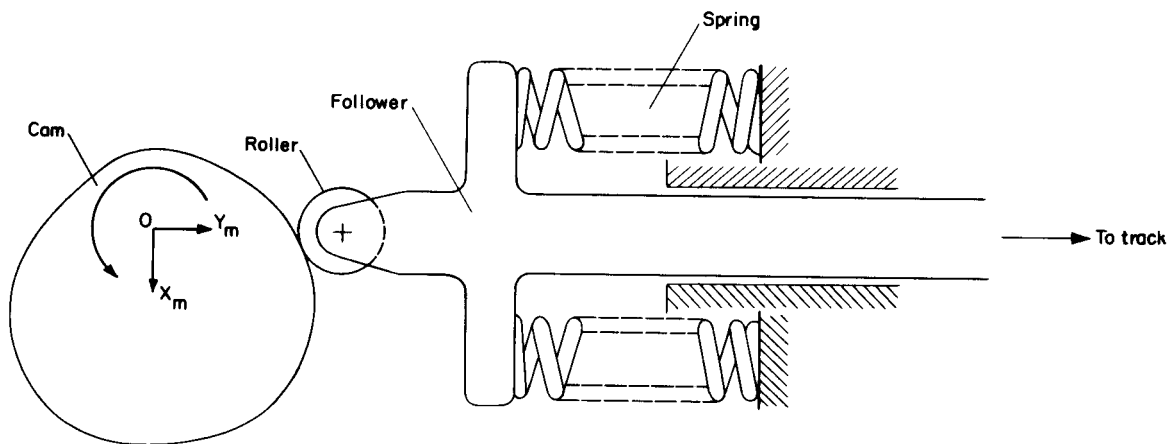
A3.1 Background

Sketch [A3.1](#) shows a schematic of a machine that is required to remove a flat card from a magazine and to place it on a moving track. The cards are removed from a magazine, one at a time, by a feeder mechanism and accelerated to the velocity of the moving track; they are then placed on the track as the feeder mechanism follows the track at constant velocity. When a card has been discharged, the feeder mechanism then decelerates to rest and returns to the magazine to commence the next cycle.



Sketch A3.1 Schematic of machine layout

The feeder mechanism, illustrated in Sketch A3.2, is operated by a disc cam which drives a translating follower through a roller. Contact between the cam and the follower is maintained by two compression springs. The Y_m -axis of the mechanism has been chosen to coincide with the direction of the follower lift. This is consistent with the notation used in Item No. 95001 (Reference 16).



Sketch A3.2 Schematic of feeder mechanism

A3.2 Operating Requirements

The feeder mechanism has the following operating requirements:

delivery rate of the product	15 cards/second
distance between the leading edges of two consecutive cards	101.5 mm
mass of the follower	0.2 kg
mass of the product	negligible
roller radius, R_r	8 mm
barrelling radius of roller, R_G	500 mm

The follower forms part of a stiff mechanism that has a period ratio in excess of 10. The follower motion is described by the Modified Sine Acceleration cam law described in Item No. 82006 (Reference 2).

A3.2.1 Rise segment

The feeder mechanism is required to accelerate from rest to the track velocity in a distance of 15.3 mm, maintain this velocity for a distance of 2.5 mm and then come to rest in a maximum deceleration distance of 10 mm. To minimise the deceleration force it is intended to utilise the full 10 mm for this portion of the motion. Consequently the feeder mechanism has moved a maximum distance of 27.8 mm.

A3.2.2 Dwell segments

For operational reasons two periods of dwell are required. One of these occurs at the end of the rise segment having a minimum duration of 0.003 seconds and the second, which occurs at the end of the return segment, has a minimum duration of 0.006 seconds.

A3.2.3 Return segment

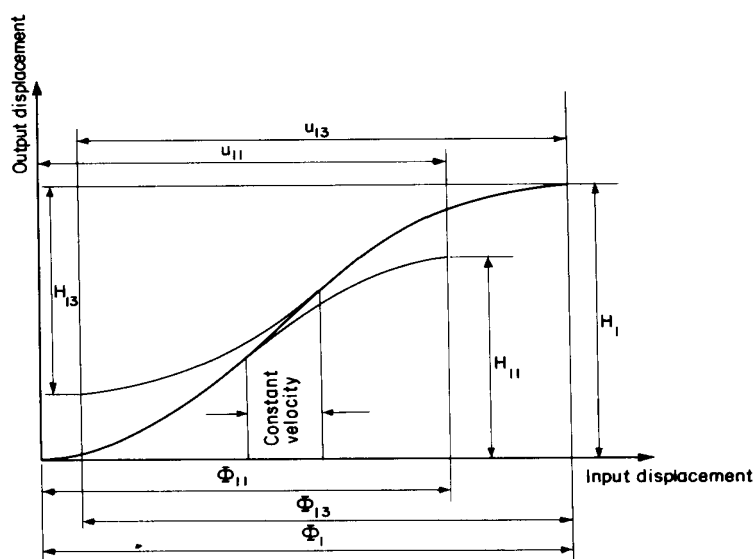
On the return segment the feeder mechanism returns the follower a distance of 27.8 mm from dwell to dwell. There are no intermediate accuracy points in this segment and so the follower displacement for the complete motion segment can be determined by substituting the boundary values into the chosen cam law.

A4. DESIGN PROCEDURE

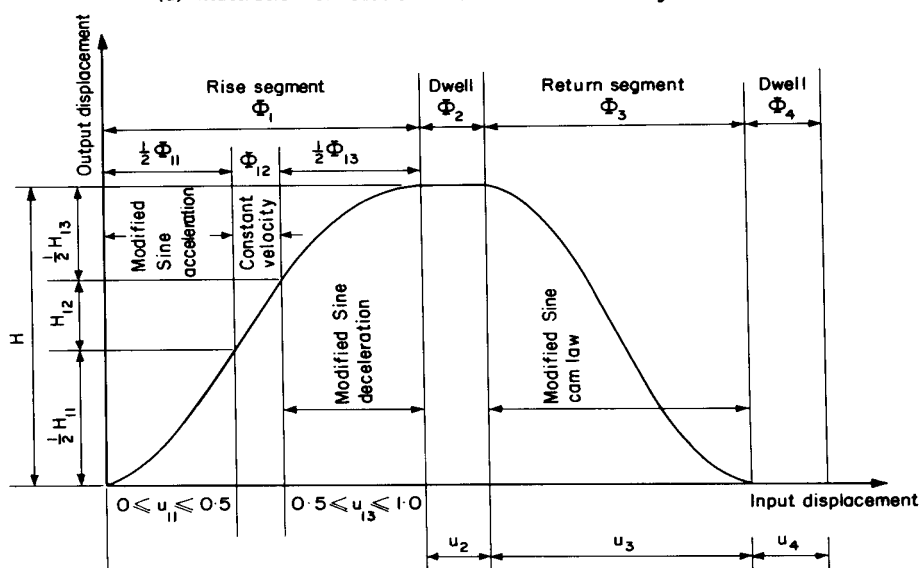
A displacement diagram of the motion characteristics, presented in Sketch A4.1, helps at this stage to illustrate the notation used in this example. Sketch A4.1(a) illustrates the notation associated with the blended profile forming the first motion segment. This profile consists of portions of 3 different motions blended to ensure smooth transitions through both blending points (Reference 6 or 10). Sketch A4.1(b) illustrates both the notation and the motion characteristics during a complete cycle of operation.

The cam displacement angle for a complete motion segment corresponding to a follower displacement H_i is denoted by Φ_i , where i identifies the segment under consideration. When a segment is divided into portions the cam displacement angle is denoted by Φ_{ij} , where j identifies to the portion. Similarly, the non-dimensional input displacements are denoted by u_i for a motion segment and u_{ij} for a segment containing portions.

For this example the displacement characteristics for one cycle of operation, Sketch A4.1(b), consist of four segments, a rise, a dwell, a return and a further dwell. The cam displacement angles corresponding to these segments are Φ_1 , Φ_2 , Φ_3 and Φ_4 , with associated follower displacements of H_1 , H_2 , H_3 and H_4 respectively. The first motion segment, or rise segment, is seen to consist of three portions where, from Sketch A4.1(a), the total input angles of the first and third portions, expressed in terms of a complete cam law ($0 \leq u \leq 1$), are Φ_{11} and Φ_{13} respectively, with corresponding output displacements of H_{11} , and H_{13} . In this example the first portion uses only half of the Modified Sine Acceleration cam law, that is for the range $0 \leq u_{11} \leq 0.5$, and so the cam displacement angle is $\frac{1}{2}\Phi_{11}$ and the output displacement is $\frac{1}{2}H_{11}$. Similarly, the third portion uses only the second half of the Modified Sine Acceleration cam law ($0.5 \leq u_{13} \leq 1.0$) and hence the input displacement and output displacement are denoted by $\frac{1}{2}\Phi_{13}$ and $\frac{1}{2}H_{13}$ respectively.



(a) Illustration of notation associated with blending



(b) Illustration of notation for complete cycle of operation

Sketch A4.1 Displacement diagram

A4.1 Rise Segment

A4.1.1 Speed requirements for the track

The follower velocity occurring during the constant velocity portion of the rise segment is equal to the track velocity. In order that the three portions describing the rise segment can be blended to give continuous motion, the follower velocity at the end of the first portion and the follower velocity at the beginning of the third portion must be equal to the velocity of the follower in the second portion. Since the follower displacements for each portion are known, the cam angular displacement can be calculated.

If the feeder mechanism delivers 15 components onto the track every second with a component spacing of

101.5 mm, then the required track velocity is given by

$$v = 0.1015 \times 15 = 1.5225 \text{ m/s.}$$

(It is necessary to retain sufficient accuracy at this stage to avoid cumulative errors in subsequent calculations.)

A4.1.2 Calculation of cam angular displacement

The motion of the rise segment is modelled by blending portions of three profiles which are described by:

- (a) the follower accelerates to track velocity with Modified Sine acceleration,
- (b) the follower has constant velocity,
- (c) the follower decelerates to rest with Modified Sine deceleration.

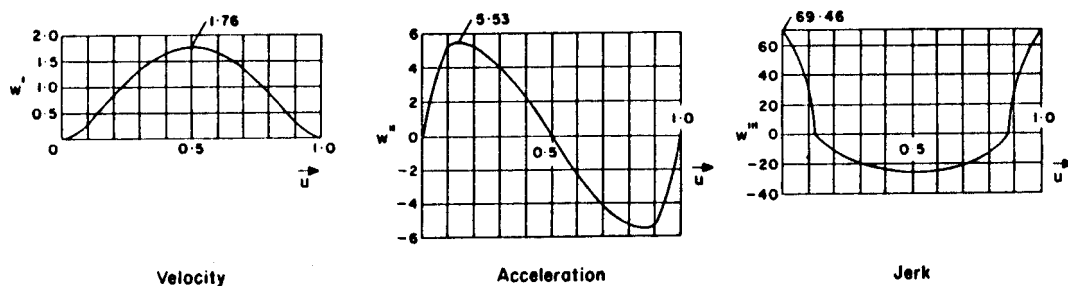
The cam law formulae and boundary conditions for the Modified Sine Acceleration cam law have been taken from Item No. 82006 and are given in Sketch A4.2.

- (a) Cam law formulae

Interval	$w [= y/H]$	$w' [= \dot{y}(\Phi\pi/180)/(\omega H)]$	$w'' [= \ddot{y}(\Phi\pi/180)^2/(\omega^2 H)]$
$0 \leq u < 0.125$	$J[u - \sin(4\pi u)/(4\pi)]$	$J[1 - \cos(4\pi u)]$	$4\pi J \sin(4\pi u)$
$0.125 \leq u < 0.875$	$-\frac{9J}{4\pi} \cos[4\pi(u - 1/8)/3] + Ju + \frac{2J}{\pi}$	$3J \sin[4\pi(u - 1/8)/3] + J$	$4\pi J \cos[4\pi(u - 1/8)/3]$
$0.875 \leq u \leq 1.0$	$\frac{J}{4\pi} \cos[4\pi(u - 7/8)] + Ju + \frac{4J}{\pi}$	$-J \sin[4\pi(u - 7/8)] + J$	$-4\pi J \cos[4\pi(u - 7/8)]$

$$[J = \pi/(\pi + 4)]$$

- (b) Cam law curves



Sketch A4.2 Modified Sine Acceleration cam law

Because the first portion of the rise segment is described by using the first half of a Modified Sine Acceleration cam law, *i.e.* for $0 \leq u_{11} \leq 0.5$, and the third portion of the rise segment is described by using the second half of a Modified Sine Acceleration cam law, *i.e.* $0.5 \leq u_{13} \leq 1.0$, the conditions that enable the three portions to be blended together are satisfied (see Sketch A4.2(b)).

Rearranging the velocity equation from Sketch A4.2(a),

$$\dot{y} = (w' \omega H_{ij} 180) / (\pi \Phi_{ij}) . \quad (\text{A4.1})$$

Equating the velocity at the end of the first portion (see Sketch A4.2(b)) to the track velocity gives

$$1.5225 = (1.76 \times 15 \times 2\pi \times 0.0306 \times 180) / (\pi \Phi_{11}) ,$$

so $\Phi_{11} = 191^\circ .$

The angular displacement of the first portion of the rise segment is thus given by

$$\frac{1}{2} \Phi_{11} = 95.5^\circ .$$

The second portion moves the follower a distance of 2.5 mm with a constant velocity of 1.5225 m/s. This corresponds to an angular displacement of

$$\Phi_{12} = \frac{0.0025}{1.5225} \times 15 \times 360 = 8.9^\circ .$$

During the third portion of the rise segment, the follower decelerates to rest in a distance of 10 mm. Using the velocity equation given in Sketch A4.2(a), the corresponding cam angular displacement is

$$1.5225 = (1.76 \times 15 \times 2\pi \times 0.020 \times 180) / (\pi \Phi_{13}) ,$$

that is $\Phi_{13} = 124.8^\circ .$

Thus the third portion of the rise segment has an angular displacement of

$$\frac{1}{2} \Phi_{13} = 62.4^\circ .$$

The total angular displacement of the cam for the rise segment is

$$\Phi_1 = \frac{1}{2} \Phi_{11} + \Phi_{12} + \frac{1}{2} \Phi_{13} = 166.8^\circ . \quad (\text{A4.2})$$

A4.1.3 Calculation of maximum follower acceleration

From Sketch A4.2 the follower acceleration (or deceleration) is given by

$$w'' = \ddot{y} \left(\frac{\pi \Phi_{ij}}{180} \right)^2 / (\omega^2 H_{ij}) . \quad (\text{A4.3})$$

From the cam law the maximum acceleration occurs when $u_{11} = 0.125$ and so

$$\left(\frac{4\pi^2}{\pi + 4}\right) \cos[4\pi(u - 1/8)/3] = \ddot{y} \left(\frac{191 \times \pi}{180}\right)^2 / [(15 \times 2\pi)^2 \times 0.0306],$$

or $\ddot{y} = 135.26 \text{ m/s}^2.$ (A4.4)

Similarly, the maximum deceleration occurs when $u_{13} = 0.875$, thus

$$\left(\frac{-4\pi^2}{\pi + 4}\right) \cos[4\pi(u - 7/8)] = \ddot{y} \left(\frac{124.8 \times \pi}{180}\right)^2 / [(15 \times 2\pi)^2 \times 0.02],$$

or $\ddot{y} = -207.07 \text{ m/s}^2.$ (A4.5)

A4.2 Dwell Segments

The angular rotation of the outer dwell that follows the rise segment is

$$\Phi_2 = 0.003 \times 15 \times 360 = 16.2^\circ. \quad (\text{A4.6})$$

Increasing this rotation to 18° will allow the cam speed to be increased by more than 10% without changing the segment angles determined directly from the timing.

The angular rotation of the inner dwell that follows the return segment is

$$\Phi_4 = 0.006 \times 15 \times 360 = 32.4^\circ. \quad (\text{A4.7})$$

Similarly, if this rotation is increased to 36° , the cam operating speed can be increased by more than 10% without changing the segment angles.

A4.3 Return Segment

The cam angular displacement for the return segment, Φ_3 , may be found from

$$\Phi_3 = 360 - (\Phi_1 + \Phi_2 + \Phi_4),$$

that is $\Phi_3 = 360 - (166.8^\circ + 18^\circ + 36^\circ) = 139.2^\circ.$ (A4.8)

A4.3.1 Acceleration of follower

During the return segment, the follower moves from a position of maximum displacement to a position of zero displacement during a cam rotation of 139.2° using a Modified Sine Acceleration cam law. Reference to Sketch A4.2(b) shows that the maximum follower acceleration and deceleration have the same absolute value and occur when $u = 0.125$ and $u = 0.875$ respectively. Rearranging the acceleration equation

(Sketch A4.2(a)) gives

$$\ddot{y} = [w''\omega^2 H] / [(\Phi\pi/180)^2], \quad (\text{A4.9})$$

where $w'' = \left(\frac{4\pi^2}{\pi + 4}\right) \cos[4\pi(u - 1/8)/3] \quad \text{for } 0.125 \leq u < 0.875,$

and $w'' = \left(\frac{-4\pi^2}{\pi + 4}\right) \cos[4\pi(u - 7/8)] \quad \text{for } 0.875 \leq u \leq 1.0.$

Hence, at $u = 0.125$ and $u = 0.875$ the maximum accelerations are

$$|\ddot{y}_{\max}| = 5.53 \times [(15 \times 2\pi)^2 \times 0.0278] / (139.2 \times \pi/180)^2.$$

The results obtained for $|\ddot{y}_{\max}|$ are shown in the following table.

u	ϕ°	$\ddot{y} \text{ (m/s}^2\text{)}$
0.125	202.2°	− 231.36
0.875	306.6°	+ 231.36

A4.4 Timing Diagrams

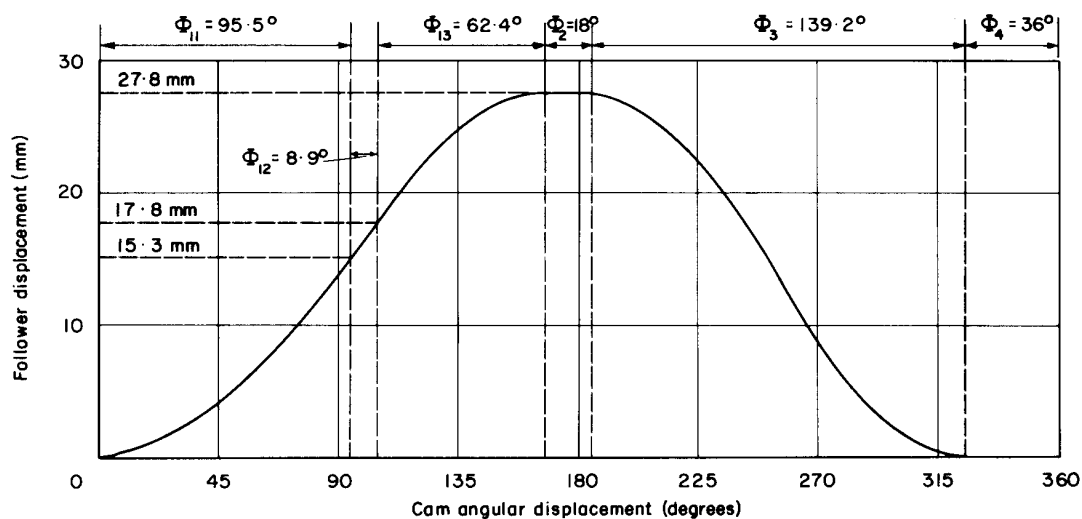
The follower displacement, velocity and acceleration diagrams are given in Sketches A4.3(a), A4.3(b) and A4.3(c) respectively. These clarify the stage reached in the design of the mechanism.

A4.5 Cam Reference Circle Dimensions

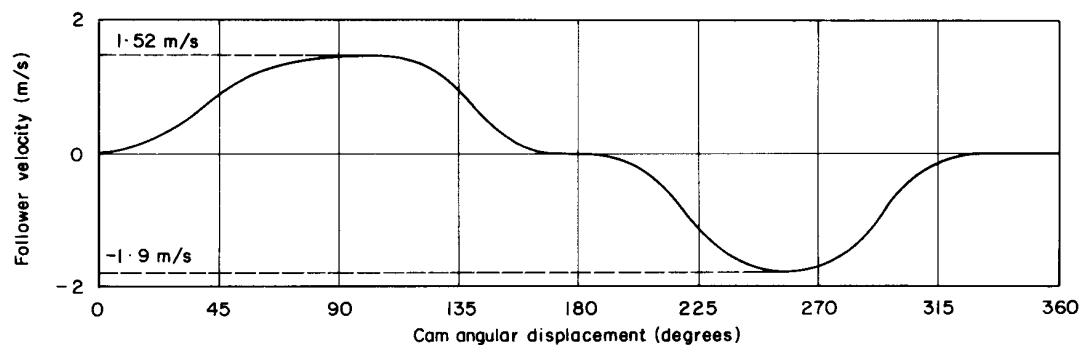
Before the profile co-ordinates of the cam and the performance characteristics of the mechanism can be determined, it is necessary to calculate the cam reference circle dimensions. From the method of instantaneous centres, Item No. 82023 (Reference 3), the distance between the cam centre of rotation and the instantaneous centre of rotation of the cam is given by

$$b = (w'_{\max} H_{ij} 180) / (\Phi_{ij} \pi), \quad (\text{A4.10})$$

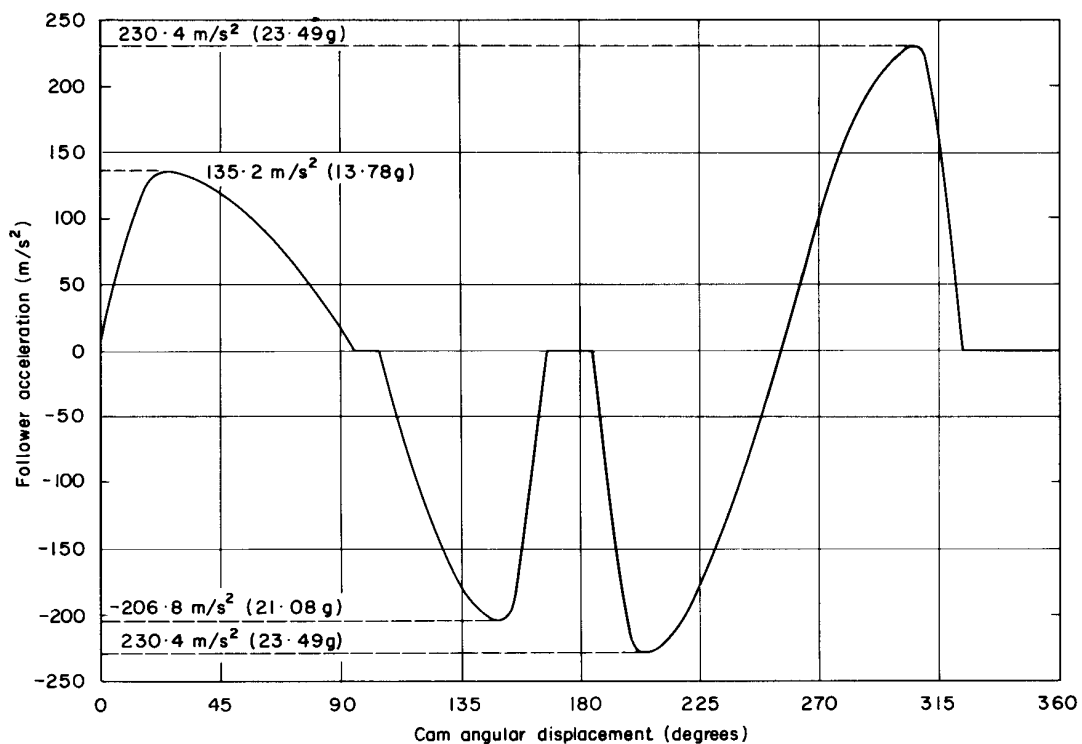
where $w'_{\max}$ is the normalised maximum follower velocity defined by the cam law for a particular segment or portion of the motion being considered.



(a) Displacement diagram



(b) Velocity diagram



(c) Acceleration diagram

Sketch A4.3 Timing diagrams

The dimension b needed to apply the method of instantaneous centres and so relate the reference circle radius to the limiting pressure angle is found for the portions of the rise and return motion segments having positive values of w'' . In the rise segment the distance b is found from the notional profile defined by $\Phi_{11} = 191^\circ$; $H_{11} = 30.6$ mm. This distance, b_1 , is given by

$$b_1 = (1.76 \times 30.6 \times 180) / (191 \times \pi) = 16.16 \text{ mm},$$

where w'_{max} has a value of 1.76 which has been obtained from Sketch A4.2(b). In the return segment the follower acceleration is positive when $0.5 \leq u_3 \leq 1.0$ and the distance b in this segment, b_3 , from Equation (A4.10), is

$$b_3 = (1.76 \times 27.8 \times 180) / (139.2 \times \pi) = 20.15 \text{ mm}.$$

Item No. 82023 recommends that it is good design practice to restrict the maximum pressure angle during these active periods to 30° . Higher values of pressure angle can cause excessive contact stresses and, in some cases, jamming of the mechanism.

The optimum follower offset necessary to minimise the contact stress is found from Item No. 82023 by

$$\text{offset} = w'_1 H_{ij} \left(\frac{180}{\pi \Phi_{ij}} \right). \quad (\text{A4.11})$$

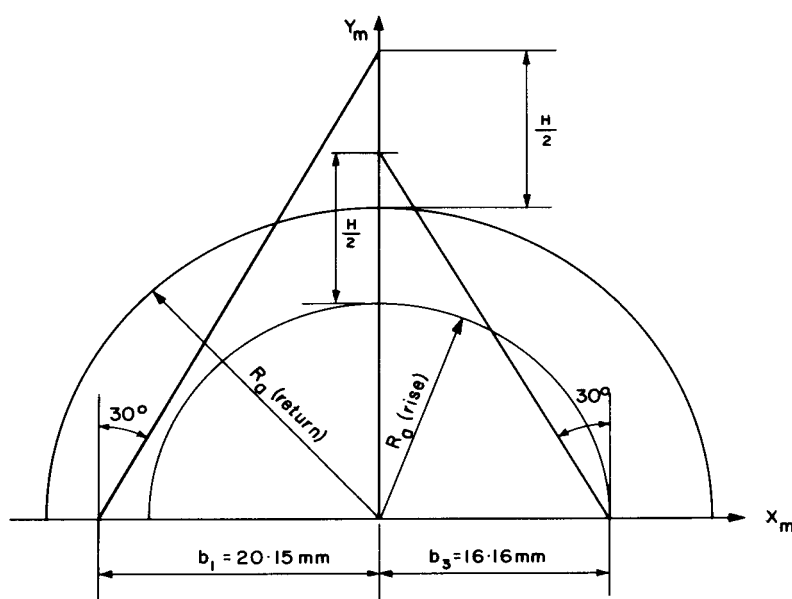
The maximum contact forces between the cam and follower during the rise and return segments occur at $u_{13} = 1.0$ and $u_3 = 0.0$. The non-dimensional velocity corresponding to these values of u is zero and hence, from Equation (A4.11), the optimum offset in the rise and return segments is zero.

A graphical construction giving an initial approximation for the prime circle radius for zero offset, based upon the values of b_1 and b_3 , where the pressure angle has, in each case, been limited to 30° , is given in Sketch A4.4.

This sketch shows that the prime circle radius in the return segment must be greater than in the rise segment in order to limit the pressure angle to a maximum of 30° throughout the motion. An initial estimate of the prime circle radius is

$$\begin{aligned} R_a &= [b_3 / \tan(\alpha_{max})] - (H/2) \\ &= [20.15 / \tan(30^\circ)] - (27.8/2) \\ &= 21 \text{ mm}. \end{aligned} \quad (\text{A4.12})$$

The maximum value of pressure angle is likely to occur near the position of maximum velocity in the return segment and so a range of values around this position can be used to check that the pressure angle does not exceed 30° .



Sketch A4.4 Graphical construction to determine the prime circle radius

The pressure angle, with zero offset, is found from Item No. 95001,

$$\alpha = \tan^{-1} \left[\left(\frac{180 H w'}{\Phi \pi} \right) / (y_{m0} + wH) \right]. \quad (\text{A4.13})$$

For the return segment the pressure angle is

$$\alpha = \tan^{-1} \left[\left(\frac{180 \times 27.8 \times w'}{139.2 \times \pi} \right) / (21 + 27.8 - 27.8 \times w) \right],$$

where the normalised displacement and velocity values, w and w' , are obtained from the cam law formulae of Sketch A4.2(a).

The maximum follower velocity occurs at a cam angle of approximately 255°. The pressure angle has been calculated for 5° increments of cam angle around this value and are given in the following table.

ϕ°	w	w'	α°
240.0	0.3222	-1.6377	-25.19
245.0	0.3824	-1.7072	-27.10
250.0	0.4445	-1.7481	-29.21
255.0	0.5076	-1.7594	-30.13
260.0	0.5705	-1.7409	-31.16
265.0	0.6323	-1.6931	-31.82
270.0	0.6918	-1.6168	-32.04
275.0	0.7481	-1.5140	-31.75
280.0	0.8003	-1.3870	-30.87
285.0	0.8475	-1.2387	-29.32
290.0	0.8891	-1.0723	-27.00
295.0	0.9244	-0.8911	-23.82
300.0	0.9530	-0.7002	-19.76

For the above values it can be seen that the pressure angle exceeds 30° , with a maximum value occurring at a cam angle of approximately 270° .

The reference circle radius can be adjusted to satisfy the condition for pressure angle by re-arranging Equation (A4.13), which becomes

$$y_{m0} = \left[\left(\frac{180 H w'}{\Phi_3 \pi} \right) / \tan(30^\circ) \right] - w H . \quad (\text{A4.14})$$

Substituting the values of H , Φ_3 and w' at $\phi = 270^\circ$ gives

$$y_{m0} = 51.27 \text{ mm} .$$

Consequently the prime circle radius is obtained from

$$R_a = y_{m0} - H_3 = 51.27 - 27.8 = 23.47 \text{ mm} .$$

Rounding up to 24 mm for the succeeding calculations, the reference circle radius, R_b , is

$$R_b = R_a - R_r = 24 - 8 = 16 \text{ mm} . \quad (\text{A4.15})$$

A4.6 Cam and Follower Characteristics

A4.6.1 Follower displacement, velocity and acceleration calculations

Using the displacement, velocity and acceleration equations for the Modified Sine Acceleration cam law, illustrated in Sketch A4.2(a), the motion characteristics of the cam and follower can be calculated in both dimensional and non-dimensional form. These calculated results are given in Table A7.1, in 5° steps, throughout one cycle of cam rotation using a reference circle radius of 16 mm.

A4.6.2 Calculation of pressure angle

Values for the pressure angle, from Equation (A4.13), are given in Table A7.2, in 5° steps, throughout the complete cycle of cam rotation, again using a reference circle radius of 16 mm. These results show that the pressure angle does not exceed 30° in any segment of the cam motion. Because of the relatively coarse increment in the pressure angle calculations this has not been proven absolutely but it will be assumed that a rigorous check proves the pressure angle to be acceptable in all segments.

A4.6.3 Calculation of the pitch curve radius of curvature

The pitch curve radius of curvature, ρ , and the cam profile radius of curvature, ρ_p , are found from the equations given in Item No. 95001 (Reference 16),

$$\rho = y_m^2 / \left(a A \cos^2 \alpha + y_m \cos \alpha \right) , \quad (\text{A4.16})$$

$$\text{where} \quad a = - \frac{A}{H} w'' \cos \alpha + w' \sin \alpha ,$$

$$\text{and} \quad \rho_p = \rho - R_r . \quad (\text{A4.17})$$

The calculated results of these variables are given in Table A7.2, in 5° steps, throughout one complete cycle of cam rotation.

A4.6.4 Profile interference

Item No. 82023 (Reference 3) states that profile interference can be avoided if the radius of the roller follower is within the range,

$$0 < R_r < \rho_{min}, \quad \text{where } \rho > 0.$$

Because the minimum pitch curve radius of the cam, ρ_{min} , given in Table A7.2, is greater than the roller radius, profile interference is avoided.

A4.7 Spring Characteristics

A4.7.1 Spring forces and dimensions

The principal components of force that act upon the follower are:

- (a) inertia force due to the acceleration (or deceleration) of the mass of the following system (including any component carried by the follower),
- (b) mass of the following system (normal practice is to include 1/3 of the spring mass),
- (c) frictional resistance,
- (d) force due to spring deflection,
- (e) force due to spring preload,
- (f) external work done.

The sum of these forces must always act towards the cam in order that the follower remains in contact with the cam.

In this example, the inertia force is dominant and hence the minimum spring force, $(S_f)_{min}$, needed to maintain contact between the cam and follower, is determined at positions of both the maximum follower deceleration in the rise segment, *i.e.* at a value of $u_{13} = 0.875$, and at the position of maximum follower acceleration in the return segment, *i.e.* when $u_3 = 0.125$.

At the position where u_{13} equals 0.875, the acceleration, $\ddot{y}$, is -207.07 m/s^2 and the minimum spring force is given by

$$(S_f)_{min} = 0.2 \times -207.07 = -41.4 \text{ N}.$$

When u_3 is equal to 0.125 and $\ddot{y}$ equals -231.36 m/s^2 , then

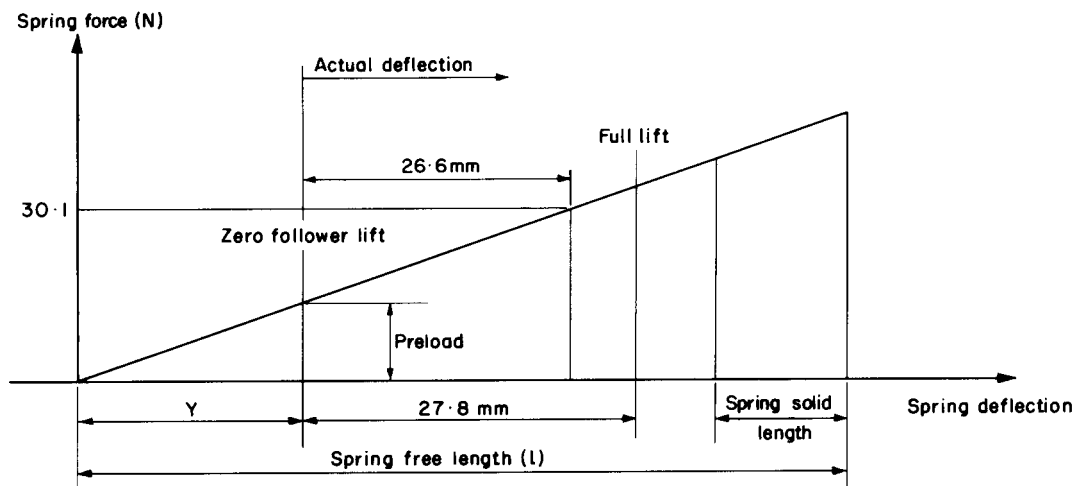
$$(S_f)_{min} = 0.2 \times -231.36 = -46.3 \text{ N}.$$

The negative signs in $(S_f)_{min}$ emphasise the direction of the force acting to separate the roller from the cam. The magnitude of the maximum spring force needed at the maximum follower acceleration, the highest

of the calculated values, is 46.3 N. Because other forces within the mechanism will act to reduce the effective spring force exerted on the follower, due particularly to friction, the actual spring force will need to be greater than this value. Initially a 30% increase in spring force will be used to overcome these additional forces. This will give a spring force of

$$1.3 \times 46.3 = 60.2 \text{ N.}$$

The mechanism uses two springs and so the force provided by each spring is required to be 30.1 N. An illustration of the spring characteristics is shown in Sketch A4.5, where Y is the initial spring displacement.



Sketch A4.5 Diagram illustrating the spring dimensions

Experience indicates (see References 24 to 27) that a suitable minimum clearance between each coil of 15% of the wire diameter, d , is required to prevent the possibility of some coils touching at minimum working length owing to irregular pitching. Additionally it is common practice to allow 3/4 of a coil at each end of a spring to allow for end coil formation. Using these factors and values from Sketch A4.5, the following equation may be formulated,

$$l - (27.8 + Y) - d \times 0.15 \times (n + 1.5) = (n + 1.5)d, \quad (\text{A4.18})$$

where n is the number of working coils.

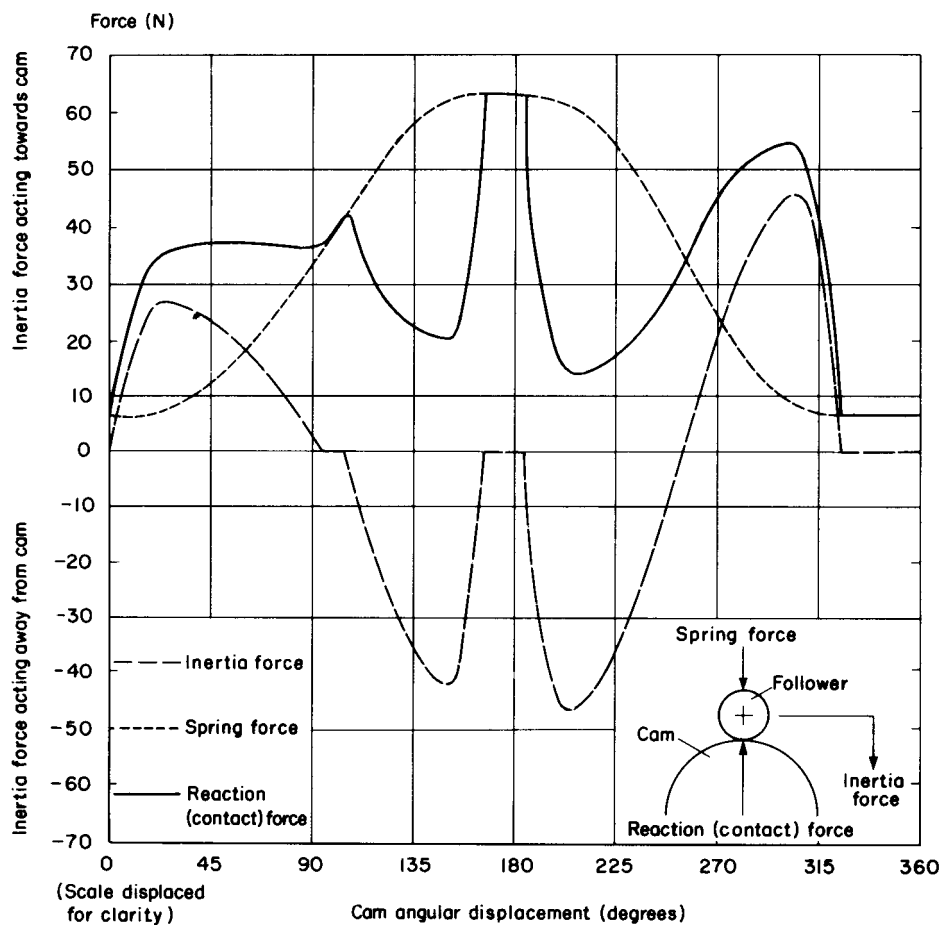
The spring rate, R , is given by

$$R = \frac{30.1}{Y + 26.6} \text{ N/mm.} \quad (\text{A4.19})$$

Choosing an initial spring displacement, Y , of 3 mm, a wire diameter, d , of 1.4 mm and a spring diameter, D , of 16 mm, the dimensions of a spring may be determined by the use of the charts and tables from Item No. 82008 (Reference 24). The following values have been calculated.

Description	Symbol	Value
spring diameter	D	16 mm
wire diameter	d	1.4 mm
spring index factor	c	11.43
number of active coils	n	10
spring rate	R	1.0169 N/mm
free length of spring	l	49.32 mm
length of spring at maximum load condition		19.72 mm
solid length of spring		16.1 mm
clearance between individual coils at maximum load		0.3148 mm
spring force at maximum follower lift	P_{max}	31.4 N
spring force at zero follower lift	P_{min}	3.05 N

Sketch A4.6 illustrates the inertia force of the follower, the reaction force between the spring and the follower and the resulting reaction (contact) force between the cam and follower throughout the cam rotation at normal operating speed.



Sketch A4.6 Comparison of the inertia force, the spring force and the reaction (contact) force

A4.7.2 Spring stress

The maximum spring static shear stress, q , may be determined using the charts of Item No. 82008, or by the equation

$$q = \frac{8 P_{max}}{\pi d^2} \times \frac{c(c + 0.2)}{(c - 1)} . \quad (\text{A4.20})$$

Substituting the above values into this equation gives,

$$\begin{aligned} q &= \frac{8 \times 31.2}{\pi \times 1.4^2} \times \frac{(11.43)[(11.43) + 0.2]}{[(11.43) - 1]} \\ &= 502.1 \text{ N/mm}^2 \quad (502.1 \text{ MN/m}^2) . \end{aligned}$$

The maximum allowable static shear stress for a carbon steel spring, which has been obtained from Figure 1 of Item No. 83002 (Reference 26), is in the range 1090 MN/m² to 1660 MN/m². It is good practice to work to a lower stress when, as in this case, the stress range is a large part of the maximum stress. Thus the margin between the working and allowable stresses is considered acceptable.

A4.7.3 Spring natural frequency

The natural frequency of a spring is given in Item No. 82008 (Reference 24), as

$$f = \frac{0.113}{D c n} \left(\frac{G}{\rho} \right)^{0.5} . \quad (\text{A4.21})$$

Assuming that the spring material has a modulus of rigidity, G , of $79 \times 10^9 \text{ N/m}^2$ and a density, ρ , of 7850 kg/m^3 , then

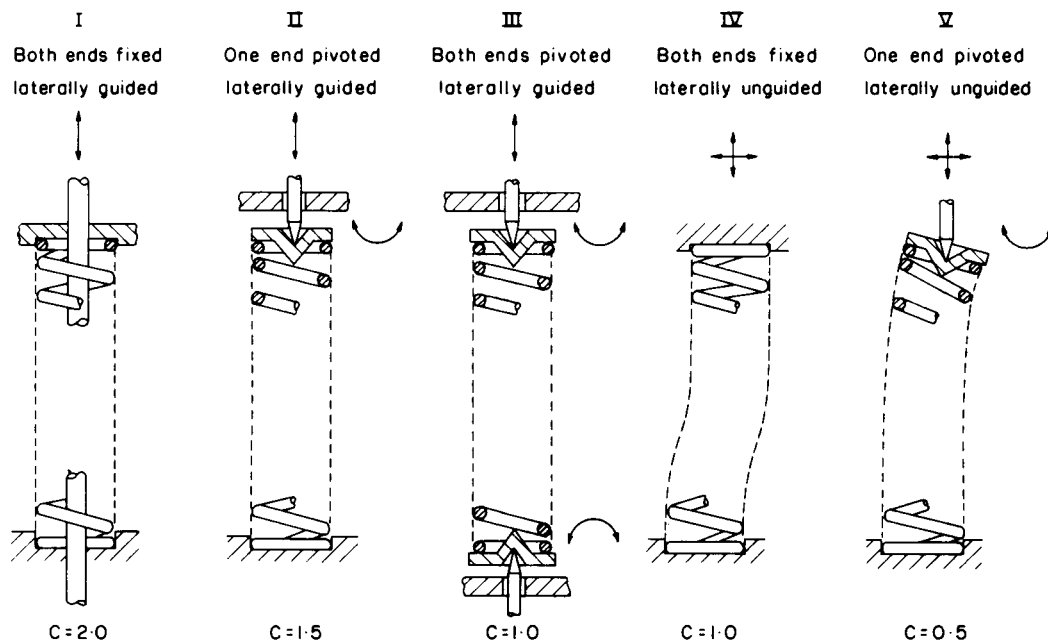
$$f = \left(\frac{0.113}{0.016 \times 11.43 \times 10} \right) \left(\frac{79 \times 10^9}{7850} \right)^{0.5} = 196 \text{ Hz} .$$

Spring surge will occur if the operating speed, multiplied by the order numbers of the follower lift harmonics, coincides with the spring natural frequency. If this occurs, the coils of the spring will oscillate along the axis of the spring. However, resonance within the spring may be considered negligible above about the 10th or 11th harmonic. Since the cam operating speed, ω , is 15 Hz, the lowest harmonic at which resonance will occur is greater than 11 and therefore spring stresses due to surge will be negligible.

A4.7.4 "Buckling instability" of the spring

The criterion for the "buckling instability" of a spring may be determined using Item No. 82008. For a spring with fixed ends the distance from the loading pivot to the plane spring seating, a , is zero, and in this case both ends of the spring are fixed and laterally guided.

An illustration of the end condition factor of the springs from Item No. 82008 is reproduced in Sketch A4.7. For the case considered in this example, the end condition factor, C , taken from the sketch, is 2. In practice C is normally reduced by 20 per cent and consequently C has been chosen to have a value of 1.6.



Sketch A4.7 Illustration of the end condition factor of the spring

The buckling characteristics of a spring are given, in Item No. 82008, as a function of Ca/l and CD/l . Springs having values of $Ca/l = 0$ and CD/l greater than about 0.4, will be stable for any deflection.

For the spring in this example,

$$\frac{CD}{l} = \frac{1.6 \times 16}{49.32} = 0.519$$

$$\text{and } \frac{Ca}{l} = \frac{1.6 \times 0}{49.32} = 0,$$

satisfying the buckling criteria.

A4.8 Roller Follower

The roller follower dimensions are given in the specification without indicating any required bearing life. Recommendations for life requirements and a procedure for calculating the size of bearing for a specified life, or the life for a specified size of bearing can be obtained from Item Nos. 81005, 81037 and 82014 (References 21, 22 and 23) or from bearing manufacturers' catalogues. These aspects will not be considered in this example.