

DESIGN FOR MINIMUM WEIGHT. STRUTS OF UNIFORM SECTION. THIN-WALLED ROUND TUBES

1. INTRODUCTION

This Item provides data that allow the dimensions of thin-walled round tubular struts of minimum weight to be determined. They can be determined for a chosen probability that overall strut buckling will occur before local buckling with a particular degree of confidence. Alternatively, the Item allows the buckling loads to be determined for struts of minimum weight that have particular constraints on their geometry. The data relate to the elastic buckling of struts with uniform wall thickness manufactured from isotropic homogeneous material.

2. NOTATION

c	confidence level, per cent		
D	outside diameter of section of strut	m	in
E	modulus of elasticity of strut material (in compression)	N/m ²	lbf/in ²
f_e	overall (Euler) buckling stress of strut (see Section 4)	N/m ²	lbf/in ²
f_b	elastic buckling stress	N/m ²	lbf/in ²
f_{bt}	theoretical elastic local buckling stress for a geometrically perfect cylinder loaded in axial compression (see Section 4)	N/m ²	lbf/in ²
K	coefficient in equation for f_{bt} (see Section 4)		
L	length of strut	m	in
l'	equivalent length of strut allowing for end conditions (see Table 3.1)	m	in
P	compressive end load on minimum weight strut at failure	N	lbf
R	mean radius of section of strut = $\frac{1}{2}(D - t)$	m	in
S	probability that δ/t will not be exceeded by population, per cent		
t	thickness of tube wall	m	in
δ	maximum inward initial deviation of mid-line of unloaded tube wall from straight axial mid-line generator of tube wall parallel to strut axis (see Data Item No. 83034*)	m	in
ν	Poisson's ratio of strut material		
ρ	factor applicable to mean of sample to estimate δ/t for values of S_{pc} (see Section 4)		

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Suffixes

c	relates to confidence level
p	relates to the percentage population below the level indicated
S	relates to cumulative probability, S_{pc}

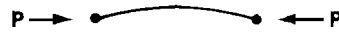
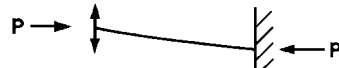
Both SI and British units are quoted but any coherent system of units may be used.

* Data Item No. 83034 "Elastic local buckling stresses of thin-walled unstiffened circular cylinders under combined axial compression and internal pressure".

3. NOTES

In Figure 1 values of the statistical factor ρ_{Sc} are plotted against S_p for various confidence levels, c . In Figures 2 to 9 values of the dimensions (R and t) for least weight are plotted as R/L and t/R , respectively, against P/EL^2 for various values of the statistical factor, ρ_{Sc} . The data are grouped for particular values of the ratio of the equivalent strut length to the actual strut length, chosen to represent the strut end conditions as shown in Table 3.1.

TABLE 3.1 Ratio l'/L for Various End Conditions

End Condition	l'/L	Figure No.		End Condition
		R/L	t/R	
Both ends fixed	0.5	2	3	
One end fixed and one end pinned	0.7	4	5	
Both ends pinned or ----- both ends fixed in rotation and one end free to translate	1.0	6	7	 
One end fixed and one end free	2.0	8	9	

A minimum weight strut is theoretically defined as one with dimensions such that overall (Euler) and local buckling occur simultaneously under load P . The equations used to produce the data in this Item are based on this assumption and are given in Section 4. The factors ρ_{Sc} , given in Figure 1, allow consideration to be given to the probable effect of initial wall irregularities, δ , of the tube on the local buckling aspect of the optimisation. The factor ρ_{Sc} to be used with Figures 2 to 9 depends on the selection of the level of irregularities in the population, S_p , and the chosen confidence level, c . Figures 2 to 9 are grouped in pairs for particular values of l'/L . The minimum weight strut geometry in terms of R/L and t/R for a particular value of P/EL^2 and l'/L can be obtained for the selected value of ρ_{Sc} . Alternatively, if R/L is specified, the thickness and buckling load of an optimum strut can be determined for l'/L and the chosen value of ρ_{Sc} .

The local buckling aspect of the optimisation considered here is based on the data given in Data Item No. 83034. The full implications of the statistical analysis of the local buckling are detailed in that Item. It follows from the statistical analysis and the assumptions of the optimisation, that the higher the level of probability (and confidence) chosen, the more likely it is that overall buckling of the strut will occur before local buckling.

The data given in this Item relate strictly to elastic local and overall buckling of long thin-walled round tubular struts. While the effect of initial wall irregularities on the local buckling is accounted for, the effect on the overall buckling of eccentric loading and bow is not. The geometry for a minimum weight strut obtained using this Item is unlikely to be a standard size tube. When selecting the appropriate standard size it is recommended that the diameter be reduced to the next standard size below the optimum and that the strut then be analysed using Data Item No. Struct. 01.01.01* increasing the thickness by standard size increments until the level of the applied load is reached (see Example 5.1).

The data in this Item have been calculated for $\nu = 0.3$ and a value of $K = 0.85$. This value of K relates to the local buckling of the tube wall when its ends are both locally simply-supported at the end attachments of the strut. Data Item No. 83034 indicates that K will vary with the geometry and local fixture of the wall ends of the tube. For struts with the wall ends clamped a value of $K = 0.92$ is suggested in Data Item No. 83034. This value has a negligible effect on the values of R/L given here but will reduce the values of t/R given by approximately 4 per cent for particular values of P/EL^2 and l'/L . These factors are valid for $L^2/(Rt) \geq 50$. Item No. 83034 should be consulted for further guidance on the appropriate value of K . Small variations in the value of K may be accommodated by multiplying the values of t/R in the figures by the approximate factor $(0.85/K)^{1/2}$.

4. FORMULATION FOR LEAST WEIGHT GEOMETRY

The dimensions of the minimum weight strut are obtained by equating the overall (Euler) buckling stress, f_e , and the local buckling stress, f_b . The local buckling stress used in this equation is obtained from Data Item No. 83034 which considers the effect of small inward initial irregularities of the tube wall, δ . The following equations were used to obtain the curves of Figures 2 to 9.

Overall buckling stress,

$$f_e = \frac{\pi^2 ER^2}{2l'^2} \quad (4.1)$$

* Data Item No. Struct. 01.01.01 "The strength of struts".

Relationship between initial inward irregularities and local buckling stress

$$\frac{\delta}{t} = \frac{2(1 - f_b/f_{bt})^2}{3[3(1 - \nu^2)]^{1/2}(f_b/f_{bt})}, \quad (4.2)$$

where the theoretical buckling stress of a perfect cylinder,

$$f_{bt} = \frac{KE}{[3(1 - \nu^2)]^{1/2}} \left(\frac{t}{R} \right). \quad (4.3)$$

The empirical factor K in Equation (4.3) has been assumed to be 0.85 when calculating the curves of Figures 2 to 9 (see Section 3).

Statistical analysis of initial inward irregularities

$$\frac{\delta}{t} = \rho_{Sc} 0.04 \left[\frac{RL}{t^2} \right]^{0.18}. \quad (4.4)$$

Equations (4.2), (4.3) and (4.4) are taken from Data Item No. 83034. The data on which that Item and the equations are based are derived from tests on very short cylinders, $L/R \leq 6.0$. The combination of these equations with Equation (4.1), for long struts, is justified by the fact that local buckling is a short wavelength phenomenon and that it may be assumed that no interaction occurs between long wavelength (overall) buckling and short wavelength (local) buckling.

5. EXAMPLES

5.1 Example 1

It is required to determine an appropriate diameter and thickness for a round tubular strut such that the strut weight is a minimum. The strut has one end fixed and the other pinned and is subjected to a load of 10 500 lbf. It is necessary to be sure that the strut will buckle overall before local buckling can occur, with probability of 95 per cent at a confidence of 99.9 per cent. The strut is to be 54 in long and manufactured from aluminium alloy with a modulus of elasticity of 10.4×10^6 lbf/in², a 0.1 per cent proof stress of 54.1×10^3 lbf/in² and a 0.2 per cent proof stress of 56.4×10^3 lbf/in².

From the data given

$$\frac{P}{EL^2} = \frac{10\,500}{10.4 \times 10^6 \times 54^2} = 3.46 \times 10^{-7},$$

$$S_p = 95 \text{ per cent}$$

$$\text{and } c = 99.9 \text{ per cent.}$$

From Figure 1, when $S_p = 95$ per cent and $c = 99.9$ per cent,

$$\rho_{Sc} = 4.55.$$

From Table 3.1, for a strut with one end fixed and one end pinned,

$$\frac{l'}{L} = 0.7.$$

Therefore, Figure 4 is used to obtain the radius and Figure 5 the thickness of the minimum weight strut.

From Figure 4, by interpolation, when $P/EL^2 = 3.46 \times 10^{-7}$ and $\rho_{Sc} = 4.55$

$$\frac{R}{L} = 0.02 ,$$

hence, $R = 0.02 \times 54 = 1.08 \text{ in} .$

From Figure 5, by interpolation, when $P/EL^2 = 3.46 \times 10^{-7}$ and $\rho_{Sc} = 4.55$,

$$\frac{t}{R} = 0.0365 ,$$

hence, $t = 0.0365 \times 1.08 = 0.0394 \text{ in} .$

This assumes that the ends of the tube walls are free to rotate at the end fittings, that is, $K = 0.85$. If the ends of the tube walls are fixed t will be reduced by approximately 4 per cent for which $t = 0.0378 \text{ in}$ (see Section 3).

It is now necessary to check that the local buckling requirement that $L^2/(Rt) \geq 50$ is satisfied.

From the data obtained from Figures 4 and 5,

$$\frac{L^2}{Rt} = \frac{54^2}{1.08 \times 0.0394} = 68\,500 .$$

The requirement is therefore satisfied.

The calculations show that in this case a strut design with the dimensions $D = 2.2 \text{ in}$ and $t = 0.0394 \text{ in}$ will have a 95 per cent probability of failing in overall buckling before local wall buckling can occur. These dimensions are, however, non-standard and a standard section strut with a radius R less than 1.08 in and a thickness greater than 0.0394 in will be chosen. In addition it is necessary to check that the form of stress-strain curve for the material and the strut eccentricity do not cause overall buckling at a lower value than $P = 10\,500 \text{ lbf}$.

Data Items Nos Struct. 01.01.01 and 76016* are used to obtain the overall buckling load of a strut. The notation defined in those Items is used where necessary in the following calculations.

The material characteristic m and the stress f_n required for use with Data Item No. Struct. 01.01.01 are given by Equations (4.1) and (4.2) respectively of Data Item No. 76016.

Hence,
$$m = \frac{\log(\epsilon_R/\epsilon'_R)}{\log(f_R/f'_R)} .$$

Here
$$m = \frac{\log(0.002/0.001)}{\log(56\,400/54\,100)} = 16.6 .$$

Also,
$$f_n = f_R \left[\frac{m\epsilon_R E}{f_R} \right]^{-1/(m-1)} .$$

Here
$$f_n = 56\,400 \left[\frac{16.6 \times 0.002 \times 10.4 \times 10^6}{56\,400} \right]^{-1/(16.6-1)} = 50\,200 \text{ lbf/in}^2 .$$

* Data Item No. 76016 "Generalisation of smooth continuous stress-strain curves for metallic materials".

The stress f_{all} required for use with Data Item No. Struct. 01.01.01 is assumed here to be the 0.2 per cent proof stress in accordance with the guidance given in that Item, since the form factor for a round tube will be relatively low and the 0.2 per cent proof stress will therefore be conservative.

Therefore
$$\frac{f_{all}}{f_n} = \frac{56\ 400}{50\ 200} = 1.12 .$$

Standard tube sections are now selected for analysis on the basis of increasing the thickness from $t = 0.394$ in. It will be seen that the chosen diameter is reduced to 2.0 in.

Standard t	Maximum D	Chosen Standard D	Equivalent R
0.048 in (18 SWG)	2.21 in	2.0 in	0.976 in
0.064 in (16 SWG)	2.22 in	2.0 in	0.968 in

Since no strut overall eccentricity has been specified the guidelines given in Data Item No. Struct. 01.01.01 will be used. That Item states that the eccentricity may be assumed to be the greater of $0.05c$ and $0.001l$ (where $l = L$ in this Item).

For a round tube

$$c = \frac{k^2}{y} \text{ and } k = \left(\frac{\pi R^3 t}{2\pi R t} \right)^{1/2} = \left(\frac{R^2}{2} \right)^{1/2} .$$

Hence, for $D = 2.0$ in and $y = 1.0$,

t (in)	R (in)	k^2 (in ²)	k (in)	c (in)	$0.05c$ (in)	$0.001L$ (in)
0.048	0.976	0.476	0.690	0.476	0.0238	0.054
0.064	0.968	0.469	0.685	0.469	0.0235	0.054

Since in both cases $0.001L \geq 0.05c$ the value of $e = 0.054$.

The following parameters for use with the figures of Data Item No. Struct. 01.01.01 can now be given.

t (in)	e/c	$(l'/k)(f_n/E)^{1/2}$	f_{all}/f_n	m
0.048	0.113	3.81	1.12	16.6
0.064	0.115	3.83	1.12	16.6

From Figures 4b, 4c, 5b and 5c of that Item, by interpolation,

for $t = 0.048$ in , $f/f_n = 0.577$

and for $t = 0.064$ in , $f/f_n = 0.574$.