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Film thicknesses in lubricated Hertzian contacts (EHL). Part 1: two-dimensional contacts (line contacts)

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THE PREPARATION OF THIS DATA ITEM

The work on this particular Item was monitored and guided by the Working Party on Elastohydrodynamic Lubrication which has the following membership:

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FILM THICKNESSES IN LUBRICATED HERTZIAN CONTACTS (EHL). PART 1: TWO-DIMENSIONAL CONTACTS (LINE CONTACTS)

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FILM THICKNESSES IN LUBRICATED HERTZIAN CONTACTS (EHL). PART 1: TWO-DIMENSIONAL CONTACTS (LINE CONTACTS)

1. NOTATION AND UNITS

The International System of Units (SI) is used in this Item. This is a coherent system. It should be noted that any other coherent system of units can be used in place of SI where the data are given in non-dimensional form.

A	viscosity parameter = $\left(\frac{\alpha^2 W_s^3}{\eta_0 u R^2} \right)^{1/2}$	–
B	elasticity parameter = $\left(\frac{W_s^2}{\eta_0 u E' R} \right)^{1/2}$	–
E_1, E_2	moduli of elasticity for materials at surfaces in contact	N m ⁻²
E'	effective modulus defined by $\frac{1}{E'} = \frac{1}{2} \left(\frac{(1 - \nu_1^2)}{E_1} + \frac{(1 - \nu_2^2)}{E_2} \right)$	N m ⁻²
h	minimum lubricant film thickness	m
$\bar{h}$	non-dimensional lubricant film thickness = $\frac{h W_s}{\eta_0 u R}$	–
k	thermal conductivity of lubricant	W m ⁻¹ K ⁻¹
p	pressure of lubricant at contact	N m ⁻²
R_1, R_2	radii of curvature of each of two bodies at contact point, taken as positive for convex surfaces and negative for concave surfaces	m
R	effective radius of curvature at contact, defined by $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ (see Section 4.1)	m
R_{q1}, R_{q2}	RMS (root mean square) roughness of contacting surfaces	m
R_{qt}	combined surface roughness of mating surfaces = $(R_{q1}^2 + R_{q2}^2)^{1/2}$	m
u_1, u_2	velocities of surfaces relative to contact point	m s ⁻¹
u'_1, u'_2	velocities of surfaces tangential to surfaces	m s ⁻¹

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u	lubricant entraining velocity = $(u_1 + u_2)/2$ (see Section 4.2)	m s^{-1}
u'_c	velocity of contact tangential to surfaces	m s^{-1}
v	linear velocity of body	m s^{-1}
W_s	load applied per unit width of contact (see Section 4.3)	N m^{-1}
X	parameter for estimating effect of inlet shear heating (see Section 5.1.2)	—
y	perpendicular distance from contact tangent to centre of rotation of body	m
α	pressure exponent of lubricant viscosity	$\text{m}^2 \text{N}^{-1}$
γ	angle between velocity of body and contact tangent	rad
η	dynamic viscosity	N s m^{-2}
η_0	lubricant dynamic viscosity at atmospheric pressure and at operating temperature (see Section 4.5)	N s m^{-2}
θ	absolute temperature	K
λ	specific film thickness (see Section 6)	—
ν_1, ν_2	Poisson's ratio for materials at surfaces in contact	—
ρ_0	density of lubricant at atmospheric pressure and at operating temperature	kg m^{-3}
ω	angular velocity of body	rad s^{-1}

2. INTRODUCTION

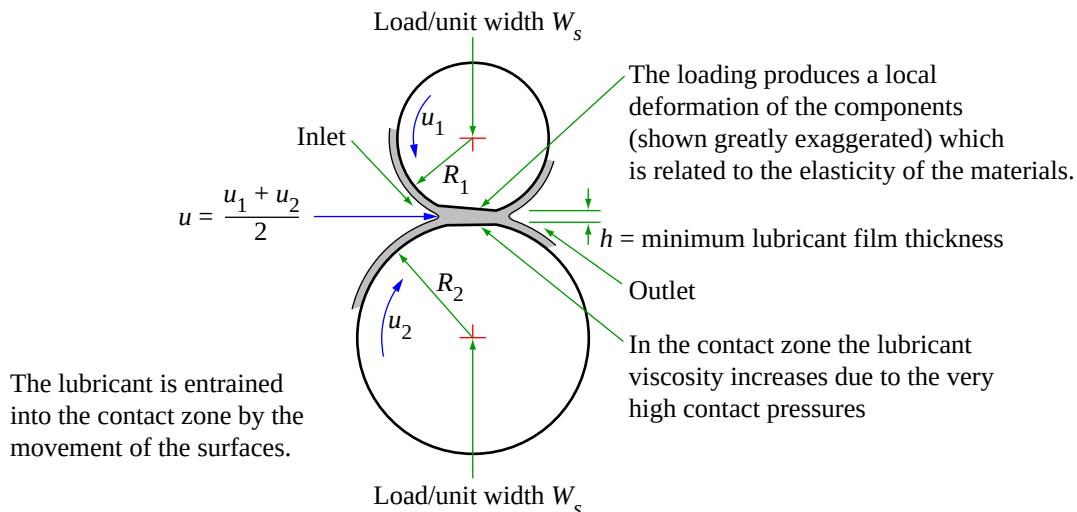
This Item provides charts and calculation methods for determining the minimum lubricant film thickness in concentrated contacts that occur, for example, in rolling bearings and gears; in such contacts, often called Hertzian contacts, the nominal contact area increases with applied load.

For the purpose of calculating the lubricant film thickness, Hertzian contacts are regarded as either two-dimensional, with nominally line contact, or three-dimensional, with nominally point contact. Nominally line contact occurs in, for example, roller bearings, roller followers on cams and spur gears while three-dimensional contacts occur in, for example, ball bearings. The charts and calculation methods presented in this Item apply to two-dimensional contacts. It is frequently possible to obtain guidance on some three-dimensional contacts using these methods (see Section 4).

The text of this Item is divided into three main parts. The first part (Section 3) describes the operation of a lubricated Hertzian contact. The second part (Sections 4 and 5) gives guidance on the specification of the design parameters, identification of the regime of operation and determination of the film thickness. The final part gives guidance on ensuring that the contact will operate with an adequate margin of safety.

3. DESCRIPTION OF CONTACT PHENOMENA

The formation of a lubricated Hertzian contact depends upon a viscous lubricant being drawn into the contact zone by the movement of the surfaces as shown in Sketch 3.1. In a typical contact the fluid pressures generated within the entrained lubricant can be a thousand times greater than those occurring in hydrodynamic journal and thrust bearings and, as a consequence of these high pressures, changes in the viscosity of the lubricant and elastic deformation of the components in contact may both be highly significant.



Sketch 3.1 A typical lubricated Hertzian contact

Normally, the lubricant film is of a uniform thickness over most of the contact but with a minimum thickness (typically 80 per cent of the uniform thickness) near the outlet of the contact zone. A knowledge of this film thickness, which can be two orders of magnitude smaller than those occurring in hydrodynamic journal and thrust bearings, is required in order to assess the likelihood of wear and surface distress.

The procedure to be followed when analysing a two-dimensional contact is shown in the flowchart of Figure 1.

4. SPECIFICATION OF PROBLEM

Certain basic information regarding the intended application is required for the design and analysis of a two-dimensional concentrated contact.

In many practical concentrated contacts the surfaces interact not at a line contact but at an elliptical contact. Provided the major axis of this elliptical contact is perpendicular to the direction of motion of the surfaces and is much larger than the minor axis, an approximate indication of the lubricant film thickness may be obtained by treating the contacting surfaces as cylindrical and applying the two-dimensional contact analysis method of this Item. The method should not be applied when the entraining direction is along the major axis of the contact ellipse.

4.1 Effective Radius of Curvature

The radius of curvature of the two cylindrical surfaces of the contacting components at the contact point must be specified. Note that a concave surface will have a negative radius of curvature. After these values have been specified, the effective radius of curvature at the contact, R , should be calculated using

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}. \quad (4.1)$$

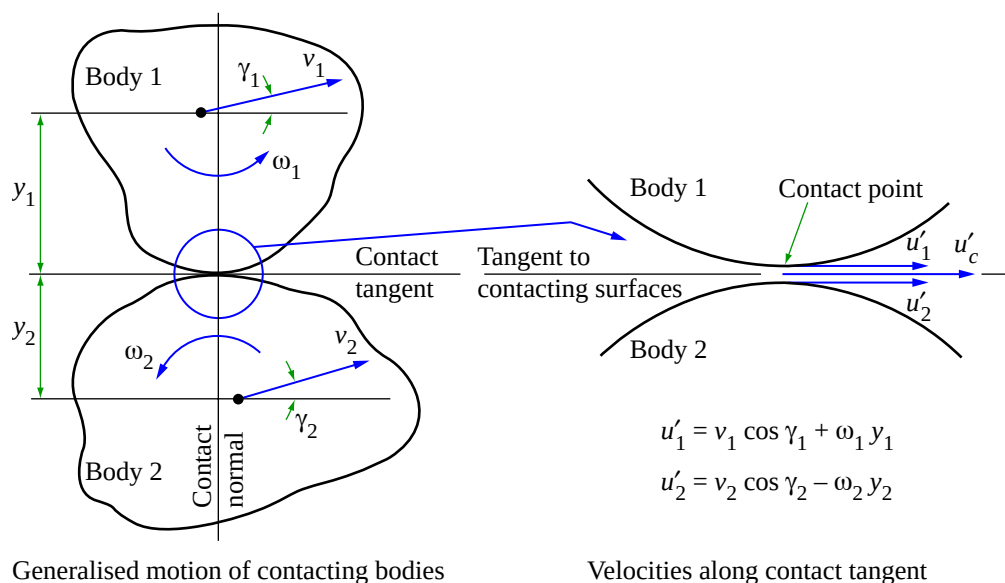
Note that where a roller contacts a flat surface the effective radius is simply equal to the radius of the roller.

4.2 Lubricant Entraining Velocity

The velocities of the two contacting surfaces, relative to the contact point, must be specified. The lubricant entraining velocity, u , is then equal to the arithmetic mean of these two values, or

$$u = \left| \frac{u_1 + u_2}{2} \right|. \quad (4.2)$$

The velocities of the two contacting surfaces should be specified in the direction of the contact tangent as illustrated in Sketch 4.1.



Sketch 4.1 Resolution of surface velocities

Care is required where the contact point itself is moving. In such cases it is often convenient to choose a co-ordinate system in which the contact point is stationary (see Examples 1 and 3 in Sections 9.1 and 9.3). Where this is not convenient, the velocity component of the contact point along the tangent, u'_c , should be specified (see Example 2 in Section 9.2). The velocities of the two contacting surfaces relative to the contact point are then given by,

$$u_1 = u'_1 - u'_c$$

and

$$u_2 = u'_2 - u'_c.$$

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Example 2 of Section 9.2 illustrates the specification of velocities and the calculation of the entraining velocity for an application where the contact point moves.

Note that in pure rolling motion (motion without sliding) $u_1' = u_2'$. Sliding has little direct effect on the film thickness.

4.3 Load

For two-dimensional contacts, the load per unit width of the contact, W_s , (measured perpendicular to the direction of motion) must be specified. Where a high aspect ratio elliptical contact is being analysed, an equivalent W_s should be established based on 70 per cent of the length of the major axis of the ellipse.

4.4 Effective Modulus

The moduli of elasticity, E_1 and E_2 , and the Poisson's ratios, ν_1 and ν_2 , for the contacting materials must be specified (see Reference 1). The effective modulus for the contact can be calculated using

$$\frac{1}{E'} = \frac{1}{2} \left(\frac{(1 - \nu_1^2)}{E_1} + \frac{(1 - \nu_2^2)}{E_2} \right). \quad (4.3)$$

Note that where similar materials contact, the effective modulus is simply

$$E' = \frac{E}{1 - \nu^2}. \quad (4.4)$$

Values of the effective modulus, $E/(1 - \nu^2)$, for typical materials are given in Table 10.1. Also given in this table are values of $(1 - \nu^2)/E$ for substitution into Equation (4.3).

4.5 Lubricant Properties

The lubricant is sometimes specified to suit the design requirements of another component. Where the designer has a free choice of lubricant, film thickness considerations may determine the choice.

For the purposes of analysing the performance of the elastohydrodynamic contact it is necessary to specify the viscosity of the lubricant at the entry to the contact, η_0 , and the pressure exponent, α , of the lubricant[†]. The viscosity, η_0 , which can be determined from a viscosity/ temperature curve for the intended lubricant, needs to be carefully considered. It should be determined for a pressure equal to the ambient pressure (not the pressure of the lubricant within the contact) and a temperature representative of the expected operating surface temperature of the contacting parts. In some high speed applications the surface temperature can be appreciably higher than the bulk lubricant temperature and in these applications an allowance should be made for this difference. Values of η_0 for some lubricants are given in Table 10.2.

For many lubricants the pressure exponent of viscosity does not vary greatly with temperature and pressure for a given lubricant. However, oils are proprietary materials and the manufacturers often cannot specify a value of α nor guarantee to hold it constant. Values of α for some lubricants are given in Table 10.2.

[†] The pressure exponent of lubricant viscosity is defined by the expression $\eta = \eta_0 \exp(\alpha p)$.

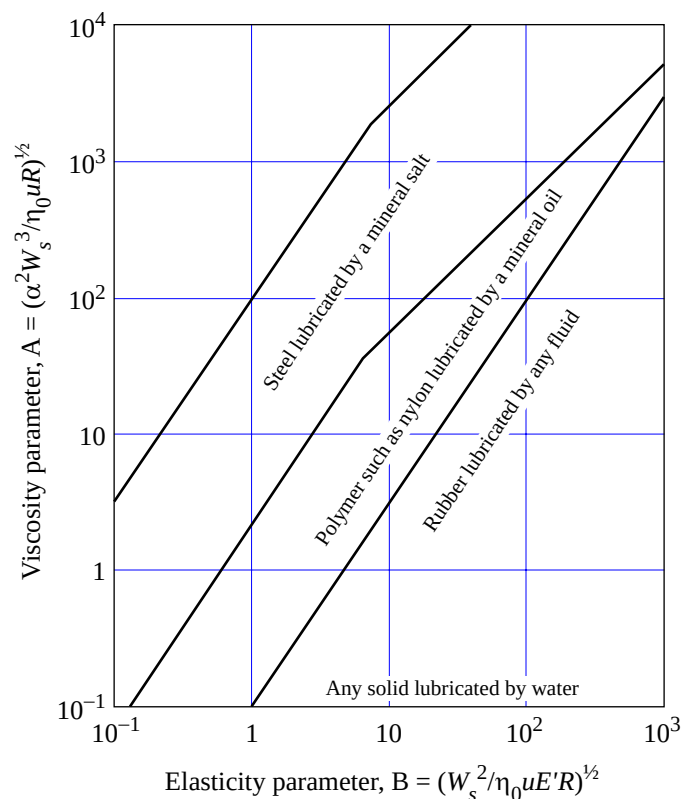
5. DETERMINATION OF LUBRICANT FILM THICKNESS

In order to determine the lubricant film thickness it is first necessary to calculate the values of the viscosity parameter[†], A, and the elasticity parameter[†], B.

$$A = \left(\frac{\alpha^2 W_s^3}{\eta_0 u R^2} \right)^{1/2} \quad (5.1)$$

$$B = \left(\frac{W_s^2}{\eta_0 u E' R} \right)^{1/2} \quad (5.2)$$

The possible values of these two parameters are limited by the practical range of the operating conditions specified in Section 4 and values of A and B falling outside the regions indicated on Sketch 5.1 are extremely rare.



Sketch 5.1 Practical operating regions

Film thickness data are presented in Figure 2 as non-dimensional film thickness, $\bar{h}$, contours on rectangular axes of the viscosity parameter, A, and the elasticity parameter, B. The value of $\bar{h}$ corresponding to the calculated values of A and B should be determined and the actual film thickness calculated using Equation (5.3).

[†] The viscosity parameter, A, and the elasticity parameter, B, are non-dimensional groups used to characterise a lubricated Hertzian contact. Other pairs of non-dimensional groups may be used to characterise the contact (see Derivation 12) but A and B (often referred to as g_1 and g_3 in the literature) have been chosen as the pair best suited to the presentation of film thickness data in this Item.

$$h = \frac{\bar{h}\eta_0 u R}{W_s}. \quad (5.3)$$

The adequacy of this calculated value of film thickness should be confirmed using the guidance given in Section 6. Where its value is considered inadequate it may be necessary to modify the contact (where this is possible) using the guidance given in Section 7.

5.1 Further Factors Influencing the Film Thickness

The lubricant film thickness estimate given by this Item assumes Newtonian lubricants, that is lubricants where viscosity is independent of shear rate, and isothermal conditions, where no significant inlet shear heating occurs. Significant reduction of the film thickness may occur where these two assumptions are not satisfied.

5.1.1 Non-Newtonian lubricants

In a typical contact the lubricant will be subjected to maximum shear rates of the order of 10^6 to 10^7 s^{-1} in the inlet region. The film thickness will be lower than the calculated value if the lubricant viscosity is significantly reduced by such shear rates (see Reference 2). In particular the increase in the viscosity of mineral oils obtained from polymer additives may vanish when the lubricant is sheared.

5.1.2 Inlet shear heating

Significant heating of a lubricant can occur due to shearing in the inlet region of contacts operating at relatively high speeds (see Reference 3). The resulting reduction in film thickness is given approximately by

$$\frac{\text{Film thickness with inlet shear heating}}{\text{Film thickness for isothermal conditions}} = (1 + 0.46 X)^{-0.45},$$

where
$$X = \frac{u^2}{k} \left(-\frac{d\eta_0}{d\theta} \right).$$

Note that $d\eta_0/d\theta$ is negative and so X is positive. The rate of change of viscosity with temperature, $d\eta_0/d\theta$, can be estimated from a chart of the viscosity/temperature characteristics of the chosen lubricant. However, for most lubricants the relationship between viscosity and temperature is highly nonlinear and it is advisable to estimate $d\eta_0/d\theta$ using the identity

$$\frac{d\eta_0}{d\theta} \equiv \eta_0 \frac{d}{d\theta} \left(\ln(\eta_0) \right).$$

The thermal conductivity of the lubricant, k , should be evaluated at ambient pressure and for mineral oils is given approximately by

$$k = \frac{(134.5 - 0.0633 \theta)}{\rho_0} \text{ W m}^{-1} \text{ K}^{-1},$$

where θ is the absolute temperature in K and ρ_0 is density in kg m^{-3} .