

## FORCE AND MOMENT COMPONENTS FOR TAKE-OFF AND LANDING CALCULATIONS

### 1. NOTATION AND UNITS

The Notation is based on the terms and symbols for flight dynamics presented in References 4 and 5 with modifications and additions to take account of ground and other forces. For an introduction to this notation and its associated axis systems and sign conventions, see Item No. 78038 (Reference 6). For a detailed review of the thinking behind the notation, see Reference 3. The units quoted are coherent sets and this avoids the necessity for introducing numerical constants into algebraic equations. Conversion factors are given in the Conversion Factors Item in Volume 1 of the Sub-series.

|                                   |                                                                                                                              | SI | British |
|-----------------------------------|------------------------------------------------------------------------------------------------------------------------------|----|---------|
| $A$                               | axial force ( $= -X^A$ ) <sup>*</sup>                                                                                        | N  | lbf     |
| $B$                               | buoyancy force (aerostatic force)                                                                                            | N  | lbf     |
| $C_A$                             | axial (aerodynamic) force coefficient,<br>$A/(\rho V^2 S/2) (= -C_X^A)$ <sup>*</sup>                                         |    |         |
| $C_D$                             | drag coefficient, $D/(\rho V^2 S/2) (= -C_{X_a}^A)$ <sup>*</sup>                                                             |    |         |
| $C_L$                             | lift coefficient, $L/(\rho V^2 S/2) (= -C_{Z_a}^A)$ <sup>*</sup>                                                             |    |         |
| $C_N$                             | normal (aerodynamic) force coefficient,<br>$N/(\rho V^2 S/2) (= -C_Z^A)$ <sup>*</sup>                                        |    |         |
| $C_X^A; C_Y^A; C_Z^A$             | body-axes airframe aerodynamic force coefficients,<br>$X^A/(\rho V^2 S/2); Y^A/(\rho V^2 S/2); Z^A/(\rho V^2 S/2)$           |    |         |
| $C_{X_a}^A; C_{Y_a}^A; C_{Z_a}^A$ | air-path axes airframe aerodynamic force coefficients,<br>$X_a^A/(\rho V^2 S/2); Y_a^A/(\rho V^2 S/2); Z_a^A/(\rho V^2 S/2)$ |    |         |
| $D$                               | drag ( $= -X_a^A$ ) <sup>*</sup>                                                                                             | N  | lbf     |
| $D_M$                             | intake momentum drag (ram drag)                                                                                              | N  | lbf     |
| $F_G$                             | engine gross thrust                                                                                                          | N  | lbf     |
| $G_B$                             | tangential force on tyre due to braking alone                                                                                | N  | lbf     |
| $G_F$                             | fluid drag force on tyre                                                                                                     | N  | lbf     |
| $G_R$                             | rolling resistance of tyre and wheel                                                                                         | N  | lbf     |
| $G_{R, \psi}$                     | rolling resistance of tyre and wheel yawed at angle<br>$\psi_{tyre}$ to direction of forward motion                          | N  | lbf     |
| $G_{skid}$                        | tangential force on tyre in non-rotating or "locked"<br>condition                                                            | N  | lbf     |

*For footnotes refer to end of Notation*

|                          |                                                                                                          |                      |                         |
|--------------------------|----------------------------------------------------------------------------------------------------------|----------------------|-------------------------|
| $G_{\psi}$               | force perpendicular to plane of rotation of wheel yawed at angle $\psi_{tyre}$                           | N                    | lbf                     |
| $H^{\Sigma}$             | resultant angular momentum                                                                               | kg m <sup>2</sup> /s | slug ft <sup>2</sup> /s |
| $I_x; I_y; I_z$          | moments of inertia with respect to x-axis; y-axis; z-axis                                                | kg m <sup>2</sup>    | slug ft <sup>2</sup>    |
| $I_{xy}; I_{yz}; I_{zx}$ | products of inertia with respect to x- and y-axes; y- and z-axes; z- and x-axes                          | kg m <sup>2</sup>    | slug ft <sup>2</sup>    |
| $J_x; J_y; J_z$          | components of angular momentum of rotating parts of aircraft or engines about x-axis; y-axis; z-axis     | kg m <sup>2</sup> /s | slug ft <sup>2</sup> /s |
| $L$                      | rolling moment                                                                                           | N m                  | lbf ft                  |
| $L$                      | lift, ( $= -Z_a^A$ ) *                                                                                   | N                    | lbf                     |
| $L_p^P$                  | propeller or rotor aerodynamic moment about $x_p$ axis                                                   | N m                  | lbf ft                  |
| $l_1, l_2, \dots$        | elements of direction cosine matrices in Table 6.2                                                       |                      |                         |
| $M^{\Sigma}$             | resultant moment                                                                                         | N m                  | lbf ft                  |
| $M$                      | pitching moment                                                                                          | N m                  | lbf ft                  |
| $M_p^P$                  | propeller or rotor "out-of-plane" aerodynamic moment about $y_p$ axis                                    | N m                  | lbf ft                  |
| $m_p$                    | resultant aerodynamic moment normal to plane of propeller or rotor disc, $[(M_p^P)^2 + (N_p^P)^2]^{1/2}$ | N m                  | lbf ft                  |
| $m_1, m_2, \dots$        | elements of direction cosine matrices in Table 6.2                                                       | N m                  | lbf ft                  |
| $N$                      | yawing moment                                                                                            | N m                  | lbf ft                  |
| $N$                      | normal force, ( $= -Z^A$ ) *                                                                             | N                    | lbf                     |
| $N_p^P$                  | propeller or rotor "out-of-plane" aerodynamic moment about $z_p$ axis                                    | N m                  | lbf ft                  |
| $n_p$                    | resultant aerodynamic force in plane of propeller or rotor disc, $[(Y_p^P)^2 + (Z_p^P)^2]^{1/2}$         | N                    | lbf                     |
| $n_1, n_2, \dots$        | elements of direction cosine matrices in Table 6.2                                                       |                      |                         |
| $p; q; r$                | angular velocity components about 0x-axis (rate of roll); 0y-axis (rate of pitch); 0z-axis (rate of yaw) | rad/s                | rad/s                   |
| $Q$                      | brake torque                                                                                             | N m                  | lbf ft                  |
| $R$                      | resultant external force, excluding gravity                                                              | N                    | lbf                     |
| $R^{\Sigma}$             | resultant external force                                                                                 | N                    | lbf                     |
| $R^A$                    | resultant airframe aerodynamic force                                                                     | N                    | lbf                     |

For footnotes refer to end of Notation

|                             |                                                                                                                                           |                |                 |
|-----------------------------|-------------------------------------------------------------------------------------------------------------------------------------------|----------------|-----------------|
| $R^G$                       | external force due to gravity                                                                                                             | N              | lbf             |
| $R^P$                       | resultant propeller or rotor aerodynamic force                                                                                            | N              | lbf             |
| $S$                         | reference wing area                                                                                                                       | m <sup>2</sup> | ft <sup>2</sup> |
| $T$                         | propeller or rotor thrust (= $X_p^P$ )                                                                                                    | N              | lbf             |
| $u; v; w$                   | components of $V$ in $x$ -direction; $y$ -direction; $z$ -direction                                                                       | m/s            | ft/s            |
| $u_a; v_a; w_a$             | components of $V$ in $x_a$ -direction; $y_a$ -direction; $z_a$ -direction (= $V, 0, 0$ )                                                  | m/s            | ft/s            |
| $u_K; v_K; w_K$             | components of $V_K$ in $x$ -direction; $y$ -direction; $z$ -direction                                                                     | m/s            | ft/s            |
| $u_{K_r}; v_{K_r}; w_{K_r}$ | components of $V_K$ in $x_r$ -direction; $y_r$ -direction; $z_r$ -direction                                                               | m/s            | ft/s            |
| $u_{K_0}; v_{K_0}; w_{K_0}$ | components of $V_K$ in $x_0$ -direction; $y_0$ -direction; $z_0$ -direction                                                               | m/s            | ft/s            |
| $u_W; v_W; w_W$             | components of $V_W$ in $x$ -direction; $y$ -direction; $z$ -direction                                                                     | m/s            | ft/s            |
| $u_{W_g}; v_{W_g}; w_{W_g}$ | components of $V_W$ in $x_g$ -direction; $y_g$ -direction; $z_g$ -direction                                                               | m/s            | ft/s            |
| $u_{W_r}; v_{W_r}; w_{W_r}$ | components of $V_W$ in $x_r$ -direction; $y_r$ -direction; $z_r$ -direction                                                               | m/s            | ft/s            |
| $u_{W_0}; v_{W_0}; w_{W_0}$ | components of $V_W$ in $x_0$ -direction; $y_0$ -direction; $z_0$ -direction                                                               | m/s            | ft/s            |
| $u_0; v_0; w_0$             | components of $V$ in $x_0$ -direction; $y_0$ -direction; $z_0$ -direction                                                                 | m/s            | ft/s            |
| $V$                         | air-path velocity (true airspeed or resultant velocity with respect to air)                                                               | m/s            | ft/s            |
| $V_K$                       | flight path velocity (resultant velocity with respect to Earth)                                                                           | m/s            | ft/s            |
| $V_W$                       | wind velocity (relative to Earth)                                                                                                         | m/s            | ft/s            |
| $X; Y; Z$                   | components of $R$ in $x$ -direction; $y$ -direction; $z$ -direction                                                                       | N              | lbf             |
| $X^A; Y^A; Z^A$             | components of $R^A$ in $x$ -direction; $y$ -direction; $z$ -direction                                                                     | N              | lbf             |
| $X_a^A; Y_a^A; Z_a^A$       | components of $R^A$ in $x_a$ -direction; $y_a$ -direction; $z_a$ -direction                                                               | N              | lbf             |
| $X_p^P; Y_p^P; Z_p^P$       | components of resultant ( $R^P$ ) propeller or rotor force in $x_p$ -direction; $y_p$ -direction; $z_p$ -direction<br>(note $X_p^P = T$ ) | N              | lbf             |

For footnotes refer to end of Notation

|                          |                                                                                                                                                                                               |     |     |
|--------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|-----|
| $x; y; z$                | body-axes co-ordinates in $x$ -direction; $y$ -direction; $z$ -direction                                                                                                                      | N   | lbf |
| $x_a; y_a; z_a$          | air-path axes co-ordinates in $x_a$ -direction; $y_a$ -direction; $z_a$ -direction                                                                                                            | m   | ft  |
| $x_{ab}; y_{ab}; z_{ab}$ | aerodynamic-body-axis co-ordinates                                                                                                                                                            | m   | ft  |
| $x_g; y_g; z_g$          | local normal Earth-axis co-ordinates                                                                                                                                                          | m   | ft  |
| $x_k; y_k; z_k$          | flight-path axis co-ordinates                                                                                                                                                                 | m   | ft  |
| $x_p; y_p; z_p$          | propeller or rotor axis co-ordinates                                                                                                                                                          | m   | ft  |
| $x_r; y_r; z_r$          | runway axis co-ordinates                                                                                                                                                                      | m   | ft  |
| $x_0; y_0; z_0$          | aircraft-carried Earth-axis co-ordinates ("North, East and Down" system)                                                                                                                      | m   | ft  |
| $\alpha_s$               | angle of downslip <sup>†</sup> ; angle between $xy$ -plane and $V$<br>( $\sin \alpha_s = w/V$ )                                                                                               | deg | deg |
| $\alpha_t$               | angle of attack <sup>†</sup> ; angle between $x$ -axis and projection of $V$ on to $zx$ -plane ( $\tan \alpha_t = w/u$ ; $\alpha_t$ has same sign as $w$ )                                    | deg | deg |
| $\beta_s$                | angle of sideslip <sup>†</sup> ; angle between $zx$ -plane and $V$<br>( $\sin \beta_s = v/V$ )                                                                                                | deg | deg |
| $\beta_t$                | flank angle of attack <sup>†</sup> ; angle between $x$ -axis and projection of $V$ on to $xy$ -plane ( $\tan \beta_t = v/u$ ; $\beta_t$ has same sign as $v$ )                                | deg | deg |
| $\gamma$                 | angle of climb (flight-path inclination angle),<br>$\sin \gamma = (-w_{K_0})/V_K = (-w_{K_g})/V_K$                                                                                            | deg | deg |
| $\gamma_a$               | air-path angle of climb (air-path inclination angle),<br>$\sin \gamma_a = (-w_0)/V = (-w_g)/V$                                                                                                | deg | deg |
| $\Delta$                 | denotes an incremental value of affixed parameter                                                                                                                                             |     |     |
| $\delta$                 | tyre deflection normal to runway surface                                                                                                                                                      | m   | ft  |
| $\delta$                 | control deflection (subscript denotes particular control)                                                                                                                                     | deg | deg |
| $\epsilon$               | "splay" angle between exhaust gross thrust vector and aircraft plane of symmetry ( $0x, 0z$ ); positive when component of $F_G$ along $0y$ is negative ( <i>i.e.</i> when $F_G$ acts to port) | deg | deg |
| $\zeta$                  | yaw-control deflection angle                                                                                                                                                                  | deg | deg |
| $\zeta_{tyre}$           | tyre steering angle (angle between plane of rotation and $0x$ body axis)                                                                                                                      | deg | deg |
| $\eta$                   | pitch-control deflection angle                                                                                                                                                                | deg | deg |

For footnotes refer to end of Notation

|                 |                                                                                                                                                                                                                                                                                      |                   |                      |
|-----------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------|----------------------|
| $\Theta$        | inclination angle <sup>‡</sup> (elevation)                                                                                                                                                                                                                                           | deg               | deg                  |
| $\Theta_p$      | propeller or rotor axis system inclination angle <sup>**</sup>                                                                                                                                                                                                                       | deg               | deg                  |
| $\Theta_r$      | angle between runway centre line and horizontal (positive uphill)                                                                                                                                                                                                                    | deg               | deg                  |
| $\xi$           | roll-control deflection angle                                                                                                                                                                                                                                                        | deg               | deg                  |
| $\rho$          | density of air                                                                                                                                                                                                                                                                       | kg/m <sup>3</sup> | slug/ft <sup>3</sup> |
| $\tau$          | angle between projection of exhaust gross-thrust vector on to aircraft plane of symmetry (0x, 0z) and 0x body axis; positive when component of $F_G$ along 0z is negative ( <i>i.e.</i> when $F_G$ contributes to lift)                                                              | deg               | deg                  |
| $\Phi$          | bank angle <sup>‡</sup>                                                                                                                                                                                                                                                              | deg               | deg                  |
| $\Phi_p$        | final rotation <sup>**</sup> required to bring propeller or rotor axis 0z <sub>p</sub> parallel with aircraft body axis 0z                                                                                                                                                           | deg               | deg                  |
| $\Phi_r$        | angle between line drawn in plane of runway, normal to centre line, and horizontal ( $\tan \Phi_r$ is runway transverse slope or cross-fall)                                                                                                                                         | deg               | deg                  |
| $\chi$          | flight-path azimuth angle (angle of track),<br>$\tan \chi = v_{K_0}/u_{K_0}$ relative to North, or<br>$\tan \chi = v_{K_g}/u_{K_g}$ relative to runway centre line                                                                                                                   | deg               | deg                  |
| $\chi_w$        | wind azimuth angle ( $\tan \chi_w = v_w/u_w$ ; North corresponds to $\chi_w = 0$ and $\chi_w$ differs by 180° from wind direction used in meteorology)                                                                                                                               | deg               | deg                  |
| $\Psi$          | azimuth angle <sup>‡</sup> ( <i>i.e.</i> aircraft heading: $\Psi = 0$ is taken in the North direction, $x_0$ , for general trajectory calculations but the projection, $x_g$ , of the runway centre line onto the horizontal plane is more usual for field-performance calculations) | deg               | deg                  |
| $\Psi_p$        | propeller or rotor axis system azimuth angle (angle between $T$ , <i>i.e.</i> 0x <sub>p</sub> axis, and projection of 0x body axis on the plane $x_p, y_p$ )                                                                                                                         | deg               | deg                  |
| $\psi_p$        | azimuth angle in plane of propeller or rotor disc ( $\psi_p = 0$ chosen to suit particular case but most conveniently taken along $y_p$ or $z_p$ axis)                                                                                                                               | deg               | deg                  |
| $\Psi_{tyre}$   | tyre yaw angle (between plane of rotation and direction of forward motion)                                                                                                                                                                                                           | deg               | deg                  |
| $\Omega^\Sigma$ | resultant angular velocity                                                                                                                                                                                                                                                           | rad/s             | rad/s                |
| $\omega$        | wheel angular velocity                                                                                                                                                                                                                                                               | rad/s             | rad/s                |

For footnotes refer to end of Notation

**Superscripts**

|        |                                                  |
|--------|--------------------------------------------------|
| $A$    | denotes (airframe) aerodynamic force or moment   |
| $F$    | denotes engine-thrust related force or moment    |
| $P$    | denotes propeller thrust related force or moment |
| $tyre$ | denotes tyre force or moment                     |
| $W$    | denotes water-reaction force or moment           |

**Subscripts**

|           |                                                       |
|-----------|-------------------------------------------------------|
| $a$       | denotes use of air-path axes                          |
| $control$ | denotes value for individual control surface          |
| $g$       | denotes use of local normal Earth axes                |
| $in$      | denotes value relative to inertial frame of reference |
| $intake$  | denotes value at location of intake                   |
| $jet$     | denotes value for exhaust jet                         |
| $k$       | denotes use of flight-path axes                       |
| $nozzle$  | denotes value at location of nozzle exit              |
| $p$       | denotes use of propeller or rotor axes                |
| $prop$    | denotes value at location of propeller or rotor       |
| $r$       | denotes use of runway axes                            |
| $tyre$    | denotes value at location of individual tyre          |
| $W$       | denotes wind                                          |
| $0$       | denotes use of North East and Down normal Earth axes  |

\* The negative signs are a result of the sign convention used whereby two of the components of  $R^A$  ( $X^A$  and  $Z^A$  in body axes;  $X_a^A$  and  $Z_a^A$  in air-path axes) usually take negative values whereas it is usual to think of aerodynamic lift and drag and normal and axial forces as positive quantities.

† This Item follows conventional practice and uses the incidence angles  $\alpha_t$  and  $\beta_s$  (consistency of definition would require the use of either  $\alpha_s$ ,  $\beta_s$  or  $\alpha_t$ ,  $\beta_t$ ). Angles  $\alpha_s$  and  $\alpha_t$  are approximately equal when they are small (as are  $\beta_s$ ,  $\beta_t$ ), see Reference 1 or Part 2 of Reference 2.

‡ Transition from the aircraft-carried normal Earth-axis system ( $0x_0, y_0, z_0$  or local normal Earth axis system ( $0x_g, y_g, z_g$ ) to the body-axis system ( $0x, y, z$ ) is effected by the rotation through the angles  $\Psi$ ,  $\Theta$ ,  $\Phi$  in that order. These angles therefore define the orientation of the body axes with respect to the aircraft-carried normal Earth axes and are called the attitude angles. See Reference 1 for further information and diagrammatic representation.

\*\* Transition from the propeller or rotor axis system ( $0x_p, y_p, z_p$ ) to the body-axis system ( $0x, y, z$ ) is effected by the rotation through the angles  $\Psi_p$ ,  $\Theta_p$ ,  $\Phi_p$  in that order. (Table 6.3 suggests a simpler treatment for cases where no “in-plane” propeller or rotor forces are involved.)

## 2. INTRODUCTION

This Item gathers together notation for axes, linear and angular displacements, aerodynamic, powerplant and ground-related forces and moments for use in take-off and landing calculations. The Item is a companion to Item No. 85029 (Reference 10) and provides a comprehensive basis for constructing equations of motion for a wide range of aircraft and manoeuvres.

Some of the information has wider application than airfield performance and in this respect may be regarded as complementary to Item No. 78038 (Reference 6). For an example of the application of the notation in such a context, see Item No. 79018 (Reference 7).

## 3. AXIS SYSTEMS AND SIGN CONVENTIONS

### 3.1 Axis Systems

Axis systems are listed in Table 6.1. They are of the right-handed, orthogonal Cartesian type. The origin is denoted "0" and the axes designated  $x, y, z$ . Axes having an origin which moves with the aircraft (usually the aircraft centre of gravity, although an exception is the propeller-axis system) are called aircraft-carried axes. The usual alternative to this is some form of "Earth-fixed" axis system. A particular choice of origin and axes is indicated by a lower-case subscript (except for body axes). Alphabetical trios are used to denote components of linear displacement and velocity, force, angular velocity and moment. For a system of body axes these components are as follows.

| Quantity            | Components | Resultant |
|---------------------|------------|-----------|
| Force               | $X Y Z$    | $R$       |
| Moment              | $L M N$    |           |
| Linear displacement | $x y z$    |           |
| Linear velocity     | $u v w$    | $V$       |
| Angular velocity    | $p q r$    | $\Omega$  |

### 3.2 Sign Conventions

Linear displacements are positive in the directions of the axes and angular displacements are positive in the clockwise direction when looking along the appropriate axis in the positive sense. Velocities, accelerations, forces, angular velocities, angular accelerations and moments are correspondingly positive in these senses.

#### 3.2.1 Control forces and moments from aerodynamic, powerplant and tyre sources

The term "control" is used here to denote all forces and moments derived from the angular displacement of surfaces and nozzles and the steering angles of tyres from some datum conditions. Two sign conventions, denoted (i) and (ii), are defined – see below. In those cases where the use of such a convention is necessary, the relationships quoted in the tables of this Item are based on convention (i) and a footnote to this effect is included in the appropriate table.

**Convention (i):** “Controls” (including nozzles and steerable wheels) are taken to have positive deflections, relative to their respective datums, when the associated moments are negative (or are intended to be so). Examples of positive angular displacements (of controls) using this sign convention are:

- (i) rear-mounted rudder deflected to **port**,
- (ii) nose-wheel steering angle to **port**.

**Convention (ii):** When the “hinge line” of a control is parallel to one of the axes of a body system, a positive deflection corresponds to a clockwise rotation when looking in the positive direction of that axis. (If not parallel to a body axis, the hinge line should be imagined to be rotated and aligned with the most relevant body axis and a positive deflection of the control in this condition remains positive with the hinge line in its proper place.) Examples of positive angular displacements (of controls) using this sign convention are:

- (i) rear-mounted rudder deflected to **port**,
- (ii) nose-wheel steering angle to **starboard**.

### 3.3 Axes Conversions

Accompanying any particular force (or group of forces) there is at least one axis system from among those in Table 6.1 relative to which Cartesian components are most conveniently (or customarily) expressed. Consequently, in any detailed calculation where aerodynamic, powerplant and tyre forces have to be considered, and especially if winds and/or runway slopes are involved, it is necessary to convert components between different axis systems. Table 6.2 presents the axes conversion formulae needed to permit forces and moments defined in terms of one axis system to be expressed as components relative to a second system. The conversions are limited to those pairs of systems where the angles relating them are familiar ones – with obvious physical significance. Consequently, for some pairs of systems it is necessary to use more than one set of conversion formulae.

## 4. POWERPLANT, AERODYNAMIC AND TYRE FORCE AND MOMENT COMPONENTS

### 4.1 Powerplant Forces and Moments

Figures 1 and 2 show two powerplant configurations and associated axes, lines of action of the principal forces and moment arms. Table 6.3 presents force components relative to air-path and body axes while Table 6.4 presents moment components relative to body axes.

### 4.2 Aerodynamic Forces and Moments

Table 6.5 gives air-path axes components of airframe and control aerodynamic forces and moments. Table 6.6 gives components in body axes. Both tables include comments on customary usage of the various components.

### 4.3 Tyre Forces and Moments

Figure 3 shows axes, lines of action of forces and associated moment arms for a tyre rolling on a non-horizontal surface. The tyre is yawed at an angle  $\psi_{tyre}$  to the direction of motion (“ $x_k$ ” flight-path axis) and is steered at an angle  $\zeta_{tyre}$  to the “ $x$ ” body axis. Table 6.7 gives the components of forces in flight-path axes – the  $0y_k$  and  $0z_k$  axes being taken in, and normal to, the runway surface respectively (see Table 6.1). Table 6.8 gives the components of moments about body axes.