

EXAMPLE OF THE USE OF DATA ITEM NO. 85001. CHOICE OF REINFORCEMENT FOR A CIRCULAR HOLE IN A FIBRE REINFORCED LAMINATED PLATE (DATA RELATING TO ONE PARTICULAR SET OF LAMINATE PROPERTIES)

1. INTRODUCTION

The program of Data Item No. 85001* provides the stresses and strains around the boundary of a hole in orthotropic material under any in-plane loading. The current Data Item shows, by example, how that program may be employed to provide curves for selection of an appropriate composite lay-up to accommodate the strains around a hole boundary. The approach can be applied to any specially orthotropic combinations of plies under in-plane loading; the examples of this Item deal only with composites with fibres in the 0° and $\pm 45^\circ$ directions. They show the way in which Data Item No. 85001 may be used to determine an appropriate reinforcement of a panel of that type in order to achieve an acceptable level of strain.

2. NOTATION

The notation shows both SI and British units. Either system, or any other coherent system of units, may be used.

E_x, E_y	moduli of elasticity of plate in x- and y-directions respectively	N/m ²	lbf/in ²
G_{xy}	shear modulus of plate in xy-plane	N/m ²	lbf/in ²
x, y	coordinates chosen such that x-axis lies along axis of orthotropy from which ply angle is measured		
ϵ_x, ϵ_y	direct strains in plate, acting parallel to x- and y-axes respectively, tension positive		
γ_{xy}	shear strain in plate, acting parallel to x- and y-axes		
ν_{xy}	Poisson's ratio of plate resulting from load applied in x-direction		
σ_x, σ_y	direct stresses in plate acting parallel to x- and y-axes respectively, tension positive	N/m ²	lbf/in ²
τ_{xy}	shear stress in plate acting parallel to x- and y-axes	N/m ²	lbf/in ²
Suffixes			
∞	values remote from hole		
max	maximum values at hole boundary		

* Data Item No. 85001 "Elastic stress and strain distributions around circular holes in infinite plates of orthotropic material. (Applicable to fibre reinforced composites.) Computer program".

3. STRESS AND STRAIN CONCENTRATION IN ORTHOTROPIC PLATES

Items such as Data Item No. 80027* provide stress concentrations for unreinforced holes in isotropic plates. In those Items the maximum circumferential stress at the hole boundary is expressed as a multiple of the uniform stress applied remote from the hole. This multiple is defined as the elastic stress concentration factor and is related to the strain concentration factor by the elastic properties of the material.

In the case of composite plates the problem is less straightforward because the properties of the composite material vary with direction. This means that some composites will have a high stress at a point around the hole boundary yet this stress is in a direction in which the composite is particularly strong. Alternatively, another composite may give a lower apparent "stress concentration" but the maximum stress is in a direction in which the material is less strong. In these circumstances there are considerable advantages in using strains rather than stresses since, in a typical laminated composite, there is less directional variation of allowable strain than there is of allowable stress. The computer program of Data Item No. 85001 calculates strains for a homogeneous orthotropic material. Therefore, strictly, a check should be made of the consequences of exposing each layer to these strains at each point around the hole. Further, the repercussions of a layer matrix material failure should be examined using a fully interactive check against a criterion of failure.

In order to simplify this process to determine a desirable composite arrangement a simple criterion, similar to that of the maximum strain criterion for a single layer (see Data Item No. 83014†), may be applied to the whole laminate. This approach has been employed in the following sections but does not obviate the need for a full layer-by-layer check on the final arrangement.

4. STRAIN DISTRIBUTIONS ROUND THE HOLE BOUNDARY FOR A PARTICULAR EXAMPLE COMPOSITE

The example chosen is a carbon epoxy composite comprising x -axis plies and varying percentages of equal numbers of plies at $+45^\circ$ and -45° to the x -axis so that the plate forms a balanced orthotropic array. In Table 4.1 the properties of this composite are given in terms of the number of $\pm 45^\circ$ plies as a percentage of the total number of plies. The example loading combinations chosen are defined in Table 4.2. They are combinations of $\sigma_{x\infty}$ and $\tau_{xy\infty}$ only (that is, $\sigma_{y\infty} = 0$), varying from unidirectional tension to pure shear. Using the program of Data Item No. 85001 the curves of Figures 1, 2 and 3 have been generated for positive ratios of $\tau_{xy\infty}/\sigma_{x\infty}$. Because the layups are symmetric about the x -axis negative values of this ratio may be dealt with by choice of the y -direction. These show the maximum values on the hole boundary of ϵ_x , ϵ_y and γ_{xy} , respectively, expressed as ratios to the value of a reference strain that corresponds to the dominant stress in the plate remote from the hole. For curves (a), (b) and (c) the reference strain is $\epsilon_{x\infty}$ while for curves (c'), (d') and (e') it is $\gamma_{xy\infty}$. Interpolation among the curves of each of the two groups ((a), (b), (c) and (c'), (d'), (e')) may be used to estimate intermediate values of $\tau_{xy\infty}/\sigma_{x\infty}$ where the curves show consistent shapes. Calculation of a curve for the specific applied stresses of interest is preferable.

Since composites have differing allowable strains in tension and compression it is always necessary to examine both positive and negative values of the strain ratio. Therefore, Figures 1 and 2 quote positive and negative values of $\epsilon_{x\max}/\epsilon_{x\infty}$ and $\epsilon_{y\max}/\epsilon_{x\infty}$. Both values should be used with the appropriately signed values of $\epsilon_{x\infty}$ in order to be able to check against the compressive and tensile criteria. For cases (c') and (d') the sign of $\gamma_{xy\infty}$ is immaterial: the sign of $\epsilon_{x\infty}$ should be used.

* Data Item No. 80027 "Elastic stress concentration factors. Single reinforced and unreinforced holes in infinite plates of isotropic materials".

† Data Item No. 83014 "Failure criteria for an individual layer of a fibre reinforced composite laminate under in-plane loading".

TABLE 4.1 Properties of Example Carbon-epoxy Plate (see Data Item No. 85001 Section 7.2)

Angle-ply percentage	E_x (GN/m ²)	E_y (GN/m ²)	G_{xy} (GN/m ²)	ν_{xy}
0	130.0	9.0	4.8	0.280
20	108.3	14.8	10.6	0.538
40	85.7	19.7	16.3	0.645
60	62.9	23.1	22.1	0.704
80	40.0	23.5	27.9	0.740
100	16.9	16.9	33.7	0.766
Allowable strains	Direct strains		Shear strains	
	Tension	Compression	0.019	
	0.013	0.010		

TABLE 4.2 Applied Stresses Remote from Hole

Loading cases	Loading		
	$\sigma_{x\infty}$	$\sigma_{y\infty}$	$\tau_{xy\infty}$
(a)	1	0	0
(b)	1	0	$\pm\frac{1}{2}$
(c) (c')	1	0	± 1
(d')	$\frac{1}{2}$	0	± 1
(e')	0	0	± 1

It should be noted that the strain ratios in Figures 1, 2 and 3 must not be regarded as implying effective merit to lay-ups, since they do not give complete design information. However, strains obtained using Figures 1 to 3 will indicate whether or not the plate will have to be reinforced.

If maximum allowable values of strain are specified in addition to the basic stiffnesses then values of the stresses remote from the hole that give maximum strains equal to these allowable values may be calculated and plotted. Examples of this approach are shown in Figures 4a to 4e. These figures, like Figures 1, 2 and 3, have been produced using Data Item No. 85001 for laminates whose properties are given in Table 4.1 and which are subject to the loading cases of Table 4.2. The figures have been drawn applying the direct strain criteria to $\varepsilon_{x\max}$ and $\varepsilon_{y\max}$ and the shear strain criterion to $\gamma_{xy\max}$. Positive and negative limits of applied direct stresses are shown, allowing for the differing compressive and tensile allowable strains. The envelope curves of least values of these limiting stresses are the allowable applied stress values. It is noteworthy that, for the particular relative values of the allowable strains given in Table 4.1, it is the compression criterion that is most often critical. It may be noted that, as the shear loading increases relative to the tensile, so the optimum angle ply percentage in the plate rises.

It is emphasised that a plate lay-up chosen on the basis of criteria applied to the whole laminate should be checked by a more complete layer-by-layer analysis.

For the purposes of illustrating the design approach this Data Item restricts the applied stresses to those defined in Table 4.2. For other combinations of applied stresses the program of Data Item No. 85001 may be used to generate curves similar to those of Figures 1 to 3 and 4a to 4e. In the same way other combinations of laminate fibre directions may be considered.

This Item has concentrated on values of ϵ_x , ϵ_y and γ_{xy} at the hole boundary. It may be preferable to transform from those strains (obtained from Data Item No. 85001) to the direct strains in the actual fibre directions in the laminate and to use them with the criteria of failure.

5. REINFORCEMENT OF PERFORATED PANELS

Data given in this Item, or similar data derived from Data Item No. 85001 for other materials, may be used to determine whether or not reinforcement is required and, if it is, the necessary additions to the panel. For the data to apply, the size of the hole must be small in comparison with the panel dimensions and the reinforcement should extend a considerable distance from the hole boundary. This Item can thus only give a lead to an acceptable lay-up for a more local reinforcement. The basic panel and the combination of panel and reinforcement must each form orthotropic composites with the same axes of orthotropy which coincide with the directions of direct and shear loading.

Figures 1, 2 or 3 may be used to examine whether the plate does in fact require reinforcement. If so, then Figures 4a to 4e may be used to determine the reinforcement necessary. The values of $\sigma_{x\infty}$ and $\tau_{xy\infty}$ in Figures 4a to 4e are stresses remote from the hole in the reinforced plate. They should be divided into the corresponding values of applied load per unit length in order to give the total thickness required for the combination of plate and reinforcement. The reinforcement should be arranged so that the final lay-up is as near as possible to the optimum arrangement indicated in Figures 4a to 4e. The numerical illustrations of Section 6 demonstrate this.

As remarked above use of Data Item No. 85001 implies a uniform reinforcement over an area large compared with that of the hole, and application to reinforcement local to the hole would need to be validated, for example by a finite element calculation. In practice, it might be reasonable to take the reinforcement found necessary at the hole boundary out to about three hole diameters and then to continue with reinforcement having properties of a similar lay-up but of about half the inner reinforcement thickness up to about five diameters. Note that there is likely to be a stress concentration around the outer perimeter of each of the reinforcements. This is not dealt with by Data Item No. 85001 and should be considered separately, for example by a finite element calculation.

Remembering that the reinforced plate has to include the original material, it will often not be easy to achieve the optimum reinforcement. This will be particularly true for the "graded" reinforcement away from the hole boundary referred to above.

6. NUMERICAL ILLUSTRATIONS

The lay-ups in the following two examples are defined by ply orderings in which the symbols 0, +, – denote respectively ply angles of 0° , $+45^\circ$ and -45° . Lay-up properties and allowable strains are as in Table 4.1.

6.1 Example 1

It is required to determine what reinforcing is necessary round two separate holes in a fibre reinforced composite plate noting that if reinforcing is required it can only be applied on one side of the plate. The plate is made up of 10 plies, each 0.25 mm thick, with the following arrangement:

$$+ - 0 0 0 0 0 0 - + \text{ (40 per cent angle plies).}$$

At hole (i) the plate is loaded by a tension of 675 kN/m in the x-direction, and at hole (ii) a similar loading of 1500 kN/m exists.

Hole (i)

The loading is 675×10^3 N/m so that

$$\sigma_x = 675 \times 10^3 / (10 \times 0.00025) = 270 \times 10^6 = 270 \text{ MN/m}^2.$$

It follows, from Table 4.1, that

$$\epsilon_{x\infty} = 270 \times 10^6 / (85.7 \times 10^9) = 0.00315.$$

Applying the strain ratios from Figures 1 to 3 gives (Case (a)):

$$\epsilon_{x\max} = 3.8 \times 0.00315 = 0.012, \quad (\text{allowable strain is } 0.013)$$

$$\epsilon_{y\max} = -2.5 \times 0.00315 = -0.008 \quad (\text{allowable strain is } -0.010)$$

$$\text{and } \gamma_{xy\max} = 4.3 \times 0.00315 = 0.014. \quad (\text{allowable strain is } 0.019)$$

These strains are all smaller than the allowable strains and so the plate does not require reinforcing.

Hole (ii)

Clearly reinforcing will be necessary in this case. From Figure 4a the maximum allowable applied stress is 295 MN/m^2 . The required total thickness is then

$$1500 \times 10^3 / (295 \times 10^6) = 5.1 \times 10^{-3} = 5.1 \text{ mm}.$$

Therefore, 21 plies are required of which about 30 per cent should be angled. An acceptable lay-up is as follows.

$$\frac{+ - 0 0 0 0 0 0 - +}{\text{original}} \quad \frac{0 + - 0 0 0 0 0 0 - +}{\text{reinforcement}}.$$

For this angle ply fraction (38 per cent), the allowable stress (Figure 4a) is 290 MN/m^2 . The actual stress is

$$1500 \times 10^3 / (21 \times 0.25 \times 10^{-3}) = 286 \times 10^6 = 286 \text{ MN/m}^2,$$