

SIMPLIFIED FORMS OF PERFORMANCE EQUATIONS

ADDENDUM A: EFFECT ON AEROPLANE LEVEL SPEED OF SMALL CHANGES IN THRUST, DRAG, WEIGHT, POWER

1. NOTATION AND UNITS (additional to Section 1 of Item No. 80032)

		SI	British
A, B	constants defined in Table 4.1 of Item No. 80032		
M_{md}	Mach number for minimum drag in 1g level flight, V_{md}/a		
V_{md}	true airspeed for minimum drag in 1g level flight ($V_{md} = [1/B][2W/\rho S]^{1/2}[K_2/K_1]^{1/4}$), see Reference 1	km/h, kn	kn
x	ratio of jet thrust power to propeller thrust power, $FV/A\eta P$, see Section 4		
y	ratio of airspeed to minimum drag speed, $V/V_{md} (= M/M_{md})$		
z	any one of independent variables $F, K_1, K_2^{1/2}, W, \eta P$		
Δ	denotes small change in the affixed parameter		
ξ_z	influence coefficient, $[z/V][\partial V/\partial z] \approx [\Delta V/V]/[\Delta z/z]$ (or $[z/M][\partial M/\partial z] \approx [\Delta M/M]/[\Delta z/z]$) for effect on V (or M) of small change in one of the variables z , with all others assumed independent of V (or M)		
ξ'_z	influence coefficient for effect on V (or M) of small change in one of the variables z , including effects on K_1, K_2, F of changes in M , see Equation (5.1)		

2. INTRODUCTION

Table 8.1 and Figures 1 to 4 give a simple method for estimating the effect on aeroplane level speed of a small change in any one of the independent variables $F, K_1, K_2, W, \eta P$. The method ignores effects of any consequent changes in the other independent variables, although a means of accommodating such effects (particularly of M on K_1 and K_2) is suggested in Section 5.1. The format of an Addendum to Item No. 80032 is used because the method is but one example of the theoretical results that may be deduced when the equations of motion of that Item are combined with a parabolic drag law, $C_D = K_1 + K_2 C_L^2$.

2.1 Aeroplane Level Speed

Level speed is estimated by the determination of the condition at which thrust equals drag using the along-flight-path equation of motion with climb and acceleration terms set equal to zero – see Equations (4.1), (4.2), and (4.11) to (4.16), in Item No. 80032. In practice this is not straightforward as most of the quantities are themselves dependent on forward speed and analytical solutions are complicated equations in V (or M) and best solved by successive approximation. Exceptions are the simplified cases where K_1, K_2 and F or ηP are constant and independent of speed – see Table 4.3 of Item No. 80032.

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3. EFFECTS OF SMALL CHANGES ON AEROPLANE LEVEL SPEED

Table 8.1 and Figures 1 to 4 give expressions and curves for the influence coefficient, ξ_z , for the effect on speed in level flight of small changes in an arbitrary parameter, z , (where z can be F , K_1 , $K_2^{1/2}$, W , ηP) expressed in the form,

$$\xi_z = \left[\frac{z}{V} \frac{\partial V}{\partial z} \right] = \left[\frac{z}{M} \frac{\partial M}{\partial z} \right]. \quad (3.1)$$

The parameter x in Table 8.1 and Figures 1 to 4 characterises the type(s) of powerplant installed – see Section 4.

All the expressions and curves for ξ_z are given in terms of the parameter y , where

$$y = \frac{V}{V_{md}} = \frac{M}{M_{md}} \quad (3.2)$$

and V_{md} is the speed for minimum drag in lg level flight (see Reference 1),

$$V_{md} = \frac{1}{B} \left[\frac{2W}{\rho S} \right]^{1/2} \left[\frac{K_2}{K_1} \right]^{1/4} = \frac{1}{B} \left[\frac{2mg}{\rho S} \right]^{1/2} \left[\frac{K_2}{K_1} \right]^{1/4}. \quad (3.3)$$

Figures 1 to 4 include arbitrary limits of $\xi_z = \pm 1$ to discourage use of the expressions in Table 8.1 at speeds approaching one or other of the following limits:

- (i) the $x = \infty$ line (“pure” jet, $F = \text{constant}$) as it approaches $y = 1$, i.e. as the “minimum drag” condition is approached, see Reference 1,
- (ii) the $x = 0$ line (“pure” propeller, $\eta P = \text{constant}$) as it approaches $y = 1/3^{1/4} = 0.7598$, i.e. as the “minimum power” condition is approached, see Reference 1.

In using the relationships of Table 8.1 and Figures 1 to 4 it is important to remember that **they apply only to small changes** and that each variable is assumed to be independent of the others and independent of V or M . See Section 5 regarding the basis of the method and its extension to include compressibility effects.

4. REPRESENTATION OF DIFFERENT POWERPLANT TYPES USING PARAMETER x

The expressions in Table 8.1 and Figures 1 to 4 use a parameter x to characterise the type(s) of powerplant installed, where

$$x = \frac{\text{jet thrust power}}{\text{propeller thrust power}} = \frac{1}{A} \left[\frac{FV}{\eta P} \right] = \frac{1}{A} \left[\frac{F}{(\eta P/V)} \right] = \frac{\text{net jet thrust}}{\text{propeller thrust}}. \quad (4.1)$$

For the case of a “pure” turbo-jet, $x = \infty$, while for a “pure” propeller-driven aeroplane, $x = 0$; Table 8.1 includes simplified versions of the expressions for ξ_z for these two cases.

4.1 Choice of Value for Parameter x

The table below gives values of “ x ” for different types of powerplant. For **turbo-props**, values were obtained by applying Equation (4.1) to appropriate engine and propeller performance data. For **unmixed-exhaust-flow turbo-fan engines**, values of x were deduced by assuming Equation (4.1) could be interpreted as,

$$x = \frac{\text{hot-stream (core) thrust}}{\text{cold-stream (fan) thrust}} \quad (4.2)$$

This implies the engine can be regarded as a combination of a constant-net-thrust hot (or “core”) stream and a “constant-power” cold (or fan) stream. The validity of this assumption depends on the method of engine control. For relatively simple systems where, for example, a fixed rating implies a constant turbine entry temperature, the assumption is reasonable. However, the capabilities of modern control systems make possible a very wide range of thrust variations with forward speed (for a nominated rating). For example, at cruise conditions, some turbo-fan engines show little or no thrust variation with change in V or M – in which case x would be taken as ∞ .

Powerplant type	x	M	Comments
“Pure” propeller	0	all values	Use for all piston-engined aeroplanes.
“Free-turbine” turbo props	0 to 0.1	0.2 to 0.6	Tendency to decrease to zero at or beyond upper end of practical speed range for given altitude and engine.
Projected future “free-turbine” propulsion units	0 to 0.15	0.4 to 0.8	As for “free-turbine” turbo props; values apply, for example, to projected “Prop fan” designs.
“Fixed-turbine” turbo props	0.05 to 0.11 0.08 to 0.12	0.2 0.6	Data for older-type turbo props; higher values apply at higher cruise altitudes.
Unmixed exhaust flow turbo-fan engines	0.3 to 0.5 0.4 to 0.7	0.2 0.8	bypass ratios $\approx$ 3 to 6 Values deduced using Equation (4.2); only applicable with simple engine control system – see Sketch 7.2. Higher values apply at higher cruise altitudes.
	0.4 to 0.6 1.0 to 1.8	0.2 0.9	
	∞	0.6 to 0.9	
Mixed exhaust flow turbo fans and turbo jets	∞	all values	Applicable if engine control system operates to maintain constant thrust with change in V or M – see Section 4.1.

5. BASIS OF METHOD AND EXTENSION TO INCLUDE COMPRESSIBILITY EFFECTS

Level speed may be estimated using Equation (4.1) or (4.2) of Item No. 80032 with the climb and acceleration terms omitted. When airframe drag coefficient is represented by a parabolic drag law ($C_D = K_1 + K_2 C_L^2$), the level-flight equation is given by Equations (4.11) to (4.16) of Item No. 80032, with $v_c = dV/dt = 0$. Expressions for ξ_z are obtained by differentiating partially any version of these equations with respect to each independent variable in turn (see Reference 2). As usual with partial derivatives, the other independent variables are held constant. The resulting expressions, quoted in Table 8.1 take no account of any consequential effects of changes in V or M . For many cases, particularly for “low-speed” aeroplanes, this assumption is valid.

Reference 2 includes expressions for the effects on level speed of small changes in ambient conditions – although different expressions are obtained depending on whether speed is expressed in terms of V or M . These expressions are not given here because changes in ambient conditions are likely to have significant effects on powerplant performance; see, instead, Item Nos RJ 2/0 and RP 2/0.

5.1 Extension of Method to Include Compressibility Effects

For cases where significant changes in F , K_1 and K_2 occur with changes in M , the expressions in Table 8.1 **must not be used alone**. However, a modified form of influence coefficient, denoted ξ'_z , can be obtained which takes account of variations in F , K_1 , K_2 with M . The general form of ξ'_z is

$$\xi'_z = \frac{\xi_z}{1 - \xi_{K_1} \left[\frac{M}{K_1} \frac{\partial K_1}{\partial M} \right] - \xi_{K_2^{1/2}} \left[\frac{M}{K_2^{1/2}} \frac{\partial K_2^{1/2}}{\partial M} \right] - \xi_F \left[\frac{M}{F} \frac{\partial F}{\partial M} \right]}. \quad (5.1)$$

In Equation (5.1), ξ_z , ξ_{K_1} , $\xi_{K_2^{1/2}}$, ξ_F are as defined in Equations (8.1) to (8.5) in Table 8.1, while the terms in square brackets are determined from thrust and drag data appropriate to the particular aeroplane at the datum value of M . The calculation procedure is demonstrated in the example given in Section 7.2.3, part “b”. Sketch 7.2 includes a comparison (at $M = 0.88$) of results (dotted line) obtained using Equations (5.1) and (8.4), with those obtained using Equation (8.4) alone (solid lines) to predict the change in level speed with change in weight for a turbo-fan engined transport. Note that although use of Equation (5.1) severely detracts from the simplicity of the method of Table 8.1, the need to adopt this procedure in regions such as the “compressibility drag rise” is clearly shown by this comparison in Sketch 7.2.

6. REFERENCES

1. ESDU Representation of drag in aircraft performance calculations (with Addenda A to E). ESDU Data Item No. 81026, Sept. 1981 (with Amendment A, Nov. 1982).
2. ESDU Derivation of influence coefficients for effects of small changes on level speed. ESDU Memor. No. 60, May 1986.