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Characteristics of atmospheric turbulence near the ground Part III: variations in space and time for strong winds (neutral atmosphere)

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CHARACTERISTICS OF ATMOSPHERIC TURBULENCE NEAR THE GROUND PART III: VARIATIONS IN SPACE AND TIME FOR STRONG WINDS (NEUTRAL ATMOSPHERE)

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CHARACTERISTICS OF ATMOSPHERIC TURBULENCE NEAR THE GROUND Part III: variations in space and time for strong winds (neutral atmosphere)

1. NOTATION* AND UNITS

		SI	British
a_{ii}	decay constant in exponent of Equation (7.5) for $\gamma_{ii}(\Delta r)$		
$C_{ii}\{r, r'; \tau\},$ $C_{ii}\{\Delta r\}$	cross-covariance function; time averaged value of product $i\{r; t\} \cdot i\{r'; t + \tau\}$	m^2/s^2	ft^2/s^2
f	longitudinal correlation function, see Equation (5.5)		
g	lateral correlation function, see Equation (5.6)		
h	gradient height, <i>i.e.</i> height of upper edge of Earth's boundary layer; see Section 3	m	ft
$i\{r; t\}, i$ $j\{r; t\}, j$	general value of gust component at point r or r' and time t where $i, j = u, v$ or w	m/s	ft/s
rL_i	general length scale of i -component of turbulence in direction of r (see Section 6)	m	ft
xL_u, yL_v, zL_w	principal longitudinal length scales of turbulence measured along $r = x, y$ or z axis respectively (see Section 6)	m	ft
n	frequency	Hz	Hz
$P\{i, j\},$ $p\{i, j\}$	joint-probability and joint-probability density functions respectively (see Section 4)	s^2/m^2	s^2/ft^2
$P_{ii}\{r, r'; n\},$ $P_{ii}\{\Delta r\}$	co-spectral density function, in-phase component of $S_{ii}\{\Delta r\}$	m^2/s	ft^2/s
$Q_{ii}\{r, r'; n\},$ $Q_{ii}\{\Delta r\}$	quad-spectral density function, out-of-phase component of $S_{ii}\{\Delta r\}$	m^2/s	ft^2/s
$r; r'$	position vectors denoting points (x, y, z) and (x', y', z') respectively	m	ft
$\tilde{r}_f$	longitudinal non-dimensional separation when Δr is in direction of i -component; $\Delta r/rL_i$		

* For notational convenience, the dependency of the various cross-correlation and cross-spectral functions on the relative spatial location of the points (x, y, z) and (x', y', z') is denoted by Δr ; in practice the various functions are also dependent on $z_m = \frac{1}{2}(z + z')$. Also, the functional dependence on τ or n is dropped from the notation used in the main text.

$\tilde{r}_g$	lateral non-dimensional separation when Δr is normal to direction of i -component; $\Delta r / (2^r L_i)$		
Δr	distance separating points r and r' ; $(\Delta x^2 + \Delta y^2 + \Delta z^2)^{1/2}$	m	ft
Δr_e	equivalent separation distance for non-zero time lag; $((\tau V_m + \Delta x)^2 + \Delta y^2 + \Delta z^2)^{1/2}$	m	ft
Δs_i	component of Δr_e resolved in direction in which gust component i is acting	m	ft
$S_{ii}\{n\}, S_{ii}$	single point spectral density function	m ² /s	ft ² /s
$S_{ii}\{r, r'; n\}, S_{ii}\{\Delta r\}$	cross-spectral density function	m ² /s	ft ² /s
t	time	s	s
u, v, w	fluctuating or gust components of wind speed along x, y and z axes respectively	m/s	ft/s
V_m	average of hourly-mean wind speed at points r and r' ; $V_m \approx V_z$ at $z = z_m$	m/s	ft/s
V_z	hourly-mean wind speed at height z over the site	m/s	ft/s
V_{10}	V_z at $z = 10$ m over site	m/s	ft/s
x, y, z	system of rectangular cartesian co-ordinates with x -axis defined in direction of mean wind and z -axis vertical	m	ft
$\Delta x, \Delta y, \Delta z$	$x - x', y - y', z - z'$	m	ft
Δx_e	equivalent longitudinal separation equal to $x - x' + V_m \tau$	m	ft
z	effective height above ground level obstructions	m	ft
z_m	mean effective height; $1/2(z + z')$	m	ft
z_0	surface roughness parameter of site	m	ft
$\gamma_{ii}^2\{r, r'; n\}$ $\gamma_{ii}^2\{\Delta r\}$	coherence function; $ S_{ii}^2\{\Delta r\} / [S_{ii} \cdot S'_{ii}]$		
η	generalised non-dimensional separation parameter given by Equation (7.10) for isotropic turbulence (see footnote to Section 2.2)		
η_1	transformed value of η given by Equation (7.12) for non-isotropic turbulence		
$\theta_{ii}\{\Delta r\}; \theta_{ii}$	phase shift or lag angle (see Section 7.2)	rad	rad
$\rho_{ii}\{r, r'; \tau\}, \rho_{ii}\{\Delta r\}$	cross-correlation function; $C_{ii}\{\Delta r\} / (\sigma_i \sigma'_i)$		

$\rho_{ii}\{\Delta r; 0\}$	zero-lag cross-correlation function		
$\rho_{ii}\{\Delta r\}_{sym}$	symmetrical or even part of cross-correlation function		
$\rho_{ii}\{\Delta r\}_{antisym}$	anti-symmetrical part of cross-correlation function		
σ_i	standard deviation of u -, v - or w - component for averaging time $\rightarrow 0$ over period $\rightarrow 1$ hr	m/s	ft/s
τ	incremental time lag	s	s
$\phi_{ii}\{\Delta r\}; \phi_{ii}$	eddy slope, $\theta_{ii}\{\Delta r\}/(2\pi n\Delta r/V_m)$	rad	rad

Suffixes

i, j	refer to u -, v - or w - component
r	refers to direction of separation between points r and r'

A prime (') denotes a value at the point r' or (x', y', z') .

2. BACKGROUND AND SUMMARY OF DATA PRESENTED

2.1 Purpose and Applicability

The fluctuating wind may be considered to be composed of a mean value on which is superimposed turbulence comprising fluctuating gust velocities in different directions. The properties of the mean wind speed (variation with height, surface roughness and averaging time) are dealt with in Items 82026, 84011 and 83045. The definitions of the statistical terms used to describe the properties of atmospheric turbulence and other background information are given in Item 74030¹. Data giving the properties of turbulence in the atmospheric boundary layer measured at single points in space (probability densities, variances, autocorrelation and spectral density functions) are given in Item 85020³.

The purpose of this Item is to provide a means of determining the statistical properties describing the relation between the same gust components (u , v or w) measured at different points in space. Data are presented for heights throughout the atmospheric boundary layer for strong winds ($V_{10} \geq 10$ m/s) or neutral atmosphere conditions.

The main application is to structures (either aircraft or ground-based) of sufficient size that the non-uniform variation of gusts acting at different points causes a reduction in the maximum fluctuating loading. This is due to the incomplete correlation of gust properties over the structure at a particular instant or at a particular frequency. The same effect may also induce a significant fluctuating torque even when the mean torque is zero. Procedures using these data for estimating along-wind loading due to buffeting by turbulence are described in Item 76001 (Superseded by Items 87035 and 88019).

In general, the statistical quantities that describe these properties in the amplitude, time and frequency domains are:

- (i) joint-probability densities,
- (ii) cross-correlation functions,
- (iii) cross-spectral densities or coherence functions.

The data presented in this Item describe the relation between the same gust components measured at different points in space. The cases considered include all three components (u , v and w) at points separated in the three principal directions (x , y , z) but guidance is given on extending the data to apply to points separated in any general direction.

2.2 Basis of the Data Presented

In general, reliable data for cross-correlations or coherence functions for strong winds (neutral atmosphere) are sparsely reported in the literature and many of these show considerable variations from each other due to

- (i) inadequate data processing in some cases, particularly those data with trends,
- (ii) non-standard conditions, *e.g.* non-neutral atmospheric stability and non-uniform terrain.

Because of this, the data presented in this Item are based on theoretical models, modified to take into account the deviations from the theory as obtained from those data that have been assessed to be the most reliable. In this way the effect of parameters such as terrain roughness, mean wind speed and height above the ground can be taken into account systematically. Additionally, extrapolation of the model to situations where no measured data are available can be made with more confidence, particularly for the high wind speeds appropriate to design conditions.

General expressions for the cross-correlation functions, derived from the modified von Karman model described in Item 85020, are given in Section 5. The corresponding Fourier transform of these relations, to provide cross-spectral density functions (or in normalised form, the coherence functions) cannot be obtained in convenient analytical form. Because of this a separate model is developed, again based on the theoretical von Karman equations for isotropic* turbulence but modified empirically to account for the significant departure from isotropy near the ground. This model is described in Section 7.

The theoretical equations on which the models are based are complicated expressions and for convenience they have been fitted with simpler expressions with little loss in accuracy.

Integral length scales of turbulence are required in estimating both cross-correlation and coherence data and equations for them are given in Section 6. These are based on measured data and are consistent with the concepts of similarity theory and theoretical relationships which enable length scales to be expressed in terms of the standard deviation (σ_i) and hourly-mean wind speed. In practice it is possible to relate all length scales to ^XL_u by simple expressions dependent on z/h .

2.2.1 Sources of data and uncertainty

The majority of available measurements (summarised in Table 2.1) are for relatively smooth terrains although some data for built-up areas are available in Derivations 6 and 16. Measurements at points with vertical separations are limited to heights below about 180 m and measurements with lateral separations are limited to heights below about 60 m. Furthermore there are comparatively few detailed measurements in strong winds (neutral atmosphere) and in particular it must be emphasised that no measurements are available for the high wind speeds associated with design conditions ($V_{10} > 20$ m/s). However, despite the limitations of the available data, the theoretical basis of the models adopted lends confidence in extrapolating to conditions that are different from those for which the applicability of the model has been confirmed. Thus, the data in this Item may be used for wind speeds up to design conditions, for heights up to at least 300 m and, tentatively, for surface roughnesses corresponding to $0.0001 \text{ m} \leq z_0 \leq 0.7 \text{ m}$.

* For isotropic turbulence the statistical properties do not change when the reference co-ordinate axes are rotated, *i.e.* the properties are independent of direction in the turbulence field. This only occurs in the layer above that containing the taller ground-based structures.

It is not possible to assess precisely the accuracy of the data presented in this Item although by taking $\pm 25\%$ on rL_i an estimate can be made of the uncertainty of the various functions that would be obtained under conditions of ideal processing and stationarity.

TABLE 2.1 Summary of Principal Data Considered

Derivation No. (see Section 8)	Velocity Components	Separation Δr	Height range of measurements (m)	Terrain
8	$u; v$	Δz	15 – 155	summary of several sources (open country, brush woods, desert brush)
9, 10, 16	$u; w$	Δy	20, 40	island; coastal waters and
	u	Δz	30 – 150	open country
12	$u; v$	Δy	3 – 18	short grass
12	$u; v$	Δz	17 – 182	open country
15, 17	$u; v$	$\Delta y, \Delta z$	10 – 60	wind from suburban terrain
18	u	Δz	1 – 8	wind from the sea
19	$u; v; w$	Δy	11	snow-covered rolling terrain
20	u	Δy	3; 57	ice; water
21	$u; v; w$	Δz	3 – 22	open country
21	u	Δy	10	open country

3. INPUT DATA

A description of the atmospheric boundary layer and the mechanisms producing turbulence is contained in Item No. 74030. The main factors that affect the production and properties of turbulence in the neutral atmospheric boundary layer (and hence the parameters that are required as input data when using this Item) are as follows.

- (i) The hourly-mean wind speed at the site in question: the variation of mean wind speed with height and terrain roughness is dealt with in Items 82026 and 84011 and the user should refer to those documents for data appropriate to strong wind or neutral atmosphere conditions.
- (ii) The longitudinal integral length scale of turbulence, ^xL_u (from which all other length scales can be determined) for the appropriate terrain, wind speed and heights above the ground over the site at which data are required: this can be obtained from Item 85020. **Note that due to their dependence on mean wind speed, length scales at design wind speeds ($V_{10} = 20$ to 35 m/s) are much larger than typical values reported in the literature which have been obtained at more moderate wind speeds (typically $V_{10} = 10$ m/s).** Note also that the effect on ^xL_u of any significant step changes in terrain roughness upwind of the site should be taken into account as described in Item 85020; this is particularly important when the site is in a built-up area.
- (iii) The atmospheric boundary layer depth, h : for the purposes of using this Data Item it is a sufficient approximation to use $h = (1/6) u_* \times 10^4$ where u_* is the friction velocity given by $u_* = V_{10}/(2.5 \ln(10/z_0))$. Note that with V_{10} and z_0 in m/s and m respectively, the unit of h is metres.

To determine the cross-covariance ($C_{ii}\{\Delta r\}$) or cross-spectral density ($S_{ii}\{\Delta r\}$) from data given in this Item, values of S_{ii} and σ_i are required. These can be obtained from Item 85020 for appropriate values of V_z , z and z_0 . For cases where the separation between points is not in the vertical direction (*i.e.* $z = z'$), it may be assumed that $S_{ii} = S'_{ii}$ and $\sigma_i = \sigma'_i$.

4. JOINT-PROBABILITY DENSITY

The probability of the occurrence of a single event or quantity, or the probability density distribution of a single fluctuating quantity (such as u) is described in Items 74030 and 85020. However, it is sometimes necessary to consider the combined probability $P\{i, j\}$ that particular values (i_p and j_p) of two components $i\{r\}$ and $j\{r'\}$ will occur together.

When these quantities are entirely unrelated, the combined probability is simply the product of the probabilities of the two separate events. For example, the joint probability of throwing a (1) followed by a (2) in successive throws of a conventional six-sided dice is $P(1, 2) = P(1) \cdot P(2) = (1/6) \times (1/6) = 1/36$.

When these quantities are completely correlated then the joint and the individual probabilities are equal.

In practice these quantities are usually partly related and the joint probability lies between these two limiting cases. It is thus not only a function of the individual probability densities $p(i)$ and $p(j)$ but depends in a more general way on i and j . The joint probability $P\{i, j\}$ that the two fluctuating quantities $i\{r\}$ and $j\{r'\}$ will be less than i_p and j_p respectively is related to the joint-probability density function in a way analogous to that for a single variable (see Items 74030 and 85020), *i.e.*

$$P\{i, j\} = \int_{-\infty}^{j_p} \int_{-\infty}^{i_p} p\{i, j\} di dj \quad (4.1)$$

where $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p\{i, j\} di dj = 1.$ (4.2)

Very few joint-probability density measurements of related gust components in the atmosphere have been made. In the absence of more specific information a Gaussian, or normal, joint-probability density function may be assumed which, for two fluctuating quantities i and j (here $i = u, v$ or w), is given by

$$p\{i, j\} = \frac{1}{2\pi\sigma_i\sigma_j[1-\rho_{ij}^2]^{1/2}} \exp - \left[\frac{i^2\sigma_j^2 - 2ij\sigma_i\sigma_j\rho_{ij} + j^2\sigma_i^2}{2\sigma_i^2\sigma_j^2[1-\rho_{ij}^2]} \right] \quad (4.3)$$

where ρ_{ij} is the corresponding cross-correlation function (see Section 5.1).

5. CROSS-CORRELATION FUNCTIONS

The cross-correlation functions characterise the relationship between fluctuating velocity components at 2 points in space (separated by Δr) and in the general case at different times (time lag τ). The zero-lag ($\tau = 0$) cross-correlations are especially important since they describe how the instantaneous fluctuating component of wind velocity varies in space. This forms the basis of calculating the maximum fluctuating wind-loading component when dynamic effects due to oscillation of the structure can be ignored.

Because changes in the gust velocity at one point (r) are not necessarily reflected immediately by similar changes in the gust velocity at another point (r'), the cross-correlation functions are not, in general,

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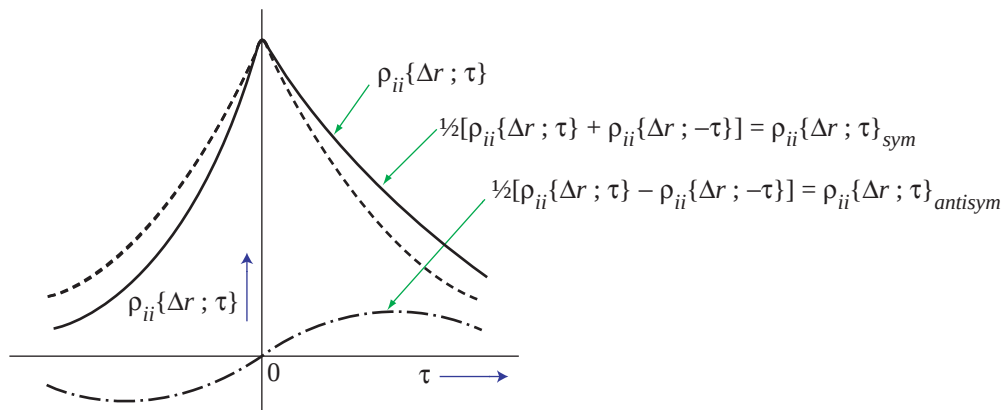
symmetrical functions of τ about the $\tau = 0$ axis. For example, changes in u at height z and time t are followed by related changes in u at a lower height z' at some time $t + \tau$ later. However, as shown in Item No. 74030, the cross-correlation function can be considered to be composed of two components, the larger one being a symmetrical function of τ and the other being an anti-symmetrical function of τ (see Sketch 5.1). These are Fourier transforms of the real (co-spectral density) and imaginary (quad-spectral density*) parts respectively of the cross-spectral density function (see Section 7). Thus the cross-correlation function can be written as

$$\rho_{ii}\{\Delta r\} = \rho_{ii}\{\Delta r\}_{sym} + \rho_{ii}\{\Delta r\}_{antisym} \quad (5.1)$$

where
$$\rho_{ii}\{\Delta r\}_{sym} = \frac{1}{\sigma_i \sigma_i'} \int_0^\infty P\{\Delta r\} \cos(2\pi n\tau) dn \quad (5.2)$$

and
$$\rho_{ii}\{\Delta r\}_{antisym} = \frac{1}{\sigma_i \sigma_i'} \int_0^\infty Q\{\Delta r\} \sin(2\pi n\tau) dn. \quad (5.3)$$

However, more convenient expressions for the zero-lag and symmetrical parts of the cross-correlation functions are given in Section 5.1.



Sketch 5.1 Illustration of symmetrical and anti-symmetrical parts of cross-correlation function

5.1 Analytical Expressions for the Cross-correlation Functions

The zero-lag ($\tau = 0$) cross-correlations, and the even or symmetrical part of the cross-correlation function for $\tau = 0$, can be obtained in terms of the basic longitudinal and lateral autocorrelation functions (f and g) given in Item 85020†. Assuming that a time lag τ can be converted to an equivalent spatial separation equal to $V_z \tau$ (Taylor's hypothesis) the symmetrical part of the cross-correlation function can be estimated from the modified equations for isotropic turbulence with appropriate values of the length scale to allow for the distortion of turbulence close to the ground. In general terms Equation (5.4) applies when $i = u, v$ or w and is valid for separations between two points in any direction with or without a time lag τ . The important zero-lag cross-correlation cases are summarised in Section 5.1.1 since these functions are of

* The quad-spectral density function is usually small and so is often ignored (see Section 7.2). It follows that the anti-symmetrical part of $\rho_{ii}\{\Delta r\}$ is also small and negligible. Furthermore, when used, for example, in calculating the fluctuating component of wind loading due to turbulence the antisymmetric and quad-coherence components integrate to zero over the structure.

† The autocorrelation functions in Item 85020 are derived from the von Karman equations for isotropic turbulence with modifications to account for observed departures from the theory close to the ground.

particular importance when evaluating fluctuating wind loads on structures.

$$\rho_{ii} \{ \Delta r \}_{sym} = \frac{f-g}{\Delta r_e^2} \Delta s_i^2 + g. \quad (5.4)$$

The longitudinal and lateral correlation functions (f and g respectively) can be evaluated from Figure 1, taken from Item 85020. The parameters Δs_i and Δr_e are equated respectively to the longitudinal and lateral separations with respect to the axis along which the component (u , v or w) acts and these are summarised in Table 5.1 and also on Figure 1.

As can be seen from Figure 1, and noted in Item 85020, the dependence on z/h is small. To a sufficiently close approximation the f and g functions can be represented by the following equations, independent of z/h .

$$f = \frac{1}{2} (f_1 + f_1^2) \quad (5.5)$$

$$g = \frac{1}{2} (g_1 + g_1^2) \quad (5.6)$$

where $f_1 = \exp[-0.822(\tilde{r}_f)^{0.77}] \quad (5.7)$

$$g_1 = \exp[-1.23(\tilde{r}_g)^{0.85}] \quad (5.8)$$

where $\tilde{r}_f = \Delta r_i / {}^rL_i$ and $\tilde{r}_g = \Delta r_i / (2 {}^rL_i)$.

The length scale parameter, ${}^rL_i$, depends on the gust components ($i = u, v$ or w) under consideration and the direction in space of the spatial separation. In evaluating the f -function, ${}^rL_i$ is taken as the longitudinal length scale ${}^xL_u$, ${}^yL_v$ or ${}^zL_w$ for the u -, v - or w - component respectively. For the g -function a compound lateral length scale is used based on weighted contributions from each lateral separation normal to the axis along which the particular gust component acts. For convenience these parameters are summarised in Table 5.1 and equations for each length scale are given in Section 6.

TABLE 5.1 Summary Chart for Cross-Correlation Functions (Equation (5.4))

Component	u	v	w
Δr^2	$= \Delta x^2 + \Delta y^2 + \Delta z^2$		
Δs_i^2	$(\tau V_m + \Delta x)^2$	Δy^2	Δz^2
Δr_e^2	$= (\tau V_m + \Delta x)^2 + \Delta y^2 + \Delta z^2$		
$\underline{f\text{-function}} \Delta r_i$	$= \Delta s_i$	$= \Delta s_i$	$= \Delta s_i$
${}^rL_i$	${}^xL_u$	${}^yL_v$	${}^zL_w$
$\underline{g\text{-function}} \Delta r_i$	$= (\Delta r_e^2 - \Delta s_i^2)^{1/2}$		
${}^rL_i$	see Equation (6.15)	see Equation (6.16)	see Equation (6.17)

5.1.1 Zero-lag cross-correlations

The zero-lag cross-correlations are of particular interest and those of more practical importance are given by Equation (5.4), or Figure 1, as follows

$$\rho_{uu}\{\Delta y; 0\} = g \text{ with } \tilde{r}_g = \Delta y / (2^y L_u) \quad (5.9)$$

$$\rho_{uu}\{\Delta z; 0\} = g \text{ with } \tilde{r}_g = \Delta z / (2^z L_u) \quad (5.10)$$

$$\rho_{uu}\{\Delta r; 0\} = g \text{ with } \tilde{r}_g = \Delta r / (2^r L_u); \Delta r = (\Delta y^2 + \Delta z^2)^{1/2} \quad (5.11)$$

$$\rho_{vv}\{\Delta z; 0\} = g \text{ with } \tilde{r}_g = \Delta z / (2^z L_v) \quad (5.12)$$

$$\rho_{ww}\{\Delta y; 0\} = g \text{ with } \tilde{r}_g = \Delta y / (2^y L_w) \quad (5.13)$$

6. LENGTH SCALES OF TURBULENCE

The length scales of turbulence are a comparative measure of the “average size” of a gust in appropriate directions and are important scaling factors in determining how rapidly gust properties vary in space. For example, ^yL_u is a length scale of the u gust component measured in the y direction. In practice it is usually defined as an integral length scale since it is determined from an integration, with respect to separation distance, of the appropriate zero-lag cross-correlation function. Thus,

$$^yL_u = \int_0^\infty \rho_{uu}\{\Delta y; 0\} d(\Delta y)$$

where $\rho_{uu}\{\Delta y; 0\}$ is the cross-correlation function for zero lag between u gust components measured at points separated in the y direction. The length scales ^xL_u , ^yL_v and ^zL_w are called longitudinal length scales because the direction of the gust components being correlated at two points is parallel to the line joining the two points. Conversely, the length scales ^yL_u , ^zL_u , ^xL_v , ^zL_v , ^xL_w and ^yL_w are called lateral length scales. The length scales increase with increasing height above the ground up to an asymptotic value at the edge of the atmospheric boundary layer (gradient height). Also, for a given height, they increase with increasing wind speed and surface roughness. The reason for this is that as wind strength increases the boundary layer height increases and eddies within the boundary layer are stretched accordingly.

At the upper edge of the boundary layer, for a given wind speed and surface roughness, all the longitudinal length scales tend to the same value which is equal to twice* the value of the lateral scales. This is why, when lateral separations and length scales are involved, the non-dimensional separation parameter (relating separation distance to typical eddy size) appears in the form $\Delta r / (2^r L_i)$. In reality, the ratio between the longitudinal and lateral length scales is 2 only in the upper part of the boundary layer; at lower heights this ratio increases and is different for each component.

* The factor of ‘2’ derives from the condition for flow continuity which imposes the relationship $g = f + \frac{1}{2} \tilde{r} \, df/d\tilde{r}$ which gives the result

$$\text{that } \int_0^\infty f \, d\tilde{r} = 2 \int_0^\infty g \, d\tilde{r}.$$