

## FRictionAL AND RETARDING FORCES ON AIRCRAFT TYRES

### PART IV: ESTIMATION OF EFFECTS OF YAW

#### 1. NOTATION AND UNITS

Tyre geometrical properties and positive directions of forces and displacements are defined in Sketch 1.1. Most equations in this Item are expressed in non-dimensional form so that any consistent set of units can be used. Because of a lack of consistency in usage, the terms “side force”, “cornering force” and “normal force” are avoided as far as possible.

|              |                                                                                                                                                                                                         | SI                     | British             |
|--------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------|---------------------|
| $C_c$        | coefficient in expression for $N$ , see Table 11.1                                                                                                                                                      |                        |                     |
| $D$          | diameter of undeflected, inflated tyre                                                                                                                                                                  | m                      | in                  |
| $G$          | resultant tangential force on tyre acting in ground plane<br>(= $(G_{R,\psi}^2 + G_{\psi}^2)^{1/2}$ for unbraked yawed rolling and<br>( $G_B^2 + G_{\psi}^2$ ) <sup>1/2</sup> for braked yawed rolling) | N                      | lbf                 |
| $G_B$        | tangential force* on tyre due to braking alone ( $G_B$ acts in ground plane and in plane of wheel rotation)                                                                                             | N                      | lbf                 |
| $G_R$        | rolling resistance* of tyre and wheel (unyawed)                                                                                                                                                         | N                      | lbf                 |
| $G_{R,\psi}$ | rolling resistance* of tyre and wheel yawed at angle $\psi_{tyre}$ to $V_K$ ( $G_{R,\psi}$ acts in ground plane and in plane of wheel rotation)                                                         | N                      | lbf                 |
| $G_{skid}$   | tangential force* on tyre in non-rotating or “locked” condition ( $G_{skid}$ acts in ground plane and parallel to $V_K$ )                                                                               | N                      | lbf                 |
| $G_{\psi}$   | force* normal to plane of rotation of tyre yawed at angle $\psi_{tyre}$ to $V_K$ ( $G_{\psi}$ acts in ground plane)                                                                                     | N                      | lbf                 |
| $h$          | half length of tyre-ground contact area, see Equation (4.4) in Item No. 86005 (Reference 26) for values                                                                                                 | m                      | in                  |
| $N$          | derivative of $G_{\psi}$ with respect to $\psi_{tyre}$ at small values of yaw, $\left[ \frac{dG_{\psi}}{d\psi_{tyre}} \right]_{\psi_{tyre} \rightarrow 0}$ , (“cornering power”)                        | N/deg                  | lbf/deg             |
| $p$          | inflation pressure <sup>†</sup> (gauge) of undeflected tyre                                                                                                                                             | N/m <sup>2</sup> , bar | lbf/in <sup>2</sup> |
| $p_r$        | tyre “rated” inflation pressure <sup>†</sup>                                                                                                                                                            | N/m <sup>2</sup> , bar | lbf/in <sup>2</sup> |
| $q$          | pneumatic caster* (or pneumatic trail); distance behind axis of rotation at which $G_{\psi}$ acts                                                                                                       | m                      | in                  |
| $s$          | braking slip ratio $[1 - (\omega/\omega_R)]$                                                                                                                                                            |                        |                     |

For footnotes refer to end of Notation Section

|               |                                                                                                                                                              |       |        |
|---------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|--------|
| $T_{sa}$      | tyre self-aligning torque, positive when it acts to reduce absolute value, $ \psi_{tyre} $ , of tyre yaw angle                                               | N m   | lbf in |
| $V_K$         | velocity of tyre along “flight” path                                                                                                                         | kn    | kn     |
| $w$           | width of undeflected, inflated tyre section                                                                                                                  | m     | in     |
| $X_k^{tyre}$  | component of tangential force on tyre acting in ground plane and parallel to flight path, positive in $V_K$ direction                                        | N     | lbf    |
| $Y_k^{tyre}$  | component of tangential force on tyre acting in ground plane and normal to flight path, positive to starboard                                                | N     | lbf    |
| $Z$           | normal (to ground) load on wheel                                                                                                                             | N     | lbf    |
| $\delta$      | tyre deflection under normal (to ground) load, see Item No. 86005 (Reference 26) for values                                                                  | m     | in     |
| $\mu_B$       | coefficient of friction, $G_B/Z$                                                                                                                             |       |        |
| $\mu_{skid}$  | locked-wheel coefficient of friction, $G_{skid}/Z$                                                                                                           |       |        |
| $\mu_\psi$    | coefficient of friction, $G_\psi/Z$                                                                                                                          |       |        |
| $\phi$        | tyre yaw-angle parameter = $\frac{N \times \psi_{tyre}}{[G_\psi]_{max} \times \left[ \frac{2}{3} + \frac{[\mu_\psi]_{max, dry}}{3[\mu_\psi]_{max}} \right]}$ |       |        |
| $\psi_{tyre}$ | tyre yaw <sup>‡</sup> (or slip <sup>‡</sup> ) angle; angle between plane of rotation and projection of $V_K$ on to ground plane                              | deg   | deg    |
| $\omega$      | wheel angular velocity                                                                                                                                       | rad/s | rad/s  |
| $\omega_R$    | wheel angular velocity in unbraked rolling                                                                                                                   | rad/s | rad/s  |
| $  $          | absolute value of enclosed parameter                                                                                                                         |       |        |

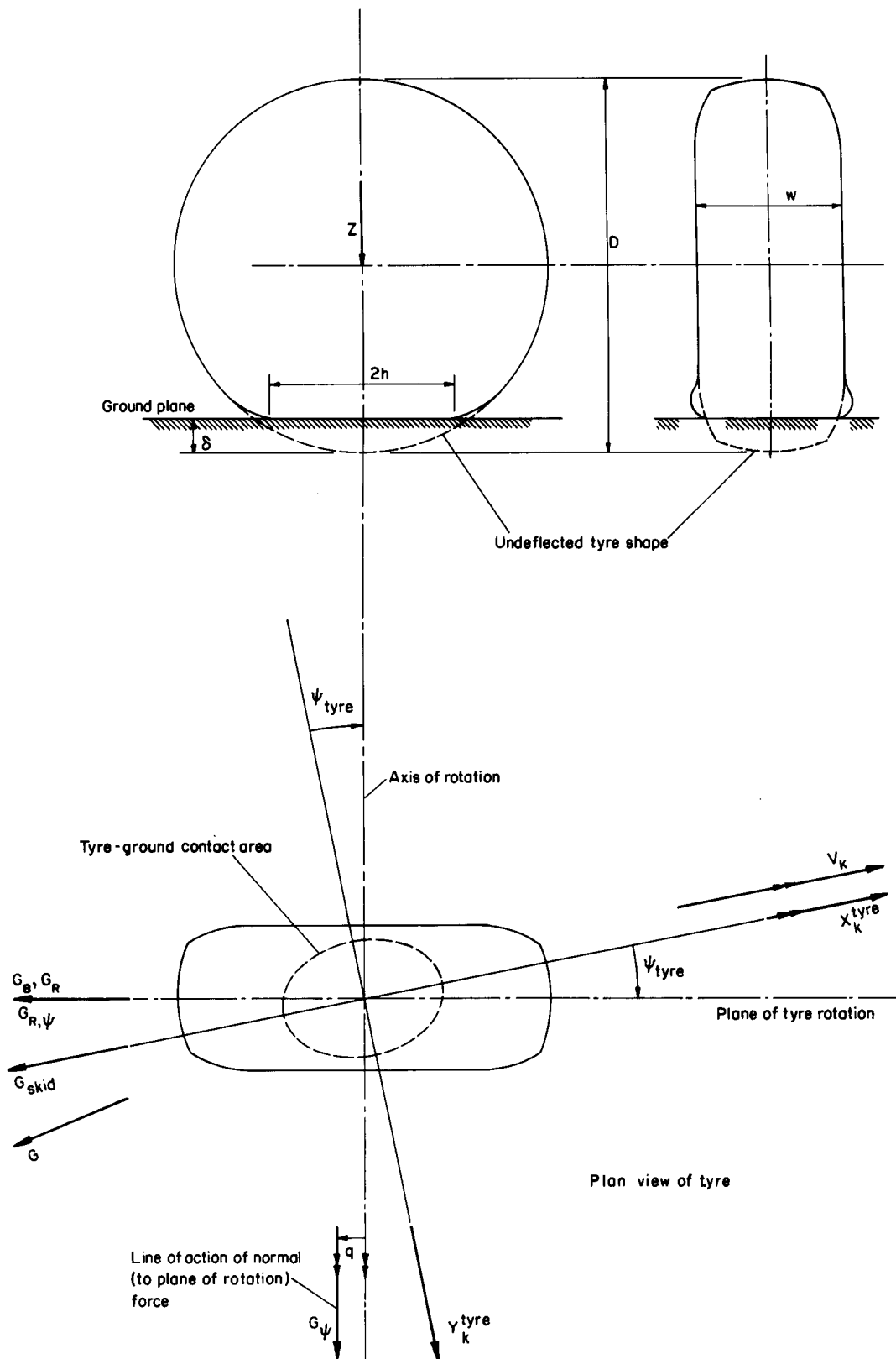
### Subscripts

|            |                                                                                                                           |
|------------|---------------------------------------------------------------------------------------------------------------------------|
| <i>dry</i> | denotes value appropriate to dry surface conditions                                                                       |
| <i>lim</i> | denotes limiting values of forces or friction coefficients during braked yawed rolling, see Sketch (6.1)                  |
| <i>max</i> | denotes maximum attainable value of force or friction coefficient in unbraked yawed rolling and in braked unyawed rolling |
| <i>wet</i> | denotes value appropriate to wet surface conditions                                                                       |

\* Sketch 1.1 shows positive directions of forces and displacements.

† Note, 1 bar =  $10^5$  N/m<sup>2</sup>.

‡ Sketch 1.1 shows positive direction. Use of the term “slip angle” is widespread but is avoided here because of the possibility of confusion with the term “braking slip” in other Data Items.



**Sketch 1.1 Notation for tyre geometry and forces**  
Arrows show positive directions

## 2. INTRODUCTION

The Item is intended for use in studies of aircraft ground handling where lateral forces are significant. Examples are estimation of values of minimum control speed (on the ground) and investigations of stopping performance with asymmetric reverse thrust and/or asymmetric braking and/or cross winds and/or low friction surfaces.

Table 11.1 gives empirically-derived expressions for several properties of aircraft-type\* tyres during unbraked yawed rolling on hard surfaces (definitions are given in Section 2.1). Sections 3 and 4 describe estimation methods for tyre forces and moments, respectively, during unbraked yawed rolling. Section 5 and Figure 1 give guidance on estimating values of the friction coefficient required in the calculations. Section 6 provides guidance on the likely combined effects of braking and yawed rolling. Section 7 indicates how the forces estimated in Sections 3, 4 and 5 are incorporated into a wider calculation scheme, such as that of Reference 25. Section 8 gives an outline of the derivation of the methods presented in the Item; more complete details are given in Reference 27. The worked example in Section 10 shows how the methods are applied to a typical transport aircraft nose wheel. All the information in the Item applies to tyres operating with zero tilt (*i.e.* plane of rotation is normal to plane of runway); some information on the effect of tilt angle is given in Reference 23.

An Introductory Item (No. 71025, Reference 8) describes how tyre **braking** forces are developed and discusses the effects of various tyre, runway surface and aircraft operational and design factors. Many of these features can be interpreted to provide a background to the development of forces during yawed rolling (whether unbraked or braked).

### 2.1 Definitions

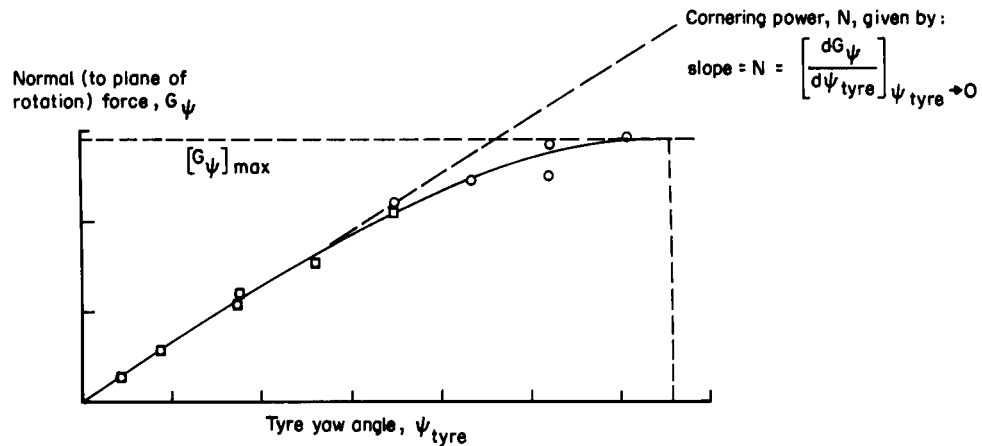
Sketch 2.1 shows an example of the variation of tyre **normal (to plane of rotation) force**,  $G_{\psi}$ , with yaw angle,  $\psi_{tyre}$ , for unbraked rolling and includes an illustration of how **cornering power**,  $N$ , is defined as the slope,  $[dG_{\psi}/d\psi_{tyre}]$ , as  $\psi_{tyre} \rightarrow 0$ . Sketch 2.2, taken from Reference 3, shows how several sets of such data are correlated using the parameter,  $G_{\psi}/[G_{\psi}]_{max}$ , plotted against a yaw-angle parameter,  $\phi$ . Section 8 describes the derivation of this correlation for dry surfaces and its adaptation to wet surfaces and to braked yawed rolling.

Tyre **self-aligning torque**,  $T_{sa}$ , acts in the plane of the tyre-ground contact area during yawed rolling. It equals the product of the normal (to plane of rotation) force and the distance behind the wheel axle at which this force acts. This distance, known as the “pneumatic caster” or “pneumatic trail”,  $q$ , is given by

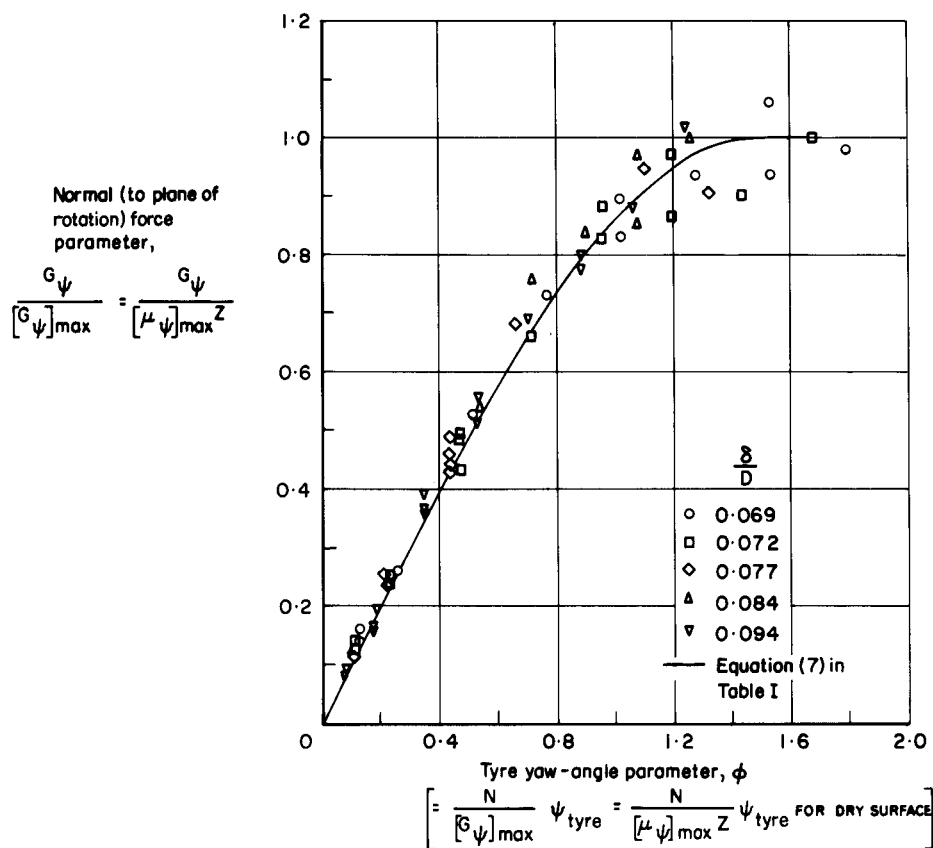
$$q = \frac{T_{sa}}{|G_{\psi}|} \quad (2.1)$$

\* In this Item, “aircraft-type tyre” implies tyres designed for aircraft use and having the following characteristics (see Item No. 71025 (Reference 8) for further details):

- (i) rated static vertical deflections of between 30 and 40 per cent of the undeflected tyre free height above the wheel flange, see Item No. 71025, Figure 1,
- (ii) tyre aspect ratios in the range 0.7 to 1.0 (ratio of tyre-section height to width when inflated but unloaded),
- (iii) cross-ply construction,
- (iv) natural rubber based tread materials (limited data on the effects of different tyre-tread materials on tyre-runway friction are given in References 15 and 18),
- (v) inflation pressures in the ranges 50 to 300 lbf/in<sup>2</sup> (3 to 21 bar approx.) although the data can probably be used reliably over a somewhat greater range, say 30 to 350 lbf/in<sup>2</sup> (2 to 24 bar approx.).



Sketch 2.1 Example of normal (to plane of rotation) force and cornering power for yawed rolling  
( $26 \times 6.6 - 12\text{PR}$  type VII tyre at low speed on hard dry surface;  $p/p_r = 1.0$ ;  $\delta/D = 0.08$ )



Sketch 2.2 Examples of variations of normal (to plane of rotation) force parameter with yaw angle parameter for steady-state yawed rolling  
( $26 \times 6.6 - 12\text{PR}$  type VII tyres at low speed on hard dry surface)

### 3. ESTIMATION OF TYRE FORCES DURING UNBRAKED YAWED ROLLING

Sections 3.1 to 3.4 describe how Equations (11.1) to (11.3) of Table 11.1 are used to estimate forces during unbraked yawed rolling.

#### 3.1 Cornering Power, $N$

Using Equations (11.1) and (11.2) the tyre cornering power,  $N$ , is estimated as a function of tyre geometry, deflection and inflation pressure. Values of tyre deflection,  $\delta$ , may be calculated using Equations (4.1) and (4.2) of Item No. 86005 (Reference 25). Values of  $N$  are required in evaluating the yaw-angle parameter,  $\phi$ , used in the present Item and provide the slope at the origin of any curve of  $G_\psi$  against  $\psi_{tyre}$ .

#### 3.2 Normal (to Plane of Rotation) Force, $G_\psi$

For cases where the yaw angle is small and the available tyre-runway friction is high (such as in dry conditions), values of normal (to plane of rotation) force can be estimated using the defining expression for cornering power to give,

$$G_\psi \approx N \times \psi_{tyre} = \left[ \frac{dG_\psi}{d\psi_{tyre}} \right]_{\psi \rightarrow 0} \times \psi_{tyre}. \quad (3.1)$$

More generally, however, and especially in cases where yaw angles are large or where the maximum value of  $G_\psi (= Z \times [\mu_\psi]_{max})$  is small, the non-linear part of the curve of  $G_\psi$  against  $\psi_{tyre}$  is required. Equation (11.3) is then used to estimate the variation of  $[G_\psi]/[G_\psi]_{max}$  with tyre yaw angle for cases where  $0 < G_\psi < [G_\psi]_{max}$ . Values of  $[G_\psi]_{max}$  are determined using known values of normal load,  $Z$ , and estimates of the maximum friction coefficient,  $[\mu_\psi]_{max}$ , from Section 5.

For yaw angles greater than that at which  $[\mu_\psi]_{max}$  occurs, no general expression is available for generating the shape of the curve of  $G_\psi$  against  $\psi_{tyre}$ . For low speeds and dry surfaces, Reference 3 recommends that  $[G_\psi]_{max}$  be used. However, an alternative procedure is to make use of the data given in Figure 2. This Figure summarises the available data for  $\mu_\psi$  at yaw angles beyond that for  $[\mu_\psi]_{max}$ . The method of presentation adopted assumes that in the limit as  $\psi_{tyre} \rightarrow 90$  deg, the value of  $\mu_\psi$  approaches that of the locked-wheel coefficient of friction,  $\mu_{skid}$ , for a smooth-tread tyre (see Section 8). A guide to the variation of  $\mu_\psi$  with  $\psi_{tyre}$  in this range of yaw angles can be obtained by combining the data in Figure 2 with values of  $\mu_{skid}$  for smooth-tread tyres taken from Figures 10 to 15 of Item No. 71026 (Reference 9).

#### 3.3 Yawed Rolling Resistance $G_{R,\psi}$

The rolling resistance,  $G_{R,\psi}$ , and the associated coefficient,  $\mu_{R,\psi}$ , **measured in the plane of rotation** of a tyre and wheel rolling with yaw angle  $\psi_{tyre}$  change little from the values obtained in unyawed conditions. Following the practice described in Section 9.2 of Item No. 85029 (Reference 24) it is suggested that whatever value, for example 0.02, is used for rolling resistance coefficient in unyawed conditions, the same value should be used for unbraked yawed rolling.

#### 4. ESTIMATION OF SELF-ALIGNING TORQUE AND PNEUMATIC CASTER (OR TRAIL) FOR UNBRAKED YAWED ROLLING

Expressions for self-aligning torque and pneumatic caster are given as Equations (11.4) to (11.9) in Table 11.1. The expressions for pneumatic caster are obtained by dividing Equations (11.4) to (11.6) by Equations (11.7) to (11.9) – as indicated in Equation (2.1). Values of  $h$ , the half length of the tyre-ground contact area, required when using those Equations, may be calculated using Equation (4.4) of Item No. 86005 (Reference 26). Values of the tyre cornering power,  $N$ , required in evaluating the yaw-angle parameter,  $\phi$ , in Equations (11.4) to (11.9) are obtained using Equations (11.1) and (11.2) – as described in Section 3.1. The derivation of Equations (11.4) to (11.9) is discussed in Section 8.6 and that Section should be consulted for advice on applicability and, in particular, regarding the effects of large yaw angles.

#### 5. ESTIMATION OF COEFFICIENT OF FRICTION FOR UNBRAKED YAWED ROLLING

To use Equations (11.3) to (11.9) in Table 11.1 it is necessary to estimate the maximum value,  $[G_\psi]_{max}$  ( $= Z \times [\mu_\psi]_{max}$ ), of the normal (to plane of rotation) force between the tyre and ground. Since the normal load,  $Z$ , is usually known or can be calculated, the major difficulty is to estimate a maximum value,  $[\mu_\psi]_{max}$ , for the coefficient of friction. For the purposes of this Item the following method of estimating values of  $[\mu_\psi]_{max}$  is proposed.

- (i) For the required tyre inflation pressure, speed, runway surface type and runway condition, use Figures 3 to 7 of Item No. 71026 (Reference 9) to estimate the maximum available **braking** coefficient of friction, *i.e.*  $[\mu_B]_{max}$  in the notation of the present Item, but denoted  $\mu_{max}$  in Item No. 71026.
- (ii) Use Figure 1 in the present Item to convert the value of  $[\mu_B]_{max}$  obtained in (i) into an estimate of  $[\mu_\psi]_{max}$  for use with the expressions in Table 11.1.

The derivation of Figure 1 is described briefly in Section 8.4 and in detail in Reference 27. Figure 1 applies to all cross-ply tyres on dry hard surfaces. However, for wet conditions it is restricted to rib-tread tyres and ungrooved hard surfaces. Advice on possible effects of both smooth-tread tyres and of grooved surfaces in wet conditions is given in Section 8.4.

Note that information on the variation of  $[\mu_\psi]$  with  $\psi_{tyre}$  at yaw angles greater than that at which  $[\mu_\psi]_{max}$  occurs is given in Figure 2 and is discussed in Sections 3.2 and 8.5.

#### 6. COMBINED BRAKING AND YAWED ROLLING

For a braked yawed tyre, values of the forces  $G_B$  and  $G_\psi$ , and of the corresponding coefficients  $\mu_B$  and  $\mu_\psi$ , lie between values corresponding to the following extremes which are represented on the axes of the right-hand part of Sketch 6.1.

- (i) For an unyawed tyre ( $\psi_{tyre} = 0$ ) the available braking friction coefficient,  $\mu_B$ , varies between zero at zero slip ( $s = 0$ ) and  $[\mu_B]_{max}$  at a corresponding value of  $s$ . See Item No. 71026 (Reference 9) for values of  $[\mu_B]_{max}$  and Section 4.2 of that Item for discussion of brake torque and energy considerations which can limit the achievable value of  $\mu_B$ .
- (ii) For an unbraked tyre ( $s = 0$ ) the value of  $\mu_\psi$  varies between zero at zero yaw ( $\psi_{tyre} = 0$ ) and  $[\mu_\psi]_{max}$  at some corresponding value of  $\psi_{tyre}$ . See Section 5 and Figure 1 of the present Item for method of estimating  $[\mu_\psi]_{max}$ .

Calculation methods for intermediate conditions of braked yawed rolling are given in Sections 6.1 and 6.2.