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Estimation of rms strain in laminated face plates of simply-supported sandwich panels subjected to random acoustic loading.

Including a simplified natural frequency prediction method

Associated software: ESDUpac A8624

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The Royal Aeronautical Society

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ESTIMATION OF R.M.S. STRAIN IN LAMINATED FACE PLATES OF SIMPLY-SUPPORTED SANDWICH PANELS SUBJECTED TO RANDOM ACOUSTIC LOADING – INCLUDING A SIMPLIFIED NATURAL FREQUENCY PREDICTION METHOD

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ESTIMATION OF R.M.S. STRAIN IN LAMINATED FACE PLATES OF SIMPLY-SUPPORTED SANDWICH PANELS SUBJECTED TO RANDOM ACOUSTIC LOADING – Including a simplified natural frequency prediction method

1. NOTATION

a	length of panel in x-direction (see Sketch 2.1)	m	in
a'	effective length of panel in x-direction, [$a' = a - 2hk$]	m	in
b	length of panel in y-direction, or arc length of curved panel (see Sketch 2.1)	m	in
b'	effective length of panel in y-direction, or arc length of curved panel, [$b' = b - 2hk$] (see Section 2.3)	m	in
E_x, E_y	face plate moduli of elasticity in x, y-directions, respectively	N/m^2	lbf/in^2
E_α, E_β	lamina moduli of elasticity in α-, β-directions, respectively	N/m^2	lbf/in^2
f_n	fundamental natural frequency of panel	Hz	Hz
G_{xy}	shear stiffness of faceplate in (x, y) plane	N/m^2	lbf/in^2
G_{xz}, G_{yz}	shear stiffness of core in (x, z) and (y, z) planes, respectively (see Section 2.4)	N/m^2	lbf/in^2
$G_{\alpha\beta}$	shear stiffness of lamina in (α, β) plane	N/m^2	lbf/in^2
h	panel thickness (see Sketch 2.1)	m	in
K_δ	damping ratio correction factor [see Equation (2.1)]		
k	multiplying factor (see Section 2.3)		
$L_{ps}(f_n)$	spectrum level of acoustic pressure at frequency f_n	dB*	dB*
m	mode number in x-direction (see Section 2.5)		
N	number of layers in face plate		
n	mode number in y-direction (see Section 2.5)		
R	internal radius of curvature	m	in
x, y, z	panel coordinate system with x- and y-axes aligned with face plate principal axes of orthotropy (see Sketch 1.1)		

For footnotes refer to end of Notation Section.

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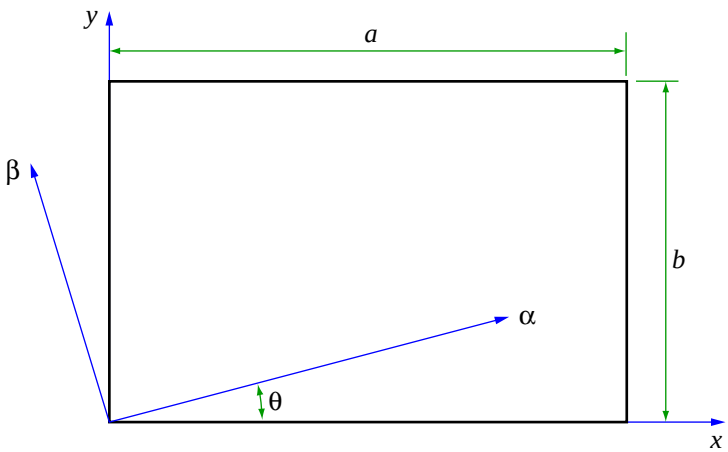
α, β	lamina coordinate system aligned with principal axes of orthotropy of lamina (see Sketch 1.1)		
δ	equivalent viscous damping ratio of fundamental mode of plate		
θ	angle of principal axis of orthotropy, α, of lamina measured from panel principal axis, x (see Sketch 1.1)	degree	degree
ν_{xy}	Poisson's ratio† for face plate resulting from load applied in x-direction		
ν_{yx}	Poisson's ratio† for face plate resulting from load applied in y-direction		
$\nu_{\alpha\beta}$	Poisson's ratio† for lamina resulting from load applied in α-direction		
ρ_c	density of core	kg/m^3	‡
ρ_f	density of face plate lamina	kg/m^3	‡

* The reference pressure for sound levels expressed in decibels is 2×10^{-5} N/m^2(*i.e.* 20 μPa) or 2.90×10^{-9} lbf/in^2.

† In some previous Items in this series, Poisson's ratio has been designated σ.

‡ A density value expressed in British units as pounds per cubic inch has to be divided by 386.4 before input into the computer program in this Item. (A force of 1 lbf acting on a mass of 1 lb produces an acceleration of 386.4 in/s^2.)



Sketch 1.1 Panel and lamina axes

2. NOTES

2.1 General

This Item provides a means of estimating the *r.m.s.* surface strains in the laminated face plates of sandwich panels subjected to random acoustic loading. The data are applicable to initially unstressed flat, or singly curved, panels and are particularly applicable to sandwich panels having fibre-reinforced laminated face

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plates. One stage within the analysis procedure is the estimation of natural frequencies of vibration so that this Item may be used to estimate the lower natural frequencies in which the predominant displacement is normal to the plane of the panel. It should be noted that the analysis procedure for natural frequencies is a simplified version of that presented in ESDU 85037*. That Item provides five natural frequencies for each combination of mode numbers (see Section 2.5), including the bending mode given by this Item. It is recommended that for a full investigation of panel natural frequencies companion ESDU 85037 should be used.

Because of the large number of lay-up configurations that may be used for the laminated face plates, it is not practical to provide strain response data in graphical form. A computer program is contained in ESDUpac A8624 with example input and output files. For sandwich panels having isotropic face plates of equal thickness, the stress response under random acoustic loading may be estimated using the graphical procedure provided in ESDU 72017†.

The programs in this Item allow the estimation of the *r.m.s.* strains in the surface of the face plates in the *x*-direction and *y*-direction at the centre of a sandwich panel. Also, natural frequencies for the panel bending modes may be estimated. The program is applicable to sandwich panels having orthotropic face plates that may be of different materials and thicknesses. The principal axes of orthotropy of the face plates must be parallel to the panel sides. The cores are assumed to have zero flexural stiffness. The panels are assumed to be simply-supported on all boundaries and may be singly-curved with shallow curvature. In practical applications simply-supported boundary conditions are seldom achieved so that effective dimensions are used to estimate panel natural frequencies and strains (see Section 2.3).

The assumption of simply-supported edges in the prediction of *r.m.s.* strains under acoustic loading has been shown to provide reasonable agreement with measurements (see Section 2.6). However, that assumption precludes the estimation of strain at the panel edges. For sandwich panels having laminated face plates, the edge detail design is likely to be specific to a particular application so that a general procedure for edge strain prediction is not practical. There is a relationship between strains at different locations on a panel so that if, for a particular panel design, that relationship is established between the panel centre strains and others of interest the latter may be evaluated.

The panel centre strains predicted using this Item provide a good indicator of the response of a particular panel design subjected to random acoustic loading. To estimate the life of a panel under acoustic loading, the predicted *r.m.s.* strains can be used to estimate endurance if suitable strain/endurance data are available. Endurance data are specific to the types of element tested, the method of testing and the method of failure identification and definition. Some endurance data for fibre-reinforced composite, laminated structural elements (but not including sandwich panel elements) are presented in ESDU 84027‡.

2.2 Method of Analysis

The method of analysis for strain response is that given in Derivation 1 and is an extension of that used for the estimation of natural frequencies in ESDU 85037. The analysis procedure is applicable to shallow cylindrical curved sandwich panels having thin face plates. It is assumed that bending is carried only by the face plates and transverse shear stresses only by the core. It is also assumed that normals to the undeformed neutral plane remain straight lines but not necessarily normal to the deformed plane, and that the transverse deflections do not vary through the thickness. Further, for the estimation of *r.m.s.* strains it is assumed that the panel responds only in its fundamental mode ($m = n = 1$) and that the loading spectral density is constant over a range of frequencies including the predominant response frequency. Usually the predominant response is at the fundamental frequency if higher frequency modes are well spaced from it.

* Natural frequencies of simply-supported sandwich panels with laminated face plates.
† Stress response of flat or singly-curved sandwich panels with cores of zero flexural stiffness subject to random acoustic loading.
‡ Endurance of fibre-reinforced composite, laminated structural elements subjected to simulated random acoustic loading.

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Also, in the estimation of strain response, it is assumed that the acoustic pressure is in phase over the whole of the panel.

In the analysis, the face plates and core are treated as orthotropic materials. The *r.m.s.* face plate stresses at the centre of the panel ($x = a/2$, $y = b/2$) are converted to strains using orthotropic stress/strain relationships. For fibre-reinforced face plates, the "apparent" elastic properties, as estimated using ESDU 85035*, are used in the analysis (these properties may also be evaluated using the program listed in this Item).

2.2.1 Spectral density of acoustic pressure

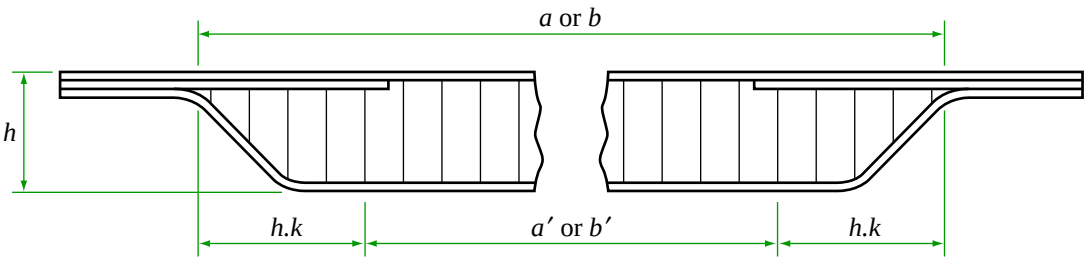
Within the program, $L_{ps}(f_n)$, the spectrum level of acoustic pressure in decibels at frequency f_n, is converted into the spectral density of acoustic pressure in units of $(\mathrm{N/m^2})^2/\mathrm{Hz}$, or $(\mathrm{lbf/in^2})^2/\mathrm{Hz}$.

2.2.2 Damping ratio, δ

For honeycomb-core sandwich panels having fibre-reinforced, laminated face plates the equivalent viscous damping ratio, δ, should be obtained by measurement. For the fundamental mode of vibration, δ typically lies within the range 0.014 to 0.055 and for a conservative estimate of *r.m.s.* strain a low value should be used. If a reference value of damping ratio is used in the *r.m.s.* strain estimation program, the *r.m.s.* strain at some other value of damping ratio may be obtained by multiplying the estimated *r.m.s.* strain by K_δ, where

$$K_\delta = \left[\frac{\text{reference damping ratio}}{\text{damping ratio}}\right]^{1/2}. \tag{2.1}$$

2.3 Effective Panel Dimensions



Sketch 2.1 Effective panel dimensions

In the theory in this Item, it is assumed that panels are simply-supported on all boundaries. This edge condition is rarely achieved in practical applications. To account for practical edge conditions it is necessary to use effective panel dimensions as shown in Sketch 2.1. The value of the multiplying factor, k, is a function of both the edge fixings and the stiffness of the support frames and is ideally chosen such that the estimated fundamental natural frequency is equal to the measured value. Limited experimental evidence suggests that for honeycomb sandwich panels with edges clamped to a "rigid" frame (*i.e.* fixed against rotation) a value of $k = 4.5$ is appropriate, and for a honeycomb panel bolted onto a frame (*i.e.* effectively simply-supported) $k = 0$. For panels for which this Item is applicable (see Section 2.6), it is expected that the value of k will lie between 0 and about 5. The estimation of k for a panel with known fundamental frequency is illustrated in the example in Section .

* Estimation of stiffnesses and apparent elastic properties of laminated flat plates.

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2.4 Core Shear Stiffnesses

The core shear stiffnesses in the (x, z) and (y, z) planes, G_{xz} and G_{yz}, respectively, are often taken to be the core shear moduli which may be obtained from manufacturers' data or from tests. A suitable test method is described in ESDU 66025 (Reference 6). For hexagonal honeycomb cores, the core shear moduli may be estimated using ESDU 67022 (Reference 7).

2.5 Mode Numbers

For simply-supported panels, the mode numbers m and n are the number of half waves across the panel in the x- and y-directions, respectively. That definition is consistent with the definition, in terms of nodal lines, used in other Items in the series.

2.6 Accuracy and Limitations of Data

In this Item a means is provided for the estimation of the surface strains, under random acoustic loading, at the centre of initially unstressed, rectangular sandwich panels having laminated face plates. Also a means of estimating the lower natural frequencies, for the bending modes in which the predominant displacement is normal to the plane of the panel, is provided. These natural frequencies closely agree with those estimated using the more exact theory in ESDU 85037.

The method of analysis is based on linear sandwich panel theory and is constrained by the assumptions detailed in Section 2.2. The theory is applicable to panels in which the displacements are small compared with the plate thickness. Also the face plates are assumed to be thin, a criterion which is met if the ratio of face plate thickness to either of the edge dimensions, a or b, is less than 0.05. The data are applicable to singly-curved panels having slight curvature. Panels may be considered to be slightly curved if the ratio of panel length, a, to radius of curvature is very much less than 1.0.

In Figures 1 and 2, a comparison is made between the estimated and measured *r.m.s.* surface strains at the centre of a sandwich panel subject to random acoustic loading. All data presented are for flat panels having fibre-reinforced, laminated face plates. For all tests, the upper and lower face plates were of identical construction, each having two or three laminae. Surface strains were estimated either by inputing the effective panel dimensions based on the measured fundamental natural frequency, or by inputing the fundamental natural frequency which was assumed to be the principal response frequency on the strain spectra. Measured damping values were used in the computation procedure. There is considerable scatter in the data plotted in Figures 1 and 2 due to an overestimation of the reported measured strains for some sandwich panels. There is no obvious explanation for this difference between estimated and measured strains. Part of the discrepancy may be due to errors in the strain gauge measurements because of the inherent difficulty of correct alignments, relative to fibre direction, and attachment of strain gauges on fibre-reinforced laminates.

3. COMPUTATION OF NATURAL FREQUENCIES AND R.M.S. STRAINS

The computer program associated with this Item, referred to as ESDUpac A8624, provides a means of estimating the bending mode natural frequencies, and strain response under random acoustic loading, of flat, or singly-curved, sandwich panels having laminated face plates and cores of zero flexural stiffness. The panels are assumed to be simply-supported on all edges. A flow chart for the program is given in Figure 3.

Known values of the elastic properties of the face plates may be input to the program. Alternatively, for fibre-reinforced laminated face plates, the elastic properties may be estimated from the individual lamina

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