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Wing pitching moment at zero lift at subcritical Mach numbers

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WING PITCHING MOMENT AT ZERO LIFT AT SUBCRITICAL MACH NUMBERS

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WING PITCHING MOMENT AT ZERO LIFT AT SUBCRITICAL MACH NUMBERS

1. NOTATION AND UNITS

		SI	British
A	aspect ratio, $b/\bar{c}$		
b	wing span	m	ft
C_{LLT}	local lift coefficient due to effective twist		
C_{m0}	pitching moment coefficient at zero total wing lift, $M_0/1/2\rho V^2 S \bar{c}$		
$(C_{m0})_1$	contribution to C_{m0} due to $(C_{m0})_\infty$, modified for planform effects		
$(C_{m0})_2$	contribution to C_{m0} due to effective twist		
c	local chord	m	ft
c_r	wing root (centre-line) chord	m	ft
c_t	wing tip chord	m	ft
$\bar{c}$	standard (geometric) mean chord, $\int_0^1 c d\eta$	m	ft
$\bar{\bar{c}}$	aerodynamic mean chord, $\int_0^1 (c^2/\bar{c}) d\eta$	m	ft
F	empirical correction factor applied to $(C_{m0i})_{\infty th}$		
f	empirical correction factor in Equation (3.9) for $(C_{m0i})_1$		
g	function of taper ratio in Equation (3.24)		
h	function of aspect ratio and sweepback in Equation (3.25)		
k	empirical factor in Equation (3.21)		
M	free-stream Mach number		
M_0	pitching moment at zero total wing lift, positive nose up	N m	lbf ft
S	wing planform area, $b\bar{c}$	m ²	ft ²
V	free-stream velocity	m/s	ft/s
x	streamwise co-ordinate, positive aft	m	ft

$x_{1/4}$	streamwise location of local quarter-chord point referred to leading edge of wing root chord	m	ft
$\bar{x}$	streamwise location of $1/4 \bar{c}$ referred to leading edge of wing root chord, see Sketch 3.1	m	ft
z_c	camber ordinate	m	ft
α_0	angle of attack at zero lift	degree	degree
β	compressibility parameter, $(1 - M^2)^{1/2}$		
δ	local geometric twist (angle of local chord relative to root chord, positive leading edge up)	degree	degree
δ_c	local camber-dependent twist in Equation (3.17)	degree	degree
δ_e	effective wing twist, $\delta + \delta_c$, see Equation (3.18)	degree	degree
δ_{et}	value of δ_e at tip	degree	degree
δ'_{et}	equivalent linear tip twist, see Equation (3.28)	degree	degree
η	spanwise distance from root as fraction of semi-span		
$\bar{\eta}$	spanwise location of $\bar{c}$, see Sketch 3.1		
η_n	$\cos (n\pi/8)$		
$\Lambda_{1/4}$	sweepback of $1/4$ -chord line	degree	degree
$\Lambda_{1/2}$	sweepback of mid-chord line	degree	degree
λ	taper ratio, c_t/c_r		
ρ	free-stream air density	kg/m ³	slug/ft ³

Subscripts

<i>expt</i>	denotes experimental value
<i>i</i>	denotes incompressible flow
<i>pred</i>	denotes predicted value
<i>r</i>	denotes value at wing root
<i>t</i>	denotes value at wing tip
<i>th</i>	denotes approximate theoretical value
0	denotes zero total wing lift

- ∞ denotes wing sectional value in two-dimensional flow
- η denotes value at spanwise location η
- η_n denotes value at η_n ($n = 1, 2, 3$), see section 3.3.2

A bar (–) over a symbol denotes an average value.

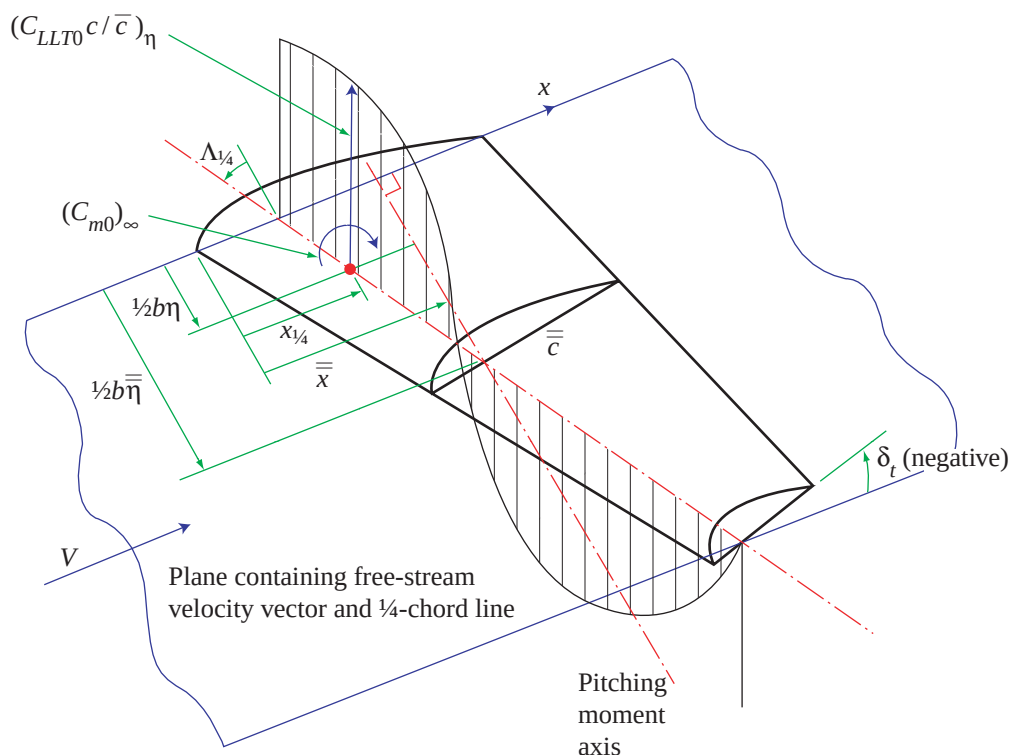
2. INTRODUCTION

This Item provides a simple semi-empirical method for estimating wing pitching moment coefficient at zero lift at subcritical Mach numbers. The method, presented in Section 3, consists of separate estimations of the low-speed contributions due to the overall effect of camber via sectional pitching moments at zero lift and due to the combined effects of geometric and camber-dependent twist, modified for the effect of Mach number in the subcritical range. The contribution due to the overall effects of camber via sectional pitching moment at zero lift (Section 3.2) is based on an approximate theoretical estimate for the wing sectional profile, corrected by means of empirical factors for the approximations involved in the theoretical estimate and for planform effects. The contribution due to the combined effects of geometric and camber-dependent twist (Section 3.3) is based on strip theory using the theoretical spanwise loading due to twist. Two forms of the method are given, a simple one for application in those cases in which the twist distribution is linear and a more general one for those cases in which the twist distribution is highly non-linear. Also given is a simpler alternative method, which is satisfactory for most cases of non-linear twist, in which the method for linear twist is employed in conjunction with an equivalent linear twist. The effect of Mach number is given in Section 3.4.

The applicability and accuracy of the method are discussed in Section 4 and an illustration of a typical use of the Item is given in worked examples in Section 6. Appendix A provides details of a computer program.

3. WING ZERO-LIFT PITCHING MOMENT

3.1 General



Hatched area is spanwise loading at zero net wing lift

Sketch 3.1 Cambered and twisted wing at zero lift

From Sketch 3.1 it can be seen that in the most general case of a cambered and twisted swept wing the wing pitching moment coefficient at zero lift can be considered to consist of two contributions. The first contribution, $(C_{m0})_1$, arises from the integration across the span of local (sectional) values, $(C_{m0})_{\infty}$, of C_{m0} , modified for planform effects. The second contribution, $(C_{m0})_2$, adds the effect of twist through another spanwise integration involving $C_{LLT0} c / \bar{c}$, the load distribution at zero total wing lift. The load distribution at zero total wing lift arises from a combination of geometric twist and what can be termed “camber-dependent” twist for those cases in which the camber line, and hence the local zero-lift angle of attack, varies across the span. The zero-lift pitching moment coefficient can therefore be written as

$$C_{m0} = (C_{m0})_1 + (C_{m0})_2 \quad (3.1)$$

where,

$$(C_{m0})_1 = f(A, \Lambda_{1/2}) \int_0^1 \frac{(C_{m0})_{\infty} c^2}{\bar{c}} d\eta \quad (3.2)$$

and

$$(C_{m0})_2 = \int_0^1 \frac{\bar{x} - x_{1/4}}{\bar{c}} \left(C_{LLT0} \frac{c}{\bar{c}} \right)_{\eta} d\eta. \quad (3.3)$$

The practical evaluation of Equations (3.2) and (3.3) for low speeds is given in Sections 3.2 and 3.3 respectively. The effect of Mach number is considered in Section 3.4.

3.2 Evaluation of $(C_{m0i})_1$

From Equation (3.2) the contribution $(C_{m0i})_1$ is given by

$$(C_{m0i})_1 = f(A, \Lambda_{1/2}) \int_0^1 \frac{(C_{m0i})_\infty c^2}{\bar{c}} d\eta \quad (3.4)$$

$$= f(A, \Lambda_{1/2}) (\bar{C}_{m0i})_\infty \quad (3.5)$$

where $(\bar{C}_{m0i})_\infty$ incorporates any effects of wing taper and is a spanwise average of the local two-dimensional values

$$(C_{m0i})_\infty = F(C_{m0i})_{\infty th} \quad (3.6)$$

Here, $(C_{m0i})_{\infty th}$ is the value obtained by using Item No. 72024 (Derivation 3), and the empirical correction factor F , in Figure 1, is a function of $(C_{m0i})_{\infty th}$ obtained from comparisons of $(C_{m0i})_{\infty th}$ with wind-tunnel test data for a large number of aerofoils in Derivations 1, 2 and 4 to 12. If the camber line is unchanged across the wing span then $(C_{m0i})_{\infty th}$ is constant, giving

$$(\bar{C}_{m0i})_\infty = F(C_{m0i})_{\infty th} \quad (3.7)$$

i.e. independent of wing taper. If, however, the camber line varies across the span a good approximation to $(\bar{C}_{m0i})_\infty$ for straight tapered wings is given by

$$(\bar{C}_{m0i})_\infty = \frac{3}{2(1 + \lambda + \lambda^2)} \left\{ [F(C_{m0i})_{\infty th}]_{0.2} (0.8 + 0.2\lambda)^2 + [F(C_{m0i})_{\infty th}]_{0.8} (0.2 + 0.8\lambda)^2 \right\} \quad (3.8)$$

which is based on the integrand of Equation (3.4) at the carefully chosen spanwise locations $\eta = 0.2$ and 0.8 .

The empirical factor, $f(A, \Lambda_{1/2})$, correcting for planform effects in Equation (3.4) was found from an analysis of wind-tunnel data on wings (Derivations 14 to 30) to be given by

$$f(A, \Lambda_{1/2}) = \frac{2A}{2A + 1} \cos \Lambda_{1/2} \quad (3.9)$$

Equation (3.4) therefore becomes, in the most general case,

$$(C_{m0i})_1 = \frac{3A \cos \Lambda_{1/2}}{(2A + 1)(1 + \lambda + \lambda^2)} \left\{ [F(C_{m0i})_{\infty th}]_{0.2} (0.8 + 0.2\lambda)^2 + [F(C_{m0i})_{\infty th}]_{0.8} (0.2 + 0.8\lambda)^2 \right\} \quad (3.10)$$

which, within a good approximation, reduces to

$$(C_{m0i})_1 = \frac{2A}{2A + 1} \cos \Lambda_{1/2} F(C_{m0i})_{\infty th} \quad (3.11)$$

when the camber line is unchanged across the span.

3.3 Evaluation of $(C_{m0i})_2$

From Equation (3.3) the contribution $(C_{m0i})_2$ is given by

$$(C_{m0i})_2 = \int_0^1 \frac{(\bar{x} - x_{1/4})}{\bar{c}} \left(C_{LLT0} \frac{c}{\bar{c}} \right)_{\eta} d\eta. \quad (3.12)$$

From Sketch 3.1 for a straight tapered wing

$$\frac{(\bar{x} - x_{1/4})}{\bar{c}} = \frac{b}{2\bar{c}} (\bar{\eta} - \eta) \tan \Lambda_{1/4} = \frac{A}{2} (\bar{\eta} - \eta) \frac{\bar{c}}{\bar{c}} \tan \Lambda_{1/4} \quad (3.13)$$

$$\text{where } \frac{\bar{c}}{\bar{c}} = \frac{3(1 + \lambda)^2}{4(1 + \lambda + \lambda^2)} \quad (3.14)$$

$$\text{and}^* \quad \bar{\eta} = \frac{1}{3} \left(\frac{1 + 2\lambda}{1 + \lambda} \right). \quad (3.15)$$

Equation (3.12) therefore becomes,

$$(C_{m0i})_2 = \frac{3(1 + \lambda)^2}{8(1 + \lambda + \lambda^2)} A \tan \Lambda_{1/4} \int_0^1 (\bar{\eta} - \eta) \left(C_{LLT0i} \frac{c}{\bar{c}} \right)_{\eta} d\eta, \quad (3.16)$$

where $\bar{\eta}$ is given by Equation (3.15). In Equation (3.16) $(C_{LLT0i} c/\bar{c})_{\eta}$ is the local loading at zero total wing lift due to the combined effects of local geometric twist, δ , and local camber-dependent twist, δ_c . The local geometric twist is defined as the angle of the local chord relative to the root (centre-line) chord, and the local camber-dependent twist is defined as the difference between local and root section values of α_{0i} , i.e.

$$\delta_c = (\alpha_{0ir})_{\infty} - (\alpha_{0i})_{\infty}. \quad (3.17)$$

Thus a combined, or effective, local twist, δ_e , can be given as

$$\delta_e = \delta + \delta_c = \delta + (\alpha_{0ir})_{\infty} - (\alpha_{0i\eta})_{\infty} \quad (3.18)$$

which can be interpreted as the angle of the local zero-lift line relative to that of the root section. An estimate of $(\alpha_{0i\eta})_{\infty}$ can be obtained from Item No. 87031 (Derivation 45).

At the tip, Equation (3.18) becomes

$$\delta_{et} = \delta_t + (\alpha_{0ir})_{\infty} - (\alpha_{0it})_{\infty}. \quad (3.19)$$

When the camber line is unchanged across the span, $(\alpha_{0ir})_{\infty} = (\alpha_{0it})_{\infty}$, and Equation (3.19) reduces to

$$\delta_{et} = \delta_t. \quad (3.20)$$

* Note that in Item No. 83040 (Reference 47) $\bar{\eta}$ is given the symbol κ .

3.3.1 Linear twist

Analysis of approximate solutions to Equation (3.16), in Derivation 34, for wings with a linear spanwise distribution of twist, has shown that the integral in that equation can be approximated by

$$\int_0^1 (\bar{\eta} - \eta) \left(C_{LLT0i} \frac{c}{c} \right)_{\eta} d\eta \approx -k \bar{\eta} \frac{A}{A+10} \delta_{et} \quad (3.21)$$

for $2 \leq A \leq 10$ and $0 \leq A \tan \Lambda_{1/4} \leq 6$ where

$$k = 0.019 \text{ for } 0 \leq \lambda \leq 0.5$$

falling to $k = 0.017$ for $\lambda = 1$.

When Equations (3.15) and (3.21) are substituted into it, Equation (3.16) becomes

$$\frac{(C_{m0i})_2}{\delta_{et}} = -\frac{k(1+\lambda)(1+2\lambda)}{8(1+\lambda+\lambda^2)} \frac{A^2}{(A+10)} \tan \Lambda_{1/4} \quad (3.22)$$

with the planform restrictions and values of k as set out below Equation (3.21).

Equation (3.22) can be written as

$$\frac{(C_{m0i})_2}{\delta_{et}} = g(\lambda) h(A, A \tan \Lambda_{1/4}) \quad (3.23)$$

$$\text{where } g(\lambda) = \frac{k(1+\lambda)(1+2\lambda)}{8(1+\lambda+\lambda^2)} \quad (3.24)$$

$$\text{and } h(A, A \tan \Lambda_{1/4}) = -\frac{A}{A+10} A \tan \Lambda_{1/4}. \quad (3.25)$$

Equations (3.24) and (3.25) are presented graphically in Figures 2 and 3.

Provided that the spanwise distribution of effective twist is linear Equation (3.22) would be expected to provide satisfactory estimates. For effective twist distributions that are highly non-linear a different approach is required.

3.3.2 Non-linear twist

For wings with a general distribution of effective twist, Derivation 34 gives

$$(C_{m0i})_2 = -\frac{3}{8} \frac{(1+\lambda)^2}{(1+\lambda+\lambda^2)} A \tan \Lambda_{1/4} \left[0.126 \left(C_{LLT0i} \frac{c}{c} \right)_{\eta_1} + 0.175 \left(C_{LLT0i} \frac{c}{c} \right)_{\eta_2} + 0.106 \left(C_{LLT0i} \frac{c}{c} \right)_{\eta_3} \right] \quad (3.26)$$

where $(C_{LLT0i}c/\bar{c})_{\eta}$ is the spanwise loading at zero total lift due to given values of effective twist, δ_e , at $\eta = \eta_n = \cos(n\pi/8)$ with $\eta_1 = 0.924$, $\eta_2 = 0.707$ and $\eta_3 = 0.383$. The spanwise loading can be obtained from the approximate method of Item No. 83040 (Reference 47). However, the use of Equation (3.26), even with the simple method of Item No. 83040 is somewhat time consuming and it has been found that the use of an equivalent linear twist δ'_{et} dependent on values of δ_e at $\eta = 0.2$ and 0.8 in conjunction with Equation (3.22) gives satisfactory agreement with values obtained using the full procedure of Equation (3.26). Thus, even for some cases of extreme discontinuous twist distribution, such as those used in Examples 8.1 and 8.3 of Item No. 83040, it is sufficient to take

$$(C_{m0i})_2 = -\frac{k(1+\lambda)(1+2\lambda)}{8(1+\lambda+\lambda^2)} \frac{A^2}{A+10} \delta'_{et} \tan \Lambda_{1/2} \quad (3.27)$$

where the equivalent linear twist is given by

$$\delta'_{et} = (\delta_{e, 0.8} - \delta_{e, 0.2})/0.6 \quad (3.28)$$

and, from Equation (3.18),

$$\delta_{e, 0.2} = \delta_{0.2} + (\alpha_{0ir})_{\infty} - (\alpha_{0i, 0.2})_{\infty} \quad (3.29)$$

$$\delta_{e, 0.8} = \delta_{0.8} + (\alpha_{0ir})_{\infty} - (\alpha_{0i, 0.8})_{\infty}. \quad (3.30)$$

3.4 Effect of Mach Number

From simple considerations of the effect of compressibility on (C_{m0i}) , the contributions $(C_{m0})_1$ and $(C_{m0})_2$ are given by

$$(C_{m0})_1 = \frac{2A+1}{2\beta A+1} (C_{m0i})_1 \quad (3.31)$$

and

$$(C_{m0})_2 = \frac{A+10}{\beta A+10} (C_{m0i})_2 \quad (3.32)$$

in which $\beta = (1-M^2)^{1/2}$ and $(C_{m0i})_1$ and $(C_{m0i})_2$ are obtained respectively from Equations (3.10) or (3.11) and (3.22), (3.26) or (3.27).

4. APPLICABILITY AND ACCURACY

The method given in this Item for estimating wing pitching moment coefficient at zero lift at subcritical Mach numbers is applicable to straight tapered wings with camber and twist provided that the local effective twist is not excessive ($\delta_e \leq 10$ degrees, say) and provided that the flow over the wing is attached and wholly subsonic at the zero-lift condition. The method has been applied successfully to wings with cranked or curved edges by means of the "equivalent wing planform" concept detailed in Item No. 76003 (Reference 46).