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## A method of determining the wave drag and its spanwise distribution on a finite wing in transonic flow

Associated software: ESDUpac A8703

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## THE PREPARATION OF THIS DATA ITEM

This Memorandum has been compiled by ESDU under a contract with Ministry of Defence (Procurement Executive). The major part of the original work was completed in 1987 and the permission of the Ministry to issue this Memorandum in 1991 is gratefully acknowledged.

The work on this particular Data Item which supersedes ESDU 83022 was monitored and guided by the Transonic Aerodynamics Committee. This Committee first met in 1960 and now has the following membership:

|                    |                                 |
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Work leading to the first issue of the computer program in Appendix A was carried out with the assistance of a student from Loughborough University of Technology, Mr F.H. Aziz, under the supervision of Mr B.M. Patel – Engineer, Transonic Aerodynamics. The current version of that program was produced by Dr G. Richards under contract to ESDU.

The person with overall responsibility for the work in this subject area is Mr D.J. Mitchell.

## A METHOD OF DETERMINING THE WAVE DRAG AND ITS SPANWISE DISTRIBUTION ON A FINITE WING IN TRANSONIC FLOW

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## A METHOD OF DETERMINING THE WAVE DRAG AND ITS SPANWISE DISTRIBUTION ON A FINITE WING IN TRANSONIC FLOW

### SUMMARY

A simple method is presented, involving quantities only on the wing surface, that may be used to estimate the spanwise distribution of wave drag and, through integration in the spanwise direction, the total wave drag on a finite wing in transonic flow.

The method and guidance on its implementation are given in Section 3 whilst its derivation is outlined in Section 4. The lack of a definitive procedure for determining the wave drag of aerofoils and wings at transonic speeds precludes a precise evaluation of the method but the extent to which it has been verified and its accuracy and applicability are discussed in Sections 5 and 6. Examples of its application are provided in Section 7.

Appendix A introduces a computer program (ESDUpac A8703) and provides associated examples of input and output formats. The program may be used to estimate wave drag either for a wing or for an aerofoil.

### 1. INTRODUCTION

The occurrence on an aircraft of shock wave drag (plus any associated increase in viscous drag) leads to a rapid increase in drag with increasing stream Mach number that imposes a constraint on the subsonic performance of military strike aircraft and on the economic cruise Mach number of civil aircraft.

Throughout the aircraft design phase it is important to be able not only to estimate the wave drag of an aircraft wing but also to identify its source so that design refinements may be introduced if necessary.

This Memorandum presents a simple method, requiring a knowledge of quantities only on the wing surface, that may be used to estimate the spanwise distribution of wave drag and through integration in the spanwise direction, total wave drag on a finite wing.

The method may be applied either to experimental data or to data generated by procedures in computational transonic aerodynamics<sup>1</sup>.

Appendix A introduces a computer program (ESDUpac A8703) and provides associated examples of input formats. The program may be used to estimate wave drag either for a wing or for an aerofoil.

[The Memorandum supersedes T.D. Memo No. 83022 which gave an earlier version of the present method, now referred to as being of “first order” and described in Section 4.2.1. The computer program in Appendix A provides output from the “improved” method and, optionally, from the “first order” method.]

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## 2. NOTATION

(In using this Item the drag coefficients must be made non-dimensional in a consistent manner.)

|                  |                                                                                           |
|------------------|-------------------------------------------------------------------------------------------|
| $A$              | gross wing aspect ratio ( $= 2s/\bar{c}$ )                                                |
| $C_{DB}$         | drag coefficient due to fuselage alone                                                    |
| $C_{DF}$         | drag coefficient due to skin friction                                                     |
| $C_{DP}$         | pressure drag coefficient                                                                 |
| $C_{DT}$         | total drag coefficient                                                                    |
| $C_{DTV}$        | drag coefficient due to trailing vortices                                                 |
| $\Delta C_{DTV}$ | $C_{DTV} - (C_L^2/\pi A)$                                                                 |
| $C_{DV}$         | drag coefficient due to streamwise momentum deficit in viscous layers emanating from wing |
| $C_{DV}(\eta)$   | local value of $C_{DV}$ at spanwise station $\eta$                                        |
| $C_{DW}$         | wave drag coefficient                                                                     |
| $C_{DW}(\eta)$   | local wave drag coefficient at spanwise station $\eta$                                    |
| $C_L$            | lift coefficient of wing or wing-body combination                                         |
| $C_p$            | pressure coefficient                                                                      |
| $C_{psh}$        | pressure coefficient at shock upstream location on aerofoil or wing surface               |
| $\bar{c}$        | mean chord ( $= \text{reference area}/(2s)$ )                                             |
| $c(\eta)$        | local streamwise chord at spanwise station $\eta$                                         |
| $F$              | function of $M_\infty$ (see Equation (4.4))                                               |
| $F_1, F_2$       | functions defined in Equations (3.2) and (3.3)                                            |
| $G$              | function of $M_{1n}$ (see Equation (4.4))                                                 |
| $G_1, G_2, G_3$  | functions defined in Equations (3.7) to (3.9)                                             |
| $H$              | total head                                                                                |
| $k$              | step integer in iteration method in Section 3.1                                           |
| $M$              | local Mach number                                                                         |

|                |                                                                                                                               |
|----------------|-------------------------------------------------------------------------------------------------------------------------------|
| $M_{Ln}$       | local Mach number of flow on wing surface normal to local isobars                                                             |
| $M_{Lsh}$      | local Mach number of flow at shock upstream location on aerofoil surface                                                      |
| $M_{Lnsh}$     | local Mach number normal to shock at shock upstream location on wing surface                                                  |
| $M_{\infty}$   | free-stream Mach number                                                                                                       |
| $N$            | number of spanwise locations at which chordwise surface pressures are available between wing root and wing tip (see Figure 5) |
| $\bar{N}$      | function defined by Equation (3.6)                                                                                            |
| $n$            | distance, non-dimensionalised by aerofoil chord, measured from and normal to aerofoil surface                                 |
| $\bar{n}$      | see Equation (4.14) and Sketch 4.1                                                                                            |
| $n_f$          | value of $n$ at point in flow field where $M_{1n}$ has fallen to unity                                                        |
| $\bar{n}_f$    | revised value of $n_f$ (see Equation (4.13) and Sketch 4.1)                                                                   |
| $n_{rat}$      | function of $x_{sh}$ defined by Equation (3.5)                                                                                |
| $Re_{\bar{c}}$ | Reynolds number based on mean chord                                                                                           |
| $s$            | gross wing semi-span                                                                                                          |
| $U_{\infty}$   | free-stream velocity                                                                                                          |
| $u$            | local velocity                                                                                                                |
| $X$            | streamwise distance measured from an arbitrary datum                                                                          |
| $x$            | chordwise distance, measured from local leading edge, non-dimensionalised by local streamwise chord                           |
| $x_{sh}$       | value of $x$ at shock position                                                                                                |
| $\bar{x}$      | value of $x$ defined by Equation (3.10) and/or Sketch 3.1                                                                     |
| $Y$            | spanwise distance measured from an arbitrary datum                                                                            |
| $z$            | coordinate, non-dimensionalised by local streamwise chord, measured from and normal to local chord                            |
| $z'$           | slope of streamwise section ( $= \partial z / \partial x$ )                                                                   |
| $z''$          | second derivative of streamwise section ( $= \partial^2 z / \partial x^2$ )                                                   |
| $\gamma$       | ratio of specific heat capacities for air                                                                                     |

|                |                                                                                                                    |
|----------------|--------------------------------------------------------------------------------------------------------------------|
| $\zeta$        | distance along shock in two-dimensional flow, non-dimensionalised by aerofoil chord                                |
| $\eta$         | spanwise distance measured from body centre line, non-dimensionalised by gross semi-span (see Figure 5)            |
| $\eta_b$       | non-dimensional spanwise location of body side ( $\equiv \eta_0$ )                                                 |
| $\kappa_w$     | wing surface curvature at the foot of the shock in streamwise plane, non-dimensionalised by local streamwise chord |
| $\bar{\kappa}$ | mean curvature of section between $x = \bar{x}$ and $x = x_{sh}$ (see Equation (3.12))                             |
| $\Lambda$      | sweep angle (positive when swept back)                                                                             |
| $\rho$         | free-stream density                                                                                                |

## Suffices

|          |                                                                |
|----------|----------------------------------------------------------------|
| $i$      | denotes $i$ 'th point in list of data ordinates                |
| $j$      | denotes $j$ 'th spanwise location                              |
| $n$      | denotes component normal to shock wave                         |
| $sh$     | pertaining to shock wave                                       |
| $T$      | denotes wing trailing edge (Equation (3.15) and Figure 1 only) |
| $0$      | denotes wing leading edge (except in Equation (3.17))          |
| $1$      | denotes conditions immediately upstream of shock               |
| $2$      | denotes conditions immediately downstream of shock             |
| $2d$     | denotes two-dimensional flow                                   |
| $3d$     | denotes three-dimensional flow                                 |
| $\infty$ | denotes conditions at upstream infinity                        |

## 3. THE METHOD

The wave drag coefficient at a local spanwise station,  $\eta$ , of a finite wing is given by

$$C_{DW}(\eta) = \frac{0.243 \cos^4 \Lambda_{sh}}{\bar{\kappa}} F_1(M_\infty \cos \Lambda_{sh}) F_2(M_{Lnsh}), \quad (3.1)$$

where

$$F_1(M_\infty \cos \Lambda_{sh}) = \left[ \frac{1 + 0.2 M_\infty^2 \cos^2 \Lambda_{sh}}{M_\infty \cos \Lambda_{sh}} \right]^3, \quad (3.2)$$

$$F_2(M_{Lnsh}) = \frac{(M_{Lnsh} - 1)^4 (2 - M_{Lnsh})}{M_{Lnsh} (1 + 0.2 M_{Lnsh}^2)} \quad (3.3)$$

and  $\bar{\kappa}$  is the mean surface curvature of a streamwise section between  $x = \bar{x}$  and  $x = x_{sh}$  (see Section 3.1 (a)). The total wave drag coefficient of the wing is then given by

$$C_{DW} = \int_{\eta_b}^1 \frac{c(\eta)}{\bar{c}} C_{DW}(\eta) d\eta. \quad (3.4)$$

Equation (3.1) may be applied to an aerofoil simply by putting  $\cos \Lambda_{sh} = 1$  in Equations (3.1), (3.2) and (3.3) whilst  $M_{Lnsh}$  is, of course, replaced by  $M_{Lsh}$ .

### 3.1 Enumeration of the Equations

#### (a) Mean surface curvature $\bar{\kappa}$

The value of  $\bar{\kappa}$  is obtained from the values of  $x_{sh}$ ,  $\Lambda_{sh}$  and  $M_{Lnsh}$  and from the ordinates of the local section, as follows.

#### (i) Calculate successively:

$$n_{rat} = 0.45 \left\{ 1 - \exp \left[ -2x_{sh} \left( 1 + x_{sh} + \frac{4}{3} x_{sh}^2 \right) \right] \right\} \quad (3.5)$$

$$\bar{N} = n_{rat} (1 - 1/M_{Lnsh}) \quad (3.6)$$

$$G_1 = [M_{Lnsh}^2 (1 - \bar{N})^2 - 1]^{0.5} \quad (3.7)$$

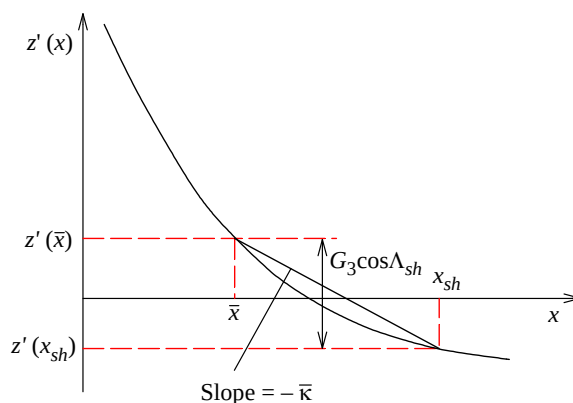
$$G_2 = [M_{Lnsh}^2 - 1]^{0.5} \quad (3.8)$$

$$G_3 = \frac{2\bar{N}G_1G_2}{(1 + 0.2M_{Lnsh}^2)(G_1 + G_2)}. \quad (3.9)$$

Thus  $G_3$  is a function of  $x_{sh}$  and  $M_{Lnsh}$ , shown in carpet form in Figure 2.

(ii) Solve the following equation to obtain  $\bar{x}$  :

$$z'(\bar{x}) = z'(x_{sh}) + G_3(M_{Lnsh}, x_{sh}) \cos \Lambda_{sh}. \quad (3.10)$$



**Sketch 3.1 Variation of aerofoil surface slope with chordwise distance**

This may be done either graphically (see Sketch 3.1) or by linear interpolation. For the latter procedure, at a given flow condition, the expression,  $z'(x_{sh}) + G_3(M_{Lnsh}, x_{sh}) \cos \Lambda_{sh}$ , is constant and equal to  $C_1$ , say. At this same flow condition, Equation (3.10) becomes

$$z'(\bar{x}) = C_1. \quad (3.11)$$

By considering the gradients of the given points on an aerofoil or spanwise station of a wing,  $\bar{x}$  can then be found by linear interpolation with the gradient at  $\bar{x}$ ,  $z'(\bar{x})$ .

Note that when calculating mean surface curvature in part (iii),

$$\begin{aligned} z'(x) - z'(x_{sh}) &= G_3(M_{Lnsh}, x_{sh}) \cos \Lambda_{sh} \\ &= \text{a constant for a given flow condition.} \end{aligned}$$

It is necessary to calculate the slope,  $z'(x)$  whilst in the first order method it is necessary to calculate the second derivative,  $z''(x)$ , of the section ordinates at an arbitrary value of  $x$ . Provided that a table of ordinates  $(x_i, z_i)$  is available at closely and smoothly spaced intervals in  $x$ , this can be done with adequate accuracy in the following way.

First, calculate the slopes  $z'_i (\equiv z'(x_i))$ , at the points  $x = x_i$  by the central difference formula

$$z'_i = \frac{z_{i+1} - z_{i-1}}{x_{i+1} - x_{i-1}} \quad (3.12)$$

and the corresponding values of  $z''(x_i)$  from

$$z''_i = \frac{z'_{i+1} - z'_{i-1}}{x_{i+1} - x_{i-1}}. \quad (3.13)$$

Then the values of  $z'(x)$  and  $z''(x)$  at intermediate points can be found by linear interpolation.

**(iii) Calculate mean surface curvature\***

$$\bar{\kappa} = \frac{z'(\bar{x}) - z'(x_{sh})}{x_{sh} - \bar{x}}. \quad (3.14)$$

**(b) Shock sweep,  $\Lambda_{sh}$**

An obvious way to determine shock sweep is to plot shock location on a scale drawing of the wing planform and simply measure the slope of the shock location curve. However this curve is often very shallow with widely spaced points and it is not easy to measure the slope reliably or to have confidence in the consistency of measurements.

An alternative approach is to plot  $x_{sh}$  vs  $\eta$ , for which any convenient scales may be used, determine the slope ( $dx_{sh}/d\eta$ ) of this curve and apply necessary corrections. A portion of a wing is depicted in Figure 1. It is required to know  $\tan \Lambda_{sh} = dX/dY$  and it may readily be shown that

$$\tan \Lambda_{sh} = \frac{c(\eta)}{s} \frac{dx_{sh}}{d\eta} + (1 - x_{sh}) \tan \Lambda_0 + x_{sh} \tan \Lambda_T. \quad (3.15)$$

**(c) Local Mach number normal to the shock,  $M_{Lnsh}$**

The local Mach number of the flow normal to the shock at the shock upstream location is given by

$$M_{Lnsh} = \left\{ 5 \left[ \frac{1 + 0.2 M_\infty^2 \cos^2 \Lambda_{sh}}{(1 + 0.7 M_\infty^2 C_{psh})^{2/7}} - 1 \right] \right\}^{0.5}, \quad (3.16)$$

which assumes that  $\gamma$ , the ratio of specific heat capacities for air, is 1.4 and that the surface flow direction just upstream of the shock is the same as it would be on an infinite yawed wing having the same sweep angle as the local value of  $\Lambda_{sh}$ . A chart allowing the rapid enumeration of Equation (3.16) is provided by Figure 1 of Reference 2, whilst Figure 2 of this same reference allows a rapid estimate to be made of the pressure coefficient corresponding to local sonic velocity, which may be an aid in identifying the existence of shock waves.

**(d) The functions  $F_1$  and  $F_2$**

Charts that provide for the rapid enumeration of Equations (3.2) and (3.3) are given in Figures 3 and 4.

\* The first-order method of Section 4.2.1 makes use, instead, of  $\kappa_w$ , the wing surface curvature at the shock location. Equation (3.13) is used to evaluate  $\kappa_w$  with linear interpolation if the shock location is not at an ordinate position,  $x_i$ .

## (e) Wave drag coefficient, $C_{DW}$

It is probable, in both experimental data and data generated by computational methods, that values of surface pressure will be available at a relatively small number of spanwise stations. In this circumstance it is recommended that the integral in Equation (3.4) should be approximated by the trapezium rule which yields:

$$C_{DW} = \frac{1}{2} \left\{ \frac{c(\eta_0)}{\bar{c}} (\eta_1 - \eta_0) C_{DW}(\eta_0) + \sum_{j=1}^N (\eta_{j+1} - \eta_{j-1}) \frac{c(\eta_j)}{\bar{c}} C_{DW}(\eta_j) + \frac{c(\eta_{N+1})}{\bar{c}} (\eta_{N+1} - \eta_N) C_{DW}(\eta_{N+1}) \right\}, \quad (3.17)$$

where  $\eta_0 = \eta_b$  is the non-dimensional spanwise location of the body side,

$\eta_1, \eta_2$  (*etc.*) are the first, second (*etc.*) spanwise locations at which surface pressures are measured, and  $\eta_{N+1} = 1$ , is the wing tip location.

The derivation of this equation and the notation used is illustrated in Figure 5.

It may be the case that chordwise pressure distributions (particularly in experimental data) are not available either at the wing root (*i.e.* body side) or wing tip. In this circumstance it is recommended that  $C_{DW}(\eta_0)$  should be taken equal to  $C_{DW}(\eta_1)$  and that  $C_{DW}(\eta_{N+1})$  should be taken equal to  $C_{DW}(\eta_N)$ .

## 4. DERIVATION

### 4.1 Possible Means of Obtaining Wave Drag

In inviscid transonic flow the total drag of a wing-body combination consists of trailing vortex drag and wave drag only, both components being transmitted to the surface in the form of normal pressure drag. In a real flow the modification of the pressure distribution due to the presence of the boundary layer will modify slightly the trailing vortex drag and lead to a third pressure drag component, namely boundary layer normal pressure drag, whilst an additional drag component will arise due to skin friction.

The total pressure drag can be determined, in principle, by integrating the streamwise component of normal pressure force. However this process has two particular shortcomings. First, it is difficult to perform the integration to adequate accuracy unless a very large number of surface pressure values are available and secondly it is impossible to distinguish between the three pressure drag components.

However, evaluation of the trailing vortex drag from a knowledge of the spanwise lift distribution by means of a Trefftz-plane analysis<sup>8</sup> does not pose large difficulties whilst the boundary layer normal pressure drag (plus the friction drag) can be determined from boundary layer and wake momentum deficit considerations based on the work of Squire and Young<sup>3</sup>.

Thus, in principle, given adequate knowledge of the surface pressure distribution the total wave drag, but not its spanwise distribution, can be determined either from experimental data or from computational methods that adequately account for viscosity through the equation

$$C_{DW} = C_{DP} + C_{DF} - C_{DV} - C_{DTV}. \quad (4.1)$$

Alternatively from experimental data the total wave drag (but, again, not its spanwise distribution) can be determined through the equation

$$C_{DW} \approx C_{DT} - C_{DV} - C_{DTV} - C_{DB}. \quad (4.2)$$

It is assumed in Equation (4.2) that the total drag and the drag of the body alone will be available from, say, balance measurements. The approximation in Equation (4.2) arises from the omission of any drag on the body due to the presence of the wing and of any interference drag there may be on the wing-body combination even after an allowance has been made for the drag of the body in the presence of the wing and the drag of the wing in the presence of the body.

However, in regard to Equation (4.1), it is the general experience that the pressure drag integration is seldom performed to adequate accuracy, particularly for swept back wings where the net pressure drag is the resultant of a large rearward (drag) force at the root and a large forward (thrust) force at the tip. This is particularly true of experimental data where the number of pressure measurements that can be made is necessarily limited. Furthermore, for both theoretical and experimental data, any uncertainty regarding the precise location of the shock waves and the pressures in their immediate vicinity contributes to the difficulty.

As a preferable alternative, the drag can be obtained by considering the flux of momentum across a large closed surface surrounding the wing and any shock waves that it produces. For inviscid flow the change in momentum may be split into two components, the transverse momentum induced in the wake by the trailing vorticity and the loss in total head produced by the passage of the flow through the shock waves. These components correspond directly to the trailing vortex drag and the wave drag. It is then possible to express

the wave drag as an integral, over a suitable surface behind\* the shock waves, that involves the jump in entropy across these shocks – and to derive wave drag in this way is equally valid in the real viscous flow. This integral can then, as a further alternative, be expressed as an integral, over the surface of each shock, of a function which assumes that the shock-upstream and shock-downstream flow conditions are related through the Rankine-Hugoniot equation and thus requires for its enumeration a knowledge of the local Mach number distribution only ahead of and normal to each shock. A numerical method based on this concept has been applied with some measure of success in two dimensional flow (for example Reference 6†) but in three dimensions, with complex flow patterns, the requirement to determine shock locations throughout the flow field is much more difficult. Nevertheless, a three-dimensional counterpart of the method of Reference 6 has recently been developed by Allwright<sup>18</sup>.

A further simplification of this last concept, that avoids the problem of shock detection in the field, has been devised by Lock<sup>8</sup>. He shows how, in two dimensions, the wave drag can be approximated by a simple formula requiring for its enumeration a knowledge of quantities only on the surface of the aerofoil; and then how this formula, using simple sweep concepts, may be adapted to determine the wave drag at a local spanwise station of a swept wing (Equation<sup>‡</sup>(3.1)) and, through integration in the spanwise direction, the total wave drag on the wing of a wing-body combination (Equation (3.4)). For a wing-body combination the ability to determine the spanwise distribution of wave drag is a particular attribute of this formula and the method of Reference 18.

## 4.2 The Wave Drag Equation in Two-Dimensional Flow

It is shown in Reference 8 that the wave drag on an aerofoil in two-dimensional flow, expressed as an integral along the shock, may, after some approximation, be written as

$$(C_{DW})_{2d} = \frac{2}{\gamma M_\infty^2} \int_{shock} \frac{\rho_1 u_{1n}}{\rho_\infty U_\infty} \log \left[ \frac{H_1}{H_2} \right] d\zeta \quad (4.3)$$

where, again taking  $\gamma = 1.4$ ,

$$\frac{\rho_1 u_{1n}}{\rho_\infty U_\infty} = \frac{M_{1n}}{M_\infty} \left[ \frac{1 + 0.2 M_\infty^2}{1 + 0.2 M_{1n}^2} \right]^3$$

$$\frac{H_1}{H_2} = \left[ \frac{5 + M_{1n}^2}{6 M_{1n}^2} \right]^{3.5} \left[ \frac{7 M_{1n}^2 - 1}{6} \right]^{2.5},$$

\* Because the flow is isentropic other than close to the shock itself the surface in question may be located some little way downstream of the shock thus obviating the need to know precisely the flow conditions at the shock location. Some success has been achieved with this concept in two-dimensional flow (References 4 and 5) and the latter reference explores its implementation in three dimensions.

† This method has so far been used only in conjunction with solutions of the potential flow equations in which the ‘shock waves’ that appear are physically unrealistic. However, a numerical technique is available<sup>7</sup> which seems to be generally successful in giving a reasonable simulation of the position of the real shock waves and of the flow field ahead of them and it is when used in conjunction with this device that the method of Reference 6 has been found to give the most plausible results.

‡ In Reference 8 the local curvature,  $\kappa_w$ , was used instead of  $\bar{\kappa}$ .

so that Equation (4.3) may be written in the form

$$(C_{DW})_{2d} = F(M_\infty) \int_{shock} G(M_{1n}) d\zeta. \quad (4.4)$$

where

$$F = \frac{1}{0.7} \left[ \frac{1 + 0.2M_\infty^2}{M_\infty} \right]^3$$

and

$$G = \frac{M_{1n}}{(1 + 0.2M_{1n}^2)^3} \left[ 3.5 \ln \left( \frac{5 + M_{1n}^2}{6M_{1n}^2} \right) + 2.5 \ln \left( \frac{7M_{1n}^2 - 1}{6} \right) \right].$$

The problem is thus reduced to one of finding an expression alternative to  $G(M_{1n})$  that can be integrated readily.

## 4.2.1 First-order method

Lock<sup>8</sup> approximates  $G(M_{1n})$  by the expression

$$G(M_{1n}) = 0.68(M_{1n} - 1)^3 - 0.85(M_{1n} - 1)^4 \quad (4.5)$$

and assumes that the shock wave lies along a normal to the aerofoil surface so that the limits of integration are  $n = 0$  and  $n = n_f$  where  $n_f$  is the value of  $n$  at the point in the flow field where  $M_{1n}$  has fallen to unity. Thus

$$(C_{DW})_{2d} = F(M_\infty) \int_0^{n_f} (0.68(M_{1n} - 1)^3 - 0.85(M_{1n} - 1)^4) dn. \quad (4.6)$$

In order to be able to express the wave drag in terms only of quantities on the aerofoil surface, Lock<sup>8</sup> expressed  $M_{1n}$  in the form of a Taylor series expansion

$$M_{1n} = M_{Lsh} + n \left[ \frac{\partial M_{1n}}{\partial n} \right]_{n=0} + \frac{n^2}{2} \left[ \frac{\partial^2 M_{1n}}{\partial n^2} \right]_{n=0} + \dots \quad (4.7)$$

and proceeded to show that the first-order term can be expressed as

$$\left[ \frac{\partial M_{1n}}{\partial n} \right]_{n=0} = -\kappa_w M_{Lsh} (1 + 0.2M_{Lsh}^2) \quad (4.8)$$

which\* again assumes that  $\gamma = 1.4$ .

\* This first-order method makes use of  $\kappa_w$ , the wing surface curvature at the shock location. Equation (3.13) is used to evaluate  $\kappa_w$  with linear interpolation if the shock location is not at an ordinate position,  $x_i$

Substituting Equation (4.8) into Equation (4.7) yields

$$M_{1n} = M_{Lsh}(1 - n\kappa_w(1 + 0.2M_{Lsh}^2)) + \dots \quad (4.9)$$

and to a first approximation

$$n_f = \frac{1 - M_{Lsh}^{-1}}{\kappa_w(1 + 0.2M_{Lsh}^2)} \quad (4.10)$$

Integration of Equation (4.6) then yields<sup>8</sup>

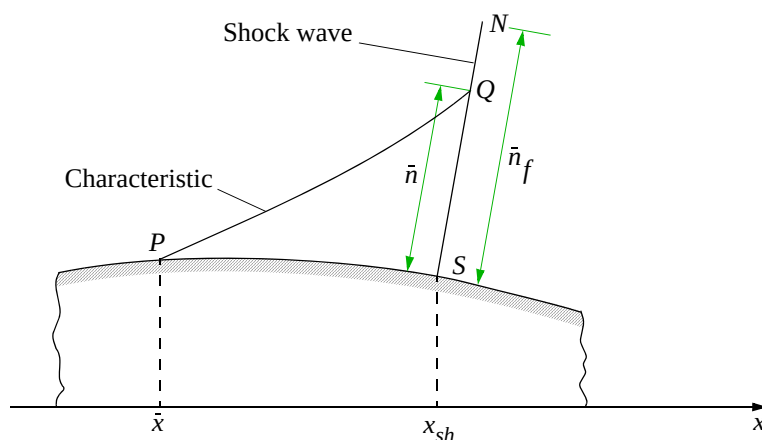
$$(C_{DW})_{2d} = \frac{0.243}{\kappa_w} \left[ \frac{(1 + 0.2M_\infty^2)}{M_\infty} \right]^3 \frac{(M_{Lsh} - 1)^4 (2 - M_{Lsh})}{M_{Lsh}(1 + 0.2M_{Lsh}^2)} \quad (4.11)$$

Experience has shown that in cases where there are rapid variations of surface curvature in the region just ahead of the shock the accuracy of this simple formula is likely to be unacceptably poor. An empirical modification has therefore been derived, which is explained in Section 4.2.2.

## 4.2.2 Improved method

As stated in Section 4.2.1, it has been found that the accuracy of the simple first order Equation (4.11) is usually inadequate whenever the chordwise variation of the surface curvature of an aerofoil or wing section is rapid or uneven in the neighbourhood of the shock wave. To improve the accuracy of the method the following empirical procedure has been devised.

The principle adopted is that the variation of the local Mach number  $M_{1n}$  along the shock, and hence the wave drag that the shock produces (cf. Equation (4.3)), is likely to depend on the shape of the surface for some little distance ahead of the foot of the shock; so that greater accuracy may be expected by replacing the local value of the curvature,  $\kappa_w$ , used in Equation (4.11) by an appropriate mean value  $\bar{\kappa}$ , the mean being taken from  $x = \bar{x}$  to  $x = x_{sh}$ , where  $x$  is to be defined empirically.



Sketch 4.1

With reference to Sketch 4.1, in which  $S$  is the foot of the shock wave,  $SN$ , and  $P$  is the point  $x = \bar{x}$  to be determined, it is further assumed that the position of  $P$  may be found by tracing the forward characteristic  $QP$  from some point  $Q$  on the shock, at a distance  $\bar{n}$  from the surface, such that the ratio  $n_{rat} = SQ/SN = \bar{n}/\bar{n}_f$  will depend only on the position of the shock ( $x - x_{sh}$ ).

The following simplifying assumptions are now introduced (additional to those already made in Section 4.2.1):

- (a) the Mach number,  $M_Q$ , at  $Q$  is given by Equation (4.9), but with  $\kappa_w$  replaced by  $\bar{\kappa}$ . Thus

$$M_Q = M_{Lsh}(1 - \bar{n}\bar{\kappa}(1 + 0.2M_{Lsh}^2)), \quad (4.12)$$

- (b) similarly, the length of the shock is now given to a better approximation by  $\bar{n}_f$ ,

$$\text{where } 1 = M_{Lsh}(1 - \bar{n}_f\bar{\kappa}(1 + 0.2M_{Lsh}^2)), \quad (4.13)$$

- (c) the variation of surface Mach number between  $S$  and  $P$  is small\* so that

$$M_P \approx M_{Lsh},$$

- (d) the slope of the characteristic line  $PQ$ , relative to the surface  $PS$ , is the mean of the values at its ends  $P$  and  $Q$ , namely

$$\frac{1}{2}[(M_P^2 - 1)^{-1/2} + (M_Q^2 - 1)^{-1/2}],$$

- (e) the change of surface slope between  $P$  and  $S$  is small.

With these assumptions, it follows that

$$\bar{n} = \frac{1}{2}(x_{sh} - \bar{x}) \left[ (M_{Lsh}^2 - 1)^{-1/2} + \left\{ M_{Lsh}^2(1 - \bar{N})^2 - 1 \right\}^{-1/2} \right] \quad (4.14)$$

$$\text{where } \bar{N} = \bar{n}\bar{\kappa}(1 + 0.2M_{Lsh}^2).$$

Since from Equation (4.13) the length of the shock is

$$\bar{n}_f = \frac{(1 - 1/M_{Lsh})}{\bar{\kappa}(1 + 0.2M_{Lsh}^2)},$$

the ratio  $n_{rat} = \bar{n}/\bar{n}_f$  is given by

$$n_{rat} = \frac{\frac{1}{2}\bar{\kappa}(x_{sh} - \bar{x})(1 + 0.2M_{Lsh}^2)(G_1 + G_2)}{(1 - 1/M_{Lsh})G_1G_2} \quad (4.15)$$

\* The effect of allowing for this variation has been investigated, and found to be negligibly small.

where

$$G_1 = [M_{Lsh}^2(1 - \bar{N})^2 - 1]^{0.5}$$

$$G_2 = (M_{Lsh}^2 - 1)^{0.5}$$

and  $\bar{N}$  simplifies to Equation (3.6), i.e.

$$\bar{N} = n_{rat}(1 - 1/M_{Lsh}).$$

The method has been calibrated by taking particular examples in which the wave drag coefficient  $C_{DW}$  is known accurately (see Section 5), in order to determine  $n_{rat}$  and so obtain an empirical relation for it. If  $C_{DW}$  is known, Equation (3.1) (with  $\Lambda_{sh} = 0$ ) can be inverted to give

$$\kappa = 0.243 F_1(M_\infty) F_2(M_{Lsh}) / C_{DW} \quad (4.16)$$

and  $\bar{x}$  can then be determined such that

$$\frac{z'(\bar{x}) - z'(x_{sh})}{x_{sh} - \bar{x}} = \kappa.$$

$$\text{or} \quad \bar{x} + z'(\bar{x})/\kappa = x_{sh} + z'(x_{sh})\kappa. \quad (4.17)$$

This equation can be solved for  $\bar{x}$ . With all quantities in Equation (4.15), other than  $n_{rat}$ , known this can then be solved to determine  $n_{rat}$  (using Newton iteration).

This has been done in a number of cases in which the accuracy of the first order formula (i.e. Equation (4.11)) was known to be poor. It was found possible to correlate the resulting values of  $n_{rat}$  with the shock position  $x_{sh}$ , following a mean curve through the data given by

$$n_{rat} = 0.45 \left[ 1 - \exp \left\{ -2x_{sh} \left( 1 + x_{sh} + \frac{4}{3}x_{sh}^2 \right) \right\} \right]. \quad (4.18)$$

With  $n_{rat}$  given by Equation (4.18), the method can now be used in a predictive manner, to calculate  $\bar{\kappa}$ , and hence  $C_{DW}$ , from a knowledge of  $M_{Lsh}$ ,  $x_{sh}$  and the surface geometry. Equation (4.15) can be written

$$\kappa(x_{sh} - \bar{x}) = G_3(M_{Lsh}, x_{sh}), \quad (4.19)$$

$$\text{where} \quad G_3 = \frac{2\bar{N}G_1G_2}{(1 + 0.2M_{Lsh}^2)(G_1 + G_2)}; \quad (4.20)$$

and since

$$\kappa(x_{sh} - \bar{x}) = z'(\bar{x}) - z'(x_{sh}),$$

the following equation for  $\bar{x}$  is obtained

$$z'(\bar{x}) = z'(x_{sh}) + G_3. \quad (4.21)$$

The mean curvature  $\kappa$  is then given by

$$\kappa = \frac{z'(\bar{x}) - z'(x_{sh})}{x_{sh} - \bar{x}} = \frac{G_3}{(x_{sh} - \bar{x})}, \quad (4.22)$$

leading finally to the revised formula for the wave drag coefficient:

$$C_{DW} = \frac{0.243}{\kappa} F_1(M_\infty) F_2(M_{Lsh}), \quad (4.23)$$

where

$$F_1(M_\infty) = \left[ \frac{1 + 0.2M_\infty^2}{M_\infty} \right]^3$$

and

$$F_2(M_{Lsh}) = \frac{(M_{Lsh} - 1)^4 (2 - M_{Lsh})}{M_{Lsh} (1 + 0.2M_{Lsh}^2)}$$

## 4.3 Adaptation to a Finite Wing

The infinite yawed wing<sup>11</sup> represents a useful concept intermediate between the two-dimensional flow on an aerofoil and the three-dimensional flow on a finite wing. The shape, surface curvature, freestream Mach number and wave drag coefficient of the streamwise section of an infinite yawed wing are related to the same quantities for its basic aerofoil (defined as the section normal to the leading edge) by the simple sweep laws:

$$\left. \begin{aligned} (z)_{3d} &= (z)_{2d} \cos \Lambda \\ (\kappa_w)_{3d} &\approx (\kappa_w)_{2d} \cos \Lambda \\ (C_{DW})_{3d} &= (C_{DW})_{2d} \cos^3 \Lambda \\ (M_\infty)_{3d} \cos \Lambda &= (M_\infty)_{2d} \end{aligned} \right\}. \quad (4.24)$$

For a finite wing,  $\Lambda$  is interpreted as the local shock sweep,  $\Lambda_{sh}$ , and then the substitution of Equations (4.24) for the relevant terms in Equation (4.23), with  $M_{Lsh}$  in Equation (4.23) interpreted as  $M_{Lsh} \cos \Lambda$  leads directly to Equation (3.1), with  $\bar{\kappa}$  obtained as explained in Section 3.1 item (a).

## 5. VERIFICATION OF THE METHOD

The verification of the method has been carried out in three stages.

### 5.1 Inviscid Two-Dimensional Flow

The first stage is concerned with two-dimensional aerofoils in inviscid flow. Here, procedures are available for calculating the wave drag of an aerofoil, by either of the techniques described in Section 4.1, to an adequate (say  $\pm 5\%$ ) standard of accuracy. Two such procedures that have been used extensively to verify the present method are