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Rudder sideforce, yawing moment and rolling moment control derivatives at low speeds: Y_ζ , N_ζ and L_ζ

Associated software: ESDUpac A8708



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RUDDER SIDEFORCE, YAWING MOMENT AND ROLLING MOMENT CONTROL DERIVATIVES AT LOW SPEEDS: Y_{ζ} , N_{ζ} AND L_{ζ}

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RUDDER SIDEFORCE, YAWING MOMENT AND ROLLING MOMENT CONTROL DERIVATIVES AT LOW SPEEDS: Y_ζ , N_ζ AND L_ζ

1. NOTATION AND UNITS (see Sketches 1.1 to 1.3)

		SI	British
a_{1F}	lift-curve slope of straight-tapered wing of aspect ratio A_F , quarter-chord sweep $\Lambda_{1/4F}$ and taper ratio c_{tF}/c_{rF}	rad^{-1}	rad^{-1}
A_F	aspect ratio of fin, taken as $2h_F^2/S_F$		
A_{Feq}	equivalent aspect ratio of fin with lift-curve slope equal to $-(Y_v)_{FR}(S_W/S_F)$ and half-chord sweep $\Lambda_{1/2F}$		
b	wing span	m	ft
b_T	tailplane span, see Sketches 1.2 and 1.3	m	ft
c_F	fin chord at mid-span of rudder, see Sketches 1.2 and 1.3	m	ft
c_R	rudder chord aft of hinge line at mid-span of rudder, see Sketches 1.2 and 1.3	m	ft
c_{rF}	fin root chord, see Sketches 1.2 and 1.3	m	ft
c_{tF}	fin tip chord, see Sketches 1.2 and 1.3	m	ft
d_{BF}	body width at station BB' defined by fin root quarter-chord station, see Sketches 1.2 and 1.3	m	ft
d_{BR}	body width at station AA' defined by inboard end of rudder hinge-line, see Sketches 1.2 and 1.3	m	ft
h_{BF}	body height at station BB' defined by fin root quarter-chord station, see Sketches 1.2 and 1.3	m	ft
h_{BR}	body height at station AA' defined by inboard end of rudder hinge-line, see Sketches 1.2 and 1.3	m	ft
h_F	height of exposed fin measured from body surface at station BB' defined by fin root quarter-chord station, see Sketches 1.2 and 1.3	m	ft
h_{FR}	height of exposed fin measured from body surface at station AA' defined by inboard end of rudder hinge-line, see Sketches 1.2 and 1.3	m	ft
h_R	spanwise extent of rudder, see Sketches 1.2 and 1.3	m	ft

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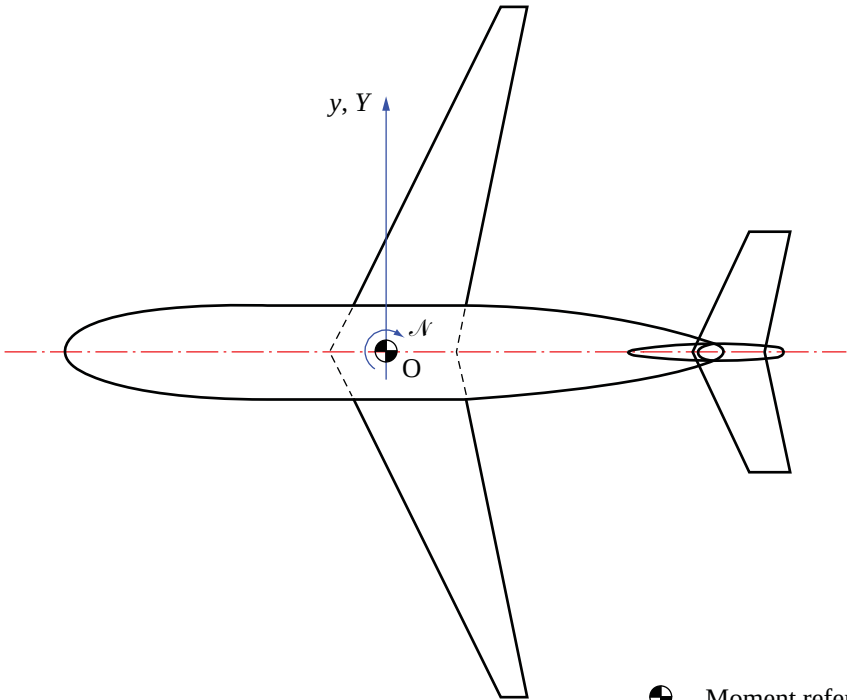
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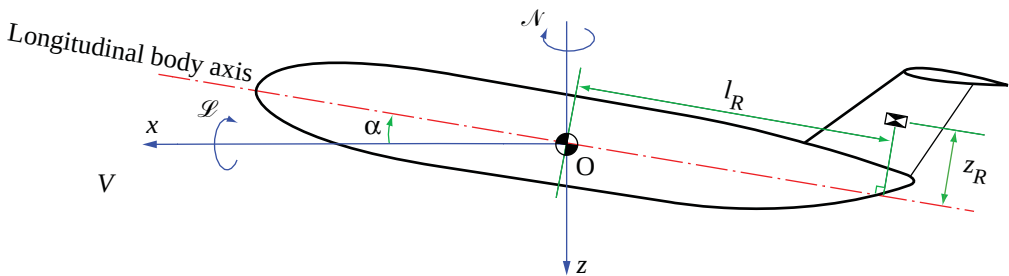
h_{Ri}	height of inboard end of rudder hinge-line above longitudinal body axis passing through moment reference point, see Sketches 1.2 and 1.3	m	ft
J_B, J_T, J_W	factors, defined in ESDU 82010 (Derivation 8), applied to a_{1F} when calculating $(Y_v)_F$ to allow for presence of body, tailplane and wing, respectively		
J_R	body interference factor in calculation of rudder control derivatives		
J_{Ro}	basic value of body interference factor J_R		
k_1, k_2	correction factors for section thickness and Reynolds number in calculation of α_δ , see Equation (3.6)		
$\mathcal{L}$	rolling moment	N m	lbf ft
L_ζ	rolling moment derivative due to rudder deflection, $(\partial \mathcal{L} / \partial \zeta) / \frac{1}{2} \rho V^2 S_W b$	rad ⁻¹	rad ⁻¹
l_R	moment arm of sideforce due to rudder measured parallel to longitudinal body axis, see Sketch 1.1	m	ft
m_F	distance from moment reference point to fin root quarter-chord point measured parallel to longitudinal body axis, see Sketches 1.2 and 1.3	m	ft
$\mathcal{N}$	yawing moment	N m	lbf ft
N_ζ	yawing moment derivative due to rudder deflection, $(\partial \mathcal{N} / \partial \zeta) / \frac{1}{2} \rho V^2 S_W b$	rad ⁻¹	rad ⁻¹
O_{xyz}	aerodynamic-body axis system, origin O at moment reference point, see Sketch 1.1		
R_F	Reynolds number based on c_F		
S_F	fin reference area, $h_F(c_{rF} + c_{tF})/2$	m ²	ft ²
S_W	wing planform (reference) area	m ²	ft ²
$(t/c)_F$	thickness to chord ratio of fin section at mid-span of rudder		
v	sideslip velocity	m/s	ft/s
V	velocity of aircraft relative to air	m/s	ft/s
Y	sideforce, positive to starboard,, see Sketch 1.1	N	lbf
$(Y_v)_F$	fin sideforce derivative due to sideslip, $(\partial Y / \partial v) / \frac{1}{2} \rho V S_W$		

$(Y_v)_{FR}$	modified fin sideforce derivative due to sideslip, defined by Equation (3.2)		
Y_ζ	sideforce derivative due to rudder deflection, $(\partial Y/\partial \zeta)/\frac{1}{2}\rho V^2 S_W$	rad ⁻¹	rad ⁻¹
$\bar{z}_F$	parameter, defined in ESDU 82010 (Derivation 8), allowing for influence of tailplane position on location of centre of pressure of fin load due to sideslip	m	ft
z_R	moment arm of sideforce due to rudder measured normal to longitudinal body axis through moment reference point, see Sketch 1.1	m	ft
z_T	height of tailplane above fin root chord, see Sketch 1.2	m	ft
z_{TR}	distance of tailplane above body surface at station AA' defined by inboard end of rudder hinge-line, see Sketch 1.2	m	ft
α	angle of attack, see Sketch 1.1	degree	degree
α_δ	control effectiveness parameter, ratio of lift-curve slope due to control deflection to lift-curve slope due to angle of attack, see Section 3.2		
$(\alpha_\delta)_{th}$	theoretical value of α_δ		
β	angle of sideslip	degree	degree
ζ	rudder deflection angle, positive deflection trailing edge to port, measured in streamwise plane, $\approx \zeta' \cos \Lambda_h$	radian	radian
ζ'	rudder deflection angle, positive deflection trailing edge to port, measured in plane normal to hinge line	radian	radian
η_i, η_o	dimensionless inboard and outboard limits of rudder, see Section 3.3		
$\Lambda_{1/4F}$	fin quarter-chord sweep angle, see Sketches 1.2 and 1.3	degree	degree
$\Lambda_{1/2F}$	fin half-chord sweep angle	degree	degree
Λ_h	rudder hinge-line sweep angle	degree	degree
Λ_F	fin taper ratio, c_{tF}/c_{rF}		
ρ	density of air	kg/m ³	slug/ft ³

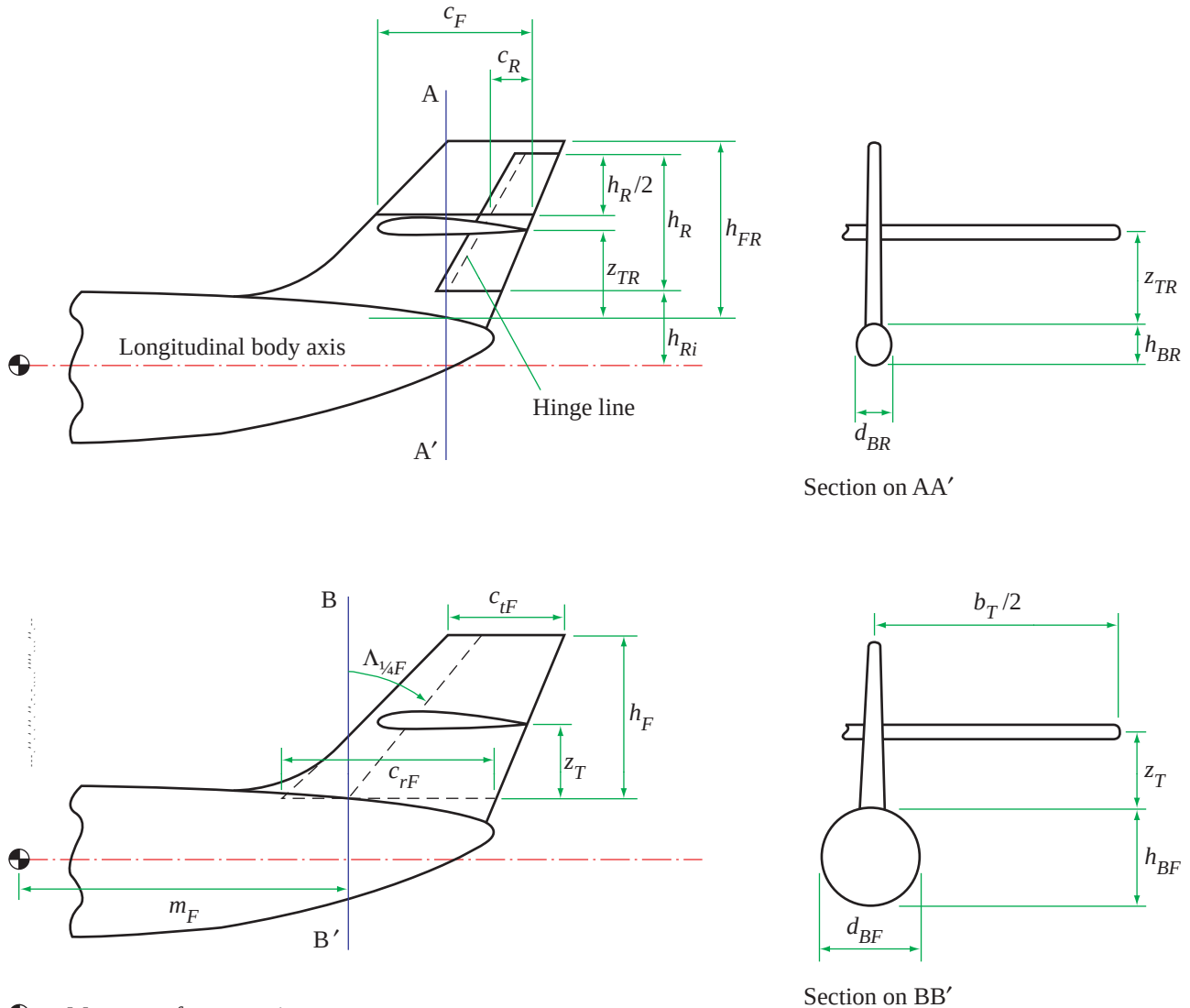
τ_F	trailing edge angle of fin section at mid-span of rudder	degree	degree
Φ_1, Φ_2	part-span correction functions		
$\Delta\Phi$	part-span correction factor		



- Moment reference point
- ⊠ Point of action of rudder sideforce
- Oxyz Aerodynamic-body axis system



Sketch 1.1 Axis system and moment arms

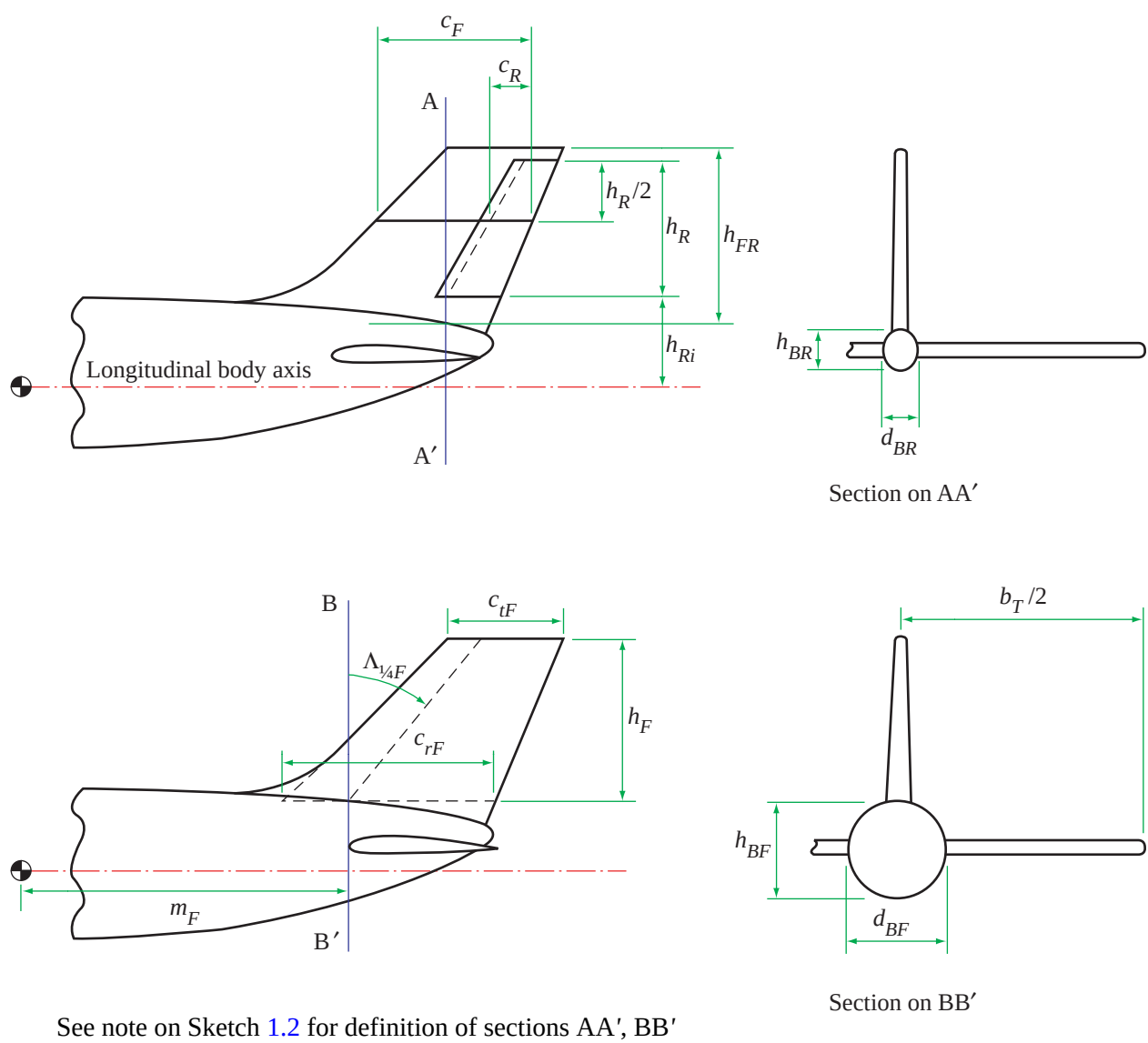


⊕ Moment reference point

AA' indicates the longitudinal station defined by the plane normal to the longitudinal body axis that passes through the inboard end of the rudder hinge-line.

BB' indicates the fin root quarter-chord station defined by the plane normal to the longitudinal axis that passes through the point at which the fin quarter-chord sweep line intersects the body surface.

Sketch 1.2 Geometry for fin-mounted tailplanes



Sketch 1.3 Geometry for body-mounted tailplanes

2. INTRODUCTION

This Item provides a method for predicting the rudder control derivatives Y_{ζ} , N_{ζ} and L_{ζ} , for aircraft at low speeds.* The method treats the rudder as a plain control with no control gap and makes use of the results for such controls fitted to a wing to obtain the control effectiveness parameter α_{δ} . The control derivatives are then obtained by combining α_{δ} with a modified fin sideforce derivative due to sideslip, $(Y_v)_{FR}$, appropriate moment arms, l_R and z_R , and a part-span correction factor, $\Delta\Phi$. The empirical factors used to modify the fin sideforce derivative have been obtained from an analysis of the wind-tunnel data contained in Derivations 9 to 15. The aircraft geometries covered by the method are those where a single fin is located on top of the aircraft rear-body, and in the plane of symmetry, with the tailplane mounted either on the fin itself or on the rear-body.

This Item is complementary to ESDU 82010 (Derivation 8) which contains a method for predicting fin sideforce, yawing moment and rolling moment derivatives due to sideslip. The method in this Item makes use of some of the data from ESDU 82010, but where necessary these have been repeated in sufficient detail to allow the present Item to be used independently. The definitions of the fin geometric parameters are consistent in the two Items, although the prediction of the control derivatives naturally requires additional parameters that are associated with the rudder geometry. In Sketches 1.2 and 1.3 the upper diagrams show the additional dimensions needed, while the lower diagrams show the dimensions that are common to both Items.

3. METHOD

The rudder control derivatives are expressed in terms of the set of aerodynamic-body axes Oxyz defined in Sketch 1.1.

The rudder sideforce derivative[†] is given by

$$Y_{\zeta} = -(Y_v)_{FR} \alpha_{\delta} \Delta\Phi \quad (3.1)$$

where $(Y_v)_{FR} = -J_R J_T a_{1F} S_F / S_W$. (3.2)

The equations for the yawing moment and rolling moment derivatives[†] are

$$N_{\zeta} = -Y_{\zeta} (l_R \cos \alpha + z_R \sin \alpha) / b \quad (3.3)$$

and $L_{\zeta} = Y_{\zeta} (z_R \cos \alpha - l_R \sin \alpha) / b$. (3.4)

3.1 Modified Fin Sideforce Derivative $(Y_v)_{FR}$

The derivative $(Y_v)_{FR}$ is obtained as a modified form of the basic fin sideforce derivative $(Y_v)_F$ that is estimated in ESDU 82010 (Derivation 8) as

$$(Y_v)_F = -J_B J_T J_W a_{1F} S_F / S_W \quad (3.5)$$

* Appendix A describes a computer program, ESDUpac A8708, that has been written for the method of this Item.

[†] The derivatives with respect to the rudder angle ζ' measured in a plane normal to the rudder hinge-line are $Y_{\zeta'} = Y_{\zeta} \cos \Lambda_h$, $N_{\zeta'} = N_{\zeta} \cos \Lambda_h$ and $L_{\zeta'} = L_{\zeta} \cos \Lambda_h$.

Here a_{1F} is the lift-curve slope of a straight-tapered wing of aspect ratio $A_F = 2h_F^2/S_F$, sweep $\Lambda_{1/4F}$ and taper ratio c_{tF}/c_{rF} and can be estimated from ESDU 70011 (Derivation 5), which gives a_{1F}/A_F in terms of the parameters $A_F \tan \Lambda_{1/2F}$, λ_F and A_F . The factors J_B , J_T and J_W are empirical correction factors and it is through changes to J_B and J_W that the sideforce derivative is modified.

In ESDU 82010 the factor J_B allows for body interference and depends on the fin-body geometry at the fin root quarter-chord station. It is a function of $h_{BF}/(h_{BF} + h_F)$ and A_F , with the body cross-section inherently assumed to be circular, i.e. $h_{BF} = d_{BF}$. For estimating $(Y_v)_{FR}$ in this Item the factor J_B is replaced by the factor J_R . This is expressed in terms of a basic value J_{Ro} , which is closely related to J_B , and an empirically determined correction factor to allow for tailplane position. Figure 1 shows J_{Ro} which has an identical form to J_B except that it is expressed as a function of the fin-body geometry at the station defined by the inboard end of the rudder hinge-line via the geometric parameter $(h_{BR} + d_{BR})/(h_{BR} + d_{BR} + 2h_{FR})$, see Sketches 1.2 and 1.3. This produces a factor that is smaller than J_B because of the tapering of the rear fuselage and allows for the possibility of a non-circular body cross-section in the way suggested in ESDU 82010. For T-tails, $z_{TR} = h_{FR}$, the correction factor for tailplane position is 1.05, for body-mounted tailplanes it is 0.80 and for tailplanes at a general height z_{TR} on the fin a linear variation is assumed between those two extremes. These results are summarised in Figure 1.

The factor J_T is used in ESDU 82010 to allow for the increase in fin effectiveness due to a tailplane. For aircraft with tailplanes mounted on the fin it is given as a function of tailplane height on the fin, z_T/h_F , and the ratio of tailplane span to fin height, b_T/h_F . For tailplanes that are mounted on the body it is given as a function of b_T/h_F and $h_{BF}/(h_{BF} + h_F)$. For estimating $(Y_v)_{FR}$ the data for J_T are used unaltered in this Item, and for convenience they are reproduced as Figures 2a and 2b.

In ESDU 82010 the factor J_W , which is a function of wing height, is used to allow for wing wake and body sidewash. As these do not affect rudder power, J_W is set equal to unity and does not appear in Equation (3.2).

3.2 Control Effectiveness α_δ

A method for estimating the effectiveness of plain controls on wings is contained in ESDU 74011 (Derivation 6) and for low speeds it gives α_δ in terms of wing aspect ratio, Reynolds number, control chord to wing chord ratio, and wing section thickness to chord ratio, the final two parameters being assumed constant across the wing span. To use this method to calculate α_δ , for the rudder it is therefore necessary to define an equivalent aspect ratio for the fin, A_{Feq} , to use in conjunction with the rudder to fin chord ratio and fin section geometry. This is achieved by finding the aspect ratio of a wing that, with a half-chord sweep of $\Lambda_{1/2F}$, has a lift-curve slope equal to $-(Y_v)_{FR} S_W/S_F \text{ rad}^{-1}$. Figure 3 presents $1/A_{Feq}$ in terms of $\Lambda_{1/2F}$ and $-(Y_v)_{FR} S_W/S_F$, together with the modified lifting-line formula that has been assumed for generating these data. (For the fin geometries that have been studied this simple approximate formula agrees to within about $\pm 5\%$ with the more accurate data that could be found iteratively from ESDU 70011, Derivation 5.) The rudder to fin chord ratio is often constant across the rudder span but, in any case, the ratio c_R/c_F at the mid-span of the rudder may be used. Similarly the fin thickness to chord ratio at rudder mid-span, $(t/c)_F$, may be taken as constant over the rudder span.

The method in ESDU 74011 presents α_δ (in a different notation) in a form equivalent to

$$\alpha_\delta = (\alpha_\delta)_{th} [1 - k_1 k_2], \quad (3.6)$$

where $(\alpha_\delta)_{th}$ is a theoretical value based on lifting-surface calculations and $[1 - k_1 k_2]$ is a correction that allows for section geometry and Reynolds number. For convenience, and to provide a consistent notation, the relevant data are reproduced in this Item as Figure 4a, which gives $(\alpha_\delta)_{th}$ in terms of $1/A_{Feq}$ and c_R/c_F , Figure 4b which gives k_1 in terms of $(t/c)_F \sec \Lambda_{1/2F}$ and c_R/c_F , and Figure 4c which gives k_2 in terms of $\log_{10} R_F$ and c_R/c_F .

The basic method of ESDU 74011 contains the simplifying restriction on section geometry that the trailing-edge angle in degrees, τ_F , is equal to $100(t/c)_F$. Most practical geometries satisfy this assumption sufficiently well for there to be no significant error in the prediction of α_δ . In the unlikely case of extreme departures, the viscous effects that are dependent on the trailing-edge angle can be more accurately allowed for by the additional use of ESDU Aero W.01.01.05 and Aero C.01.01.03 (Derivations 3 and 4) to calculate an appropriate factor to replace $[1 - k_1 k_2]$, as described in ESDU 74011.

3.3 Part-span Factor $\Delta\Phi$

If the rudder does not extend across the complete span of the fin it is necessary to calculate a part-span factor $\Delta\Phi$.

For a T-tail, as shown in diagram (i) in Figure 5a, the rudder is contained between the tailplane and the body and if the spanwise loading across the fin is assumed to be fairly uniform, then $\Delta\Phi$ can be taken simply as the ratio of rudder span to fin span, h_R/h_{FR} . If there is a fin extension above the tailplane, as shown in diagram (ii), then the function Φ_1 , which represents the proportion of the load carried by the fin below the tailplane, multiplies h_R/z_{TR} . The function Φ_1 depends on the tailplane height, z_{TR} , and is based on calculations made for a rectangular fin and tailplane of typical span using the trailing horseshoe vortex model of Derivation 2 with an ad hoc allowance for the presence of the body; it only provides a simple approximation to the division of the fin load but one that should be adequate for the purposes of estimating rudder control derivatives.

If the tailplane is mounted on the body, or if the tailplane is low on the fin so that the rudder is completely above the tailplane or extends continuously on either side of it, as shown in diagrams (iii), (iv) and (v) in Figure 5b, then use is made of the method in ESDU 74012 (Derivation 7). That method was designed for application to wings with part-span controls and evaluates $\Delta\Phi$ as $\Phi_2(\eta_o) - \Phi_2(\eta_i)$, where Φ_2 is a function calculated from lifting-surface theory that depends on wing planform geometry, and η_o and η_i denote the dimensionless outboard and inboard limits of the control. For estimating rudder control derivatives, η_o and η_i are taken as fractions of the fin span h_{FR} as shown in Figure 5b. The numerical data in the Figure have been reproduced directly from ESDU 74012, but are expressed in terms of the fin parameters A_{Feq} , $\Lambda_{1/2F}$ and λ_F .

3.4 Moment Arms l_R, z_R

The moment arm l_R is measured parallel to the body longitudinal axis and is obtained by adding a quarter of the fin chord at rudder mid-span to the semi-empirical moment arm calculated in ESDU 82010 for the fin sideforce due to sideslip, so that

$$l_R = (m_F + 0.7\bar{z}_F \tan \Lambda_{1/4F}) + 0.25c_F. \quad (3.7)$$

The bracketed expression is the moment arm from ESDU 82010. The distance m_F is measured from the moment reference point to the fin root quarter-chord station (see Sketches 1.2 and 1.3), and the parameter $\bar{z}_F$ which is a function of tailplane height z_T is given in Figure 6 which has been reproduced directly from ESDU 82010.

The moment arm z_R is measured normal to the longitudinal body axis. For a rudder contained between the tailplane and the body, as in diagrams (i) and (ii) on Figure 5a, z_R is measured to the rudder mid-span, so

$$z_R = h_{Ri} + 0.5h_R, \quad (3.8)$$

where h_{Ri} is the distance of the inboard end of the rudder hinge-line above the moment reference point and h_R is the rudder span.

If the tailplane is body-mounted or if the tailplane is low on the fin so that the rudder is completely above the tailplane or extends on either side of it, as shown in diagrams (iii), (iv) and (v) in Figure 5b, then z_R is measured to 40% of the rudder span, so

$$z_R = h_{Ri} + 0.4h_R, \quad (3.9)$$

where the 0.4 factor has been determined empirically.

4. ACCURACY AND APPLICABILITY

Predicted and experimental values of the rudder control derivatives are compared in Sketches 4.1 to 4.3. In general, Y_ζ is predicted to within ± 0.04 , N_ζ to within ± 0.02 and L_ζ to within ± 0.01 ; the data shown are all for zero angle of attack. The error bands are half those associated with the prediction of the fin derivatives due to sideslip in ESDU 82010, which is a consistent reflection of the possible uncertainty introduced via the estimate of $(Y_v)_{FR}$.

The method has been based mainly on analysis of data for civil transport aircraft, where the fin is reasonably well aft of the wing. Some additional data for light aircraft and combat aircraft have also been utilised to support the analysis. Attention has been restricted to low speeds and angles of attack, sideslip and rudder deflection for which the rudder sideforce and moments vary linearly with ζ , typically $0 \leq \alpha \leq 10^\circ$, $|\beta| \leq 10^\circ$ and $|\zeta| \leq 10^\circ$. For some configurations Y_ζ decreases by about 10% or so as α increases from 0 to 10° , with a consequent influence on the variation of N_ζ and L_ζ with α . In most cases the rudder characteristics depart only slowly from the linear as ζ increases from 10° to 20° or so. The extent of the linear range depends on the control nose shape. It has been assumed that there is no load on the tailplane, and the data are restricted to conventional controls with a sharp trailing-edge.

The control gap between rudder and fin has been assumed to be insignificant in terms of its effect on rudder control derivatives. Some information on the effect of significant gaps on control effectiveness is contained in ESDU Aero C.01.01.04 (Derivation 1).

Table 4.1 summarises the ranges of parameters covered by the experimental data. The method should only be used with caution for configurations with geometries that are significantly different from those listed. The different types of fin and rudder geometries that have been studied are illustrated in the diagrams in Figures 5a and 5b.