

ELASTIC WRINKLING OF SANDWICH COLUMNS AND BEAMS WITH UNBALANCED LAMINATED FIBRE REINFORCED FACE PLATES

(Face plates of AsBoDs, AsBlDs and AsBsDs types)

1. FACE PLATE WRINKLING BEHAVIOUR, SOLUTIONS AND LIMITATIONS

1.1 Introduction

This Data Item presents ESDUpac program A8713 for calculating the theoretical wrinkling loads of balanced laminated plates and of two classes of unbalanced laminated plates which form the face plates of sandwich columns or beams. The seven possible classes of laminated plates are described in detail in Data Item No. 94003.* The solution and program provided deal with the classes coded as AsBoDs, AsBlDs and AsBsDs in Data Item No. 94003. These classes are described in detail in Section 1.3.

The program is available in two formats: (a) a Fortran source code with example input and output files, and an executable program for PCs. Both versions can be found on the Compact Disc and on disk in the Sub-series Software Volume.

This Data Item consists of three parts. Section 1 describes the theoretical solution to the title problem and its limitations in application to practical columns and beams. Section 2 describes the input and output for the Fortran program of ESDUpac A8713 together with example calculations. Section 3 gives notes on the method of solution used, the resulting accuracy and program testing.

At Amendment C the sign convention of Data Item No. 94003 has been adopted. This sign convention is now consistent with that used in Data Items associated with ESDUview and/or ESDUpac programs for the analysis of composites. The convention has changed the signs in some of the equations and some of the elements of the coupling stiffness matrix for unbalanced face plates. Nevertheless, the compressive wrinkling loads are unaltered.

One possible mode of failure of a sandwich column or beam with thin face plates is the localised buckling (wrinkling) of the compression faces as described in Section 1.5 and illustrated in Sketches 1.2 to 1.4. For fibre reinforced face plates made up from a number of thin layers, it is possible that the face plates will each form an unbalanced lay-up about their local mid-planes. This lack of balance is further accentuated by the layer of glue that is required to bond each face plate to the core material and which can have a significant effect on the wrinkling loads. The following sections detail the forms of face plate stiffness sub-matrices dealt with by this Data Item and illustrate the influence of glue layer, core thickness and face plate thickness on the wrinkling loads of columns and beams.

* Data Item No. 94003 "Stiffnesses of laminated flat plates".

1.2 Notation

Both SI and British units are quoted but any coherent system of units may be used.

1.2.1 Face plate overall stiffnesses

A	in-plane stiffness matrix for face plate	N/m	lbf/in
A_{ij}	element of A	N/m	lbf/in
A_e	effective in-plane stiffness term for face plate	N/m	lbf/in
B	coupled in-plane and flexural stiffness matrix for face plate	N	lbf
B_{ij}	element of B	N	lbf
B_e	effective in-plane and flexural stiffness term for face plate	N	lbf
D	flexural stiffness matrix for face plate	N m	lbf in
D_{ij}	element of D	N m	lbf in
D_e	effective flexural stiffness term for face plate	N m	lbf in

1.2.2 Layer properties and reference axes

C	reference number for layer type (see Section 2.3.3)		
$E_{\alpha k}, E_{\beta k}$	moduli of elasticity of k th layer in α_k - and β_k -directions respectively	N/m ²	lbf/in ²
$G_{\alpha\beta k}$	shear modulus of k th layer in $(\alpha, \beta)_k$ directions	N/m ²	lbf/in ²
t_k	thickness of k th layer	m	in
ψ_k	angle of principal axes of orthotropy, α_k , of k th layer measured from plate reference axis, 0x, as shown in Sketch 1.1	degree	degree
$\nu_{\alpha\beta k}, \nu_{\beta\alpha k}$	Poisson's ratio for k th layer resulting from load applied in α_k -direction and in β_k -direction, respectively		
$0(\alpha, \beta)$	principal axes of orthotropy of a layer (see Sketch 1.1)		

1.2.3 Core properties

E_c	modulus of elasticity of core material in z-direction	N/m ²	lbf/in ²
G_c	shear modulus of core material in (x, z) plane	N/m ²	lbf/in ²

1.2.4 Column/beam loading and geometry and reference axes

h	depth of core measured between mid-planes of face plates	m	in
h_t	total depth of sandwich between exterior faces of face plates	m	in
N	wrinkling load per face plate per unit width of column or beam (see Sketches 1.2 to 1.4)	N/m	lbf/in
t	thickness of face plate including thickness of face plate to core adhesive	m	in
t_a	effective thickness of face plate to core adhesive layer	m	in
λ	length of half wave of wrinkle along column or beam length	m	in
$O(x, y, z)$	column or beam reference axes (see Sketch 1.1)		

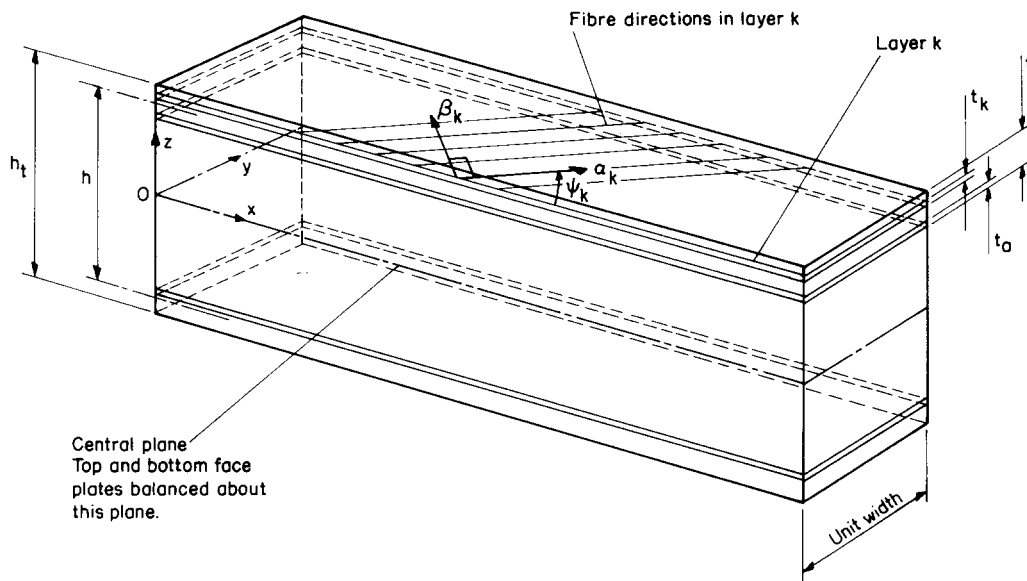
1.2.5 Suffixes

i, j	numbers signifying position in row and column, respectively, of element in matrix
k	layer identification number
w_b	relates to wrinkling load for compression face plate of beam subject to bending (see Sketch 1.4)
w_{ca}	relates to antisymmetric wrinkling (see Sketch 1.3)
w_{cs}	relates to symmetric wrinkling (see Sketch 1.2)

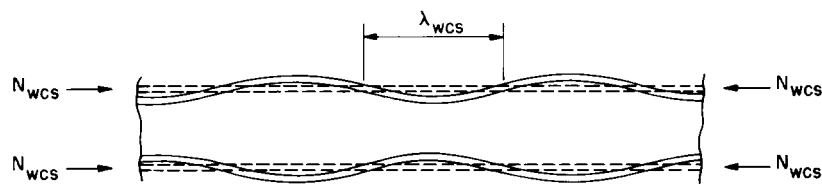
1.2.6 Mathematical notation and sign convention

sym	indicates matrix is symmetric about leading diagonal
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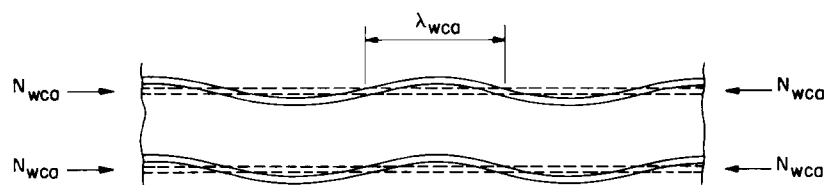
Tensile stresses and loadings are positive, compressive stresses and loadings are negative.



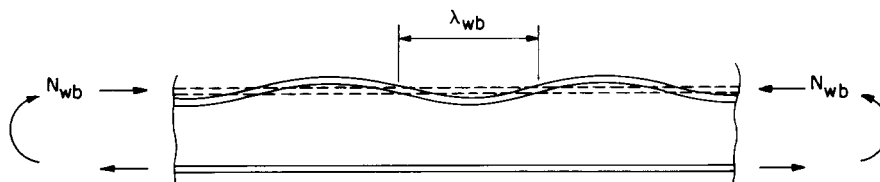
Sketch 1.1 Sandwich construction



Sketch 1.2 Column face plate symmetric wrinkling



Sketch 1.3 Column face plate antisymmetric wrinkling



Sketch 1.4 Beam face plate wrinkling

1.3 Class of Unbalanced Plates for Which a Solution is Given

Data Item No. 94003 shows how the stiffness matrix forms for the various types of unbalanced plate arise. This present Data Item deals only with plates in which the **A** and **D** stiffness sub-matrices are of the following forms

$$\mathbf{A} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ & A_{22} & 0 \\ (sym) & & A_{33} \end{bmatrix}, \mathbf{D} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ & D_{22} & 0 \\ (sym) & & D_{33} \end{bmatrix} \quad (1.1)$$

and the **B** coupling matrix is either zero or is one of the forms,

$$\mathbf{B} = \begin{bmatrix} B_{11} & 0 & 0 \\ & B_{22} & 0 \\ (sym) & & 0 \end{bmatrix} \text{ or } \mathbf{B} = \begin{bmatrix} B_{11} & B_{12} & 0 \\ & B_{22} & 0 \\ (sym) & & B_{33} \end{bmatrix}. \quad (1.2)$$

These forms accord with the reference codes AsBoDs, AsBIDs and AsBsDs defined in Data Item No. 94003.

It should be noted that for the column as a whole, the pair of face plates must form a balanced lay-up about the central plane of the column as shown in Sketch 1.1. This is not a requirement in the case of the beam because one of the faces (the tension face) is assumed to form a rigid boundary for the buckling condition.

1.4 Statement of Basic Assumptions

Section 1.6 sets out the equations that are the basis for the buckling solution provided by the program of Section 3. The solution is based upon elastic, thin plate, small deflection theory, in which the laminated face plates are assumed to behave as homogeneous orthotropic plates whose axes of orthotropy are aligned with the edges of the beam or column. For columns the face plates are assumed to form a balanced lay-up about the central plane of the sandwich. In the beam, the tension face plate is assumed to provide a rigid boundary for the core against wrinkling of the compression face plate.

The theory only takes account of direct in-plane and flexural deformations as governed by the first 2×2 terms of the **A**, **B** and **D** stiffness sub-matrices. No account is taken of through-the-thickness shear deformations. The faces are assumed to be initially flat, and to be held flat by the core until they wrinkle. The core is assumed to give continuous support to the faces and the possibility of localised buckling within honeycomb type cells (often called inter-cell buckling) is not considered. Also, the core is taken to have zero longitudinal stiffness but finite direct and shear stiffnesses in the z -direction. These assumptions make the analysis more applicable to honeycomb type cores than to solid foam cores, but should give conservative results for the latter. The applied load is assumed to lie in the mid-plane of the face plate (which might include a layer of glue) and this is the plane to which **B** and **D** are related.

It should be noted that the length of the column does not appear in any of the equations. This is because it is convenient to work in terms of the half wavelength of the wrinkle, λ , it being assumed that the column is long enough for λ to achieve its minimum value. The program provides the value of λ for each case and this should be compared to the strut length. For a sandwich beam, the end load carried by each of the faces will be obtained from the bending moment carried by the beam. In this case, therefore, it is strictly a

requirement that the bending moment should be constant over a significant portion of the beam, that is, approximately one buckle half wavelength. If this is not the case, then a conservative wrinkling load can be obtained by analysing the section where the maximum bending moment occurs.

The equations and program given in this Data Item are based upon the theoretical work of Derivations 1 and 2 of Section 1.7 which employ the assumptions described above. Because of this, wrinkling loads obtained should only be used as a guide for design purposes. Wherever possible, verification of final designs should be obtained from wrinkling tests on specimens that are representative of the actual method of manufacture and configuration.

The equations and program can be applied to the simple case of sandwiches with ordinary, isotropic, metallic face plates. For these cases empirical design formulae based on many test results are available in Reference 3. It is recommended that these should be consulted before a design is finalised.

1.5 Wrinkling Characteristics

Sketches 1.2, 1.3 and 1.4 show the forms of face plate wrinkling that can occur. Note that symmetric wrinkling of a column is independent of core shear modulus, G_c , see Equation (1.6). Figure 1 shows how the various buckling loads vary with core thickness for a particular sandwich column and beam, with a lay-up of the AsBIDs type. For the core with $G_c = 26.6 \text{ N/mm}^2$ the general trend for all three cases is that the buckling loads are seen to decrease as the core thickness is increased from a value of 10 mm. The load required for beam wrinkling is between the other two when $h \geq 7 \text{ mm}$, and the load for antisymmetric column wrinkling is the highest. The curve for antisymmetric wrinkling shows an interesting characteristic for values of core thickness less than 10 mm. In this region, N_{wca} actually decreases quite suddenly, whilst the corresponding values of λ_{wca} increase. The reason for this behaviour is that the length of λ_{wca} in relation to the column geometry is such that the wrinkling modes are now influenced by those that would be developed when overall Euler buckling occurs at relatively low column loads. For values of h below 2.67 mm, Euler buckling dominates and a minimum value of λ_{wca} does not exist. This type of behaviour is not possible in the case of symmetric column wrinkling. For the core with the lowest value of G_c a similar pattern is evident but with the peak in N_{wca} at a higher value of h .

Figures 2a, 2b and 2c show how an adhesive layer influences the wrinkling loads for a particular combination of face plates and core. Each figure shows curves for three values of h_t both with, and without, an adhesive layer. Note that introducing the adhesive layer reduces the core thickness since it is h_t that is held constant for each curve. The secondary effect of t_a in reducing core thickness has negligible influence on the curves shown since t_a is small in relation core thickness.

Generally, the effect of the adhesive is to increase all the wrinkling loads. The increase is largest when both h_t and t are small. When t is large the influence of t_a is reduced and, in the example Figures 2a, 2b and 2c, at $t = 0.6 (= 24t_a)$ the adhesive increases the buckling loads by less than 10 per cent. An exception to these general trends occurs on Figure 2b. This shows that at larger t and smaller h_t values the curves begin to fall-off with increasing t . This is because the core thickness is being significantly reduced and the peak value of N_{wca} , shown on Figure 1, is being approached at high t and small h_t .

Because of the fact that in the symmetric column case there is symmetry of deformation about the centre line, it might be expected that the solution for the column could be used to obtain the solution for the beam by taking the central plane as the fixed tension face of the beam with appropriate adjustment of the core thickness. However, for the range of geometries considered in Figure 1, it is found that the beam buckling loads so obtained are below the values given by Equation (1.9) below. One reason for this probably lies in the difference between the boundary displacements assumed at the column centre plane and the beam tension face.

1.6 Basic Equations

1.6.1 Effective stiffness terms

In the formulae that follow for the wrinkling loads, the effective stiffness terms A_e , B_e and D_e are given in terms of A_{ij} , B_{ij} and D_{ij} by the following expressions.

$$A_e = A_{11} + \frac{(2A_{12}B_{12}B_{22} - A_{12}^2D_{22} - B_{12}^2A_{22})}{(A_{22}D_{22} - B_{22}^2)}, \quad (1.3)$$

$$B_e = B_{11} + \frac{(A_{12}D_{12}B_{22} - A_{12}B_{12}D_{22} - B_{12}D_{12}A_{22} + B_{12}^2B_{22})}{(A_{22}D_{22} - B_{22}^2)} \quad (1.4)$$

and

$$D_e = D_{11} + \frac{(2B_{12}D_{12}B_{22} - B_{12}^2D_{22} - D_{12}^2A_{22})}{(A_{22}D_{22} - B_{22}^2)}. \quad (1.5)$$

1.6.2 Symmetric wrinkling of columns

The wrinkling load per unit width is given by

$$N_{wcs} = -2 \left[\frac{2E_c(D_e - B_e^2/A_e)}{h} \right]^{\frac{1}{2}} \quad (1.6)$$

and the corresponding half wavelength is

$$\lambda_{wcs} = \pi \left[\frac{(D_e - B_e^2/A_e)h}{2E_c} \right]^{\frac{1}{4}}. \quad (1.7)$$

1.6.3 Antisymmetric wrinkling of columns

The wrinkling load per unit width is given by the following equation which is to be minimised with respect to λ_{wca}

$$N_{wca} = - \frac{\left\{ \frac{h^2 A_e}{2} + 2hB_e + 2D_e \right\} \left(\frac{\pi}{\lambda_{wca}} \right)^2 + (A_e D_e - B_e^2) \left\{ \frac{h}{G_c} \left(\frac{\pi}{\lambda_{wca}} \right)^4 + \frac{h^3}{12E_c} \left(\frac{\pi}{\lambda_{wca}} \right)^6 \right\}}{2 \left\{ 1 + \frac{hA_e}{2G_c} \left(\frac{\pi}{\lambda_{wca}} \right)^2 + \frac{h^3 A_e}{24E_c} \left(\frac{\pi}{\lambda_{wca}} \right)^4 \right\}}. \quad (1.8)$$

1.6.4 Wrinkling of beams

The wrinkling load per unit width in the compression face plate is given by the following equation which

is to be minimised with respect to λ_{wb}

$$N_{wb} = - \left[\frac{E_c}{h} \left(\frac{\lambda_{wb}}{\pi} \right)^2 + \frac{A_e E_c}{G_c} + \left\{ \frac{A_e h^2}{3} + B_e h + D_e \right\} \left(\frac{\pi}{\lambda_{wb}} \right)^2 + (A_e D_e - B_e^2) \left\{ \frac{h^3}{12 E_c} \left(\frac{\pi}{\lambda_{wb}} \right)^6 + \frac{h}{G_c} \left(\frac{\pi}{\lambda_{wb}} \right)^4 \right\} \right] \times \left[1 + \frac{A_e h}{G_c} \left(\frac{\pi}{\lambda_{wb}} \right)^2 + \frac{A_e h^3}{12 E_c} \left(\frac{\pi}{\lambda_{wb}} \right)^4 \right]^{-1} \quad (1.9)$$

1.7 Derivation and Reference

1.7.1 Derivation

This section lists selected sources that have assisted in the preparation of this Item.

1. WEBBER, J.P.H.
KYRIAKIDES, S.
LEE, C.T. On the wrinkling of honeycomb sandwich columns with laminated cross-ply faces. *The Aeronautical Journal*, Vol. 80, pp. 264-272, June 1976.
2. GUTIERREZ, A.J.
WEBBER, J.P.H. Flexural wrinkling of honeycomb sandwich beams with laminated faces *International Journal of Solids and Structures*, Vol. 16, pp. 645-651, 1980.

1.7.2 Reference

The reference given is a recommended source of information supplementary to that in this Item.

3. SULLINS, R.T.
SMITH, G.W.
SPIER, E.E. Manual for structural stability analysis of sandwich plates and shells. NASA CR-1457, Section 2.2, December 1969.

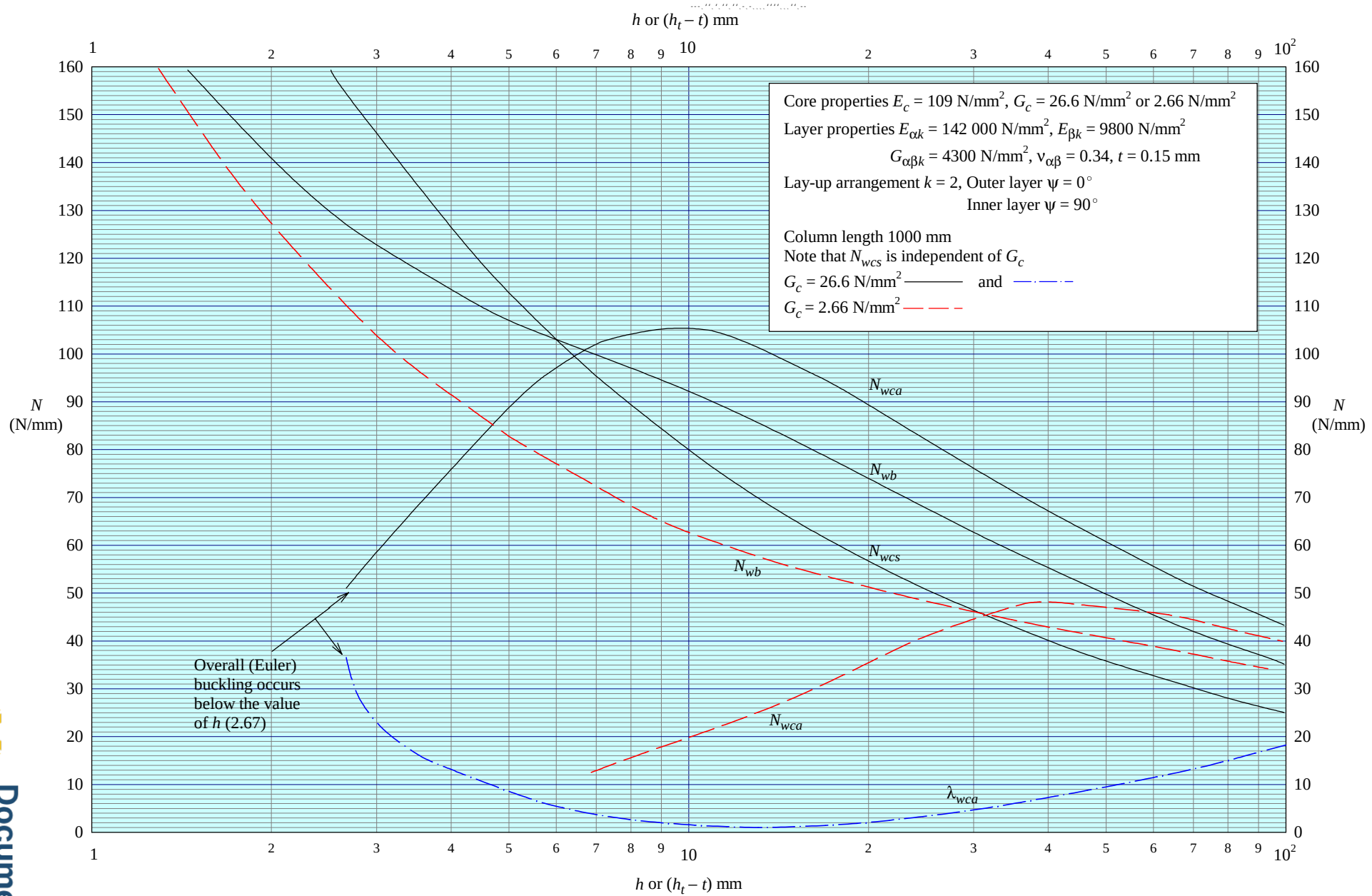


FIGURE 1 ILLUSTRATING EFFECT OF DEPTH OF CORE