

EXAMPLE OF TAKE-OFF FIELD LENGTH CALCULATIONS FOR A CIVIL TRANSPORT AEROPLANE

1. NOTATION AND UNITS

The notation is based on the system introduced in Item No. 78038 (Reference 4), and extended in Item No. 85030 (Reference 6) to cover ground-performance applications. Units quoted are a coherent set so as to avoid introducing numerical constants into equations (however, speeds are expressed in knots in the tables and figures). Conversion factors to SI Units are given in the Conversion Factors Item in Volume 1.

C_D	drag coefficient, $D/(\rho V^2 S/2) (= -C_{X_a}^A)^*$	
$C_{D_{fa}}$	value of C_D out of ground effect	
$C_{D_{ge}}$	value of C_D in ground effect	
C_L	lift coefficient $L/(\rho V^2 S/2) (= -C_{Z_a}^A)^*$	
$C_{L_{fa}}$	value of C_L out of ground effect	
$C_{L_{ge}}$	value of C_L in ground effect	
$C_{L_{LIM}}$	limiting value of C_L for airborne segment	
$C_X^A; C_Z^A$	body-axes airframe aerodynamic force coefficients, $X^A/(\rho V^2 S/2); Z^A/(\rho V^2 S/2)$	
$C_{X_a}^A; C_{Z_a}^A$	air-path axes airframe aerodynamic force coefficients, $X_a^A/(\rho V^2 S/2); Z_a^A/(\rho V^2 S/2)$	
C_y	fin side-force coefficient for flight with asymmetric thrust, $[(F_{N_{E1}} + F_{N_{E2}})y_{nozzle}/(\rho V^2 x_{fin} S_{fin}/2)]$	
$\bar{c}$	aerodynamic mean chord, see Sketch 4.1	ft
D	drag, $(= -X_a^A)^*$	lbf
D_M	momentum drag (ram drag), per engine	lbf
D_{spill}	spillage drag, per intake	lbf
D_{wm}	windmilling drag, per engine	lbf
d_{tyre}	diameter of undeflected, inflated tyre	ft

For footnotes see end of Notation section

E_{brake}	energy absorbed by brake, per brake unit	ft lbf
$E_{max,stop}$	maximum certificated brake energy for emergency use, per brake unit	ft lbf
F_{CUT}	factor to describe thrust and momentum drag variations following throttle cut, see Equation (C2.4)	
F_G	gross thrust, per engine	lbf
F_N	net thrust, $(F_G - D_M)$, per engine	lbf
$F_{N_{CUT}}$	net thrust, per engine, during run down after throttle cut	lbf
$F_{N_{RUNUP}}$	net thrust, per engine, during run up	lbf
F_{RUNUP}	factor to describe net thrust variation during run up, $[F_{N_{RUNUP}} - F_{N_{IDLE}}] / [F_{N_{max}} - F_{N_{IDLE}}]$	
$f [\dots]$	denotes functional relationship	
f_{ge}	factor for variation of $\Delta C_{D_{ge}}$ with height, see Table 7.2	
G_B	tangential force on tyre due to braking alone	lbf
G_R	rolling resistance of tyre and wheel	lbf
g	acceleration due to gravity	ft/s ²
H^Σ	resultant angular momentum	slug ft ² /s
H_p	pressure height	ft
h_{screen}	take-off screen height, see Sections 3.5 and 3.6	ft
L	lift*, $(= -Z_a^A)$	lbf
M^Σ	resultant moment	lbf ft
M	pitching moment	lbf ft
m	mass	slug
n	normal load factor	
n_E	number of engines	
n_{EF}	number of engines failing	

For footnotes see end of Notation section

n_{mbrake}	number of brake units on main undercarriage legs	
n_{mtyre}	total number of tyres on main undercarriage legs	
n_{nbrake}	number of brake units on nose undercarriage	
n_{ntyre}	number of tyres on nose undercarriage	
p	pressure	lbf/ft ²
Q	brake torque	lbf ft
Q_{lim}	limiting value of brake torque	lbf ft
Q_{max}	maximum value of Q_{lim}	lbf ft
q	angular velocity component about 0y-axis (rate of pitch)	rad/s
R	resultant external force, excluding gravity	lbf
R^A	resultant airframe aerodynamic force	lbf
R^G	external force due to gravity	lbf
R^Σ	resultant external force	lbf
S	wing reference area	ft ²
S_{fin}	fin reference area	ft ²
s	distance	ft
T	temperature	K
t	time	s
$u; w$	components of V in x-direction; z-direction	ft/s
$u_a; w_a$	components of V in x_a -direction; z_a -direction (= $V, 0$)	ft/s
$u_g; w_g$	components of V in x_g -direction; z_g -direction	ft/s
$u_K; w_K$	components of V_K in x-direction; z-direction	ft/s
$u_{K_g}; w_{K_g}$	components of V_K in x_g -direction; z_g -direction	ft/s
$u_{K_r}; w_{K_r}$	components of V_K in x_r -direction; z_r -direction	ft/s

For footnotes see end of Notation section

$u_W; w_W$	components [†] of V_W in x-direction; z-direction	ft/s
$u_{W_g}; w_{W_g}$	components [†] of V_W in x_g -direction; z_g -direction	ft/s
$u_{W_r}; w_{W_r}$	components [†] of V_W in x_r -direction; z_r -direction	ft/s
V	air-path velocity (true airspeed or resultant velocity with respect to air)	ft/s
V_c	calibrated airspeed	ft/s
V_{EF}	engine failure speed [‡]	ft/s
V_e	equivalent airspeed	ft/s
V_K	flight path velocity (resultant velocity with respect to Earth)	ft/s
V_{LOF}	lift-off speed [‡]	ft/s
V_{MC}	minimum control speed [‡] (in the air)	ft/s
V_{MCG}	minimum control speed on the ground [‡]	ft/s
V_{MU}	minimum unstick speed [‡]	ft/s
V_R	rotation speed [‡]	ft/s
V_S	stall speed [‡]	ft/s
V_W	wind velocity (relative to Earth)	ft/s
V_1	take-off decision speed [‡]	ft/s
V_2	take-off safety speed [‡]	ft/s
$X; Z$	components of R in x-direction; z-direction	lbf
$X^A; Z^A$	components of R^A in x-direction; z-direction	lbf
$X_a^A; Z_a^A$	components of R^A in x_a -direction; z_a -direction	lbf
$X^\Sigma; Z^\Sigma$	components of R^Σ in x-direction; z-direction	lbf
$x; y; z$	body-axes co-ordinates in x-direction; y-direction; z-direction (aircraft-carried), see Sketch 1.1	ft

For footnotes see end of Notation section

$x_a; y_a; z_a$	air-path axes co-ordinates in x_a -direction; y_a -direction; z_a -direction (aircraft-carried), see Sketch 1.1	ft
x_{fin}	body axis co-ordinate of fin sideforce reference point, see Sketch 4.1	ft
$x_g; y_g; z_g$	normal Earth-axis co-ordinates (Earth-fixed), see Sketch 1.1	ft
$x_k; y_k; z_k$	flight-path axis co-ordinates (aircraft-carried), see Sketch 1.1	ft
$x_r; y_r; z_r$	runway axis co-ordinates (Earth-fixed), see Sketch 1.1	ft
z_{datum}	body axis co-ordinate of ground for zero tyre and undercarriage leg deflections, <i>i.e.</i> tyres are inflated but unloaded, undercarriage legs are unloaded and fully extended	ft
α	angle of attack; angle between x -axis and projection of V on to zx -plane ($\tan \alpha = w/u$; α has same sign as w)	deg
α_{LIM}	limiting value of α in airborne segment	deg
γ	angle of climb (flight-path inclination angle), $\sin \gamma = (-w_{K_g})/V_K$	deg
γ_a	air-path angle of climb (air-path inclination angle), $\sin \gamma_a = (-w_g)/V$	deg
Δ	denotes increment in affixed parameter	
$\Delta C_{D_{asym}}$	change in C_D due to flight with asymmetric engine thrust	
$\Delta C_{D_{uc}}$	change in C_D due to undercarriages	
$\Delta C_{D_{ge}}$	change in C_D due to ground effect	
$\Delta C_{D_{spill}}$	change in C_D due to spillage, per intake	
$\Delta C_{D_{spoil}}$	change in C_D due to deployment of spoilers	
$\Delta C_{D_{wm}}$	change in C_D due to engine windmilling, per engine	
$\Delta C_{L_{ge}}$	change in C_L due to ground effect	
$\Delta C_{L_{spoil}}$	change in C_L due to deployment of spoilers	
ΔH_{screen}	vertical distance between aeroplane lift-off point (at V_{LOF}) and screen height, h_{screen} , see Section 3.6	ft
ΔT_{H_p}	temperature increment applied at constant pressure height	K

For footnotes see end of Notation section

$\Delta t_{a, brake}$	time from decision to application of brakes by pilot or auto system	s
$\Delta t_{a, cut}$	time from decision to throttle cut of live engine(s)	s
$\Delta t_{a, spoil}$	time from decision to selection of spoilers by pilot or auto system	s
$\Delta t_{e, cut}$	time from engine throttle cut (see Table 7.3)	s
$\Delta t_{e, fail}$	time for thrust of failed engine to fall to zero	s
$\Delta t_{eff, brake}$	time from brake application to achieve effective braking	s
$\Delta t_{eff, spoil}$	time from selection to achieve effective spoiler deployment	s
Δt_{rec}	demonstrated engine-failure recognition time	s
Δt_{ucr}	undercarriage retraction time	s
δ	relative pressure, p/p_0	
$\delta_{mtyre}, \delta_{ntyre}$	deflections of tyres on main and nose undercarriages, normal to runway surface	ft
$\delta_{muc}, \delta_{nuc}$	deflections of main and nose undercarriage legs from unloaded, fully extended condition	ft
δ_{tyre}	tyre deflection normal to runway surface	ft
ε	“splay” angle between gross thrust vector and aircraft plane of symmetry (0x, 0z); positive when component of F_G along 0y is negative (i.e. when F_G acts to port)	deg
Θ	inclination angle (elevation)	deg
Θ_g	angle between ground and 0x body axis (ground attitude angle, = $\Theta - \Theta_r$)	deg
$\Theta_{g datum}$	value of Θ_g for zero tyre and undercarriage deflections	deg
Θ_r	angle between runway centre line and horizontal (positive uphill)	deg
θ	relative temperature, T/T_0	
μ_{eff}	effective tyre-runway coefficient of friction	
μ_R	coefficient of rolling resistance	
ρ	density	slug/ft ³

For footnotes see end of Notation section

σ	relative density, ρ/ρ_0	
τ	angle between projection of gross-thrust vector on to aircraft plane of symmetry and 0x body axis; positive when component of F_G along 0z is negative (i.e. F_G contributes to lift)	deg
Ω^Σ	resultant angular velocity	rad/s

Superscripts

A	denotes (airframe) aerodynamic force or moment
F	denotes engine-thrust related force or moment
G	denotes gravitational force or moment
$tyre$	denotes tyre related force or moment
$mtyre$	denotes force or moment on main undercarriage tyre
$ntyre$	denotes force or moment on nose undercarriage tyre

Subscripts

ALL	denotes all engines operating
a	denotes use of air-path axes
$aero$	denotes aerodynamic forces reference point
E_1, E_2	denotes engine 1, 2....
EF_1, EF_2	denotes engine 1, 2... sustaining failure at V_{EF}
g	denotes use of local normal Earth axes
$i, i + 1$	denotes successive values in step-by-step calculation
$intake$	denotes value at centre of engine intake plane
jet	denotes value for exhaust jet
k	denotes use of flight-path axes
LOF	denotes value at lift-off
max	denotes maximum value
min	denotes minimum value

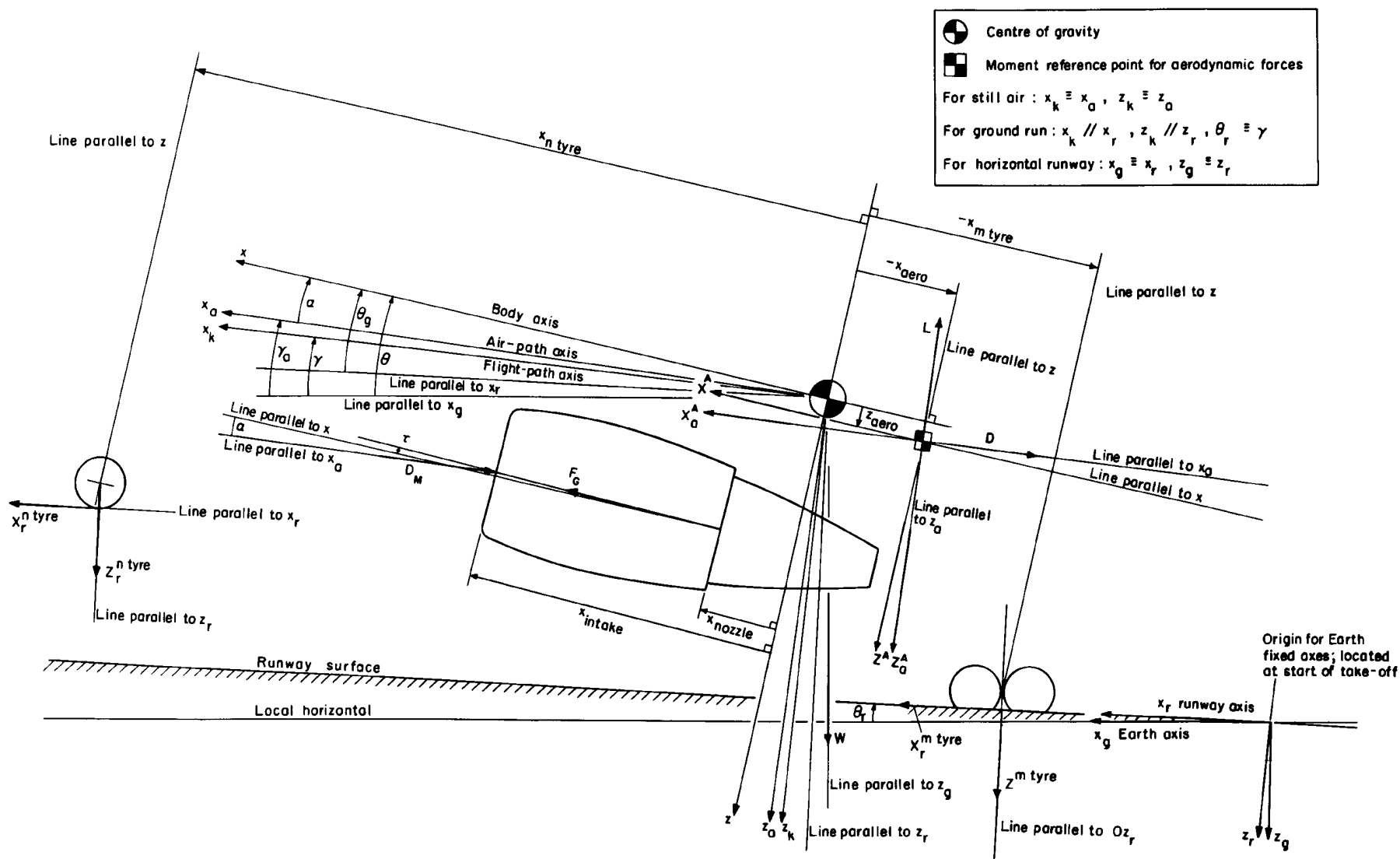
For footnotes see end of Notation section

<i>mtyre</i>	denotes value at main undercarriage tyre
<i>ntyre</i>	denotes value at nose undercarriage tyre
<i>nozzle</i>	denotes value at centre of engine nozzle exit
<i>R</i>	denotes value at rotation speed, V_R
<i>r</i>	denotes use of runway axes
<i>rev</i>	denotes use of reverse thrust
<i>screen</i>	denotes value at take-off screen
<i>START</i>	denotes value at start of ground run
<i>std</i>	denotes value in International Standard Atmosphere, ISA
<i>STOP</i>	denotes value at end of accelerate-stop
<i>W</i>	denotes wind
<i>0</i>	denotes value at sea level in International Standard Atmosphere, ISA

* The negative signs result from the sign convention used whereby the aerodynamic force components X^A and Z^A (in body axes) and X_a^A and Z_a^A (in air-path axes) usually take negative values whereas it is usual to think of aerodynamic lift and drag and normal and axial forces as positive quantities.

† Because of the axis systems used here (see Sketch 1.1) the wind component of greatest interest in this Item, *i.e.* that which acts along the runway, is **positive** for a **tailwind**. This convention differs by 180 deg. from the wind direction used in meteorology.

‡ Defined in terms of **calibrated airspeed**; for formal definition see Reference 8; for summary, see Table 13.1 of Reference 5.



Sketch 1.1 Axis systems

2. INTRODUCTION

This Data Item gives detailed examples of calculations of take-off distances and accelerate-stop distances for a large transport aeroplane. The calculation methods used are representative of one approach to the task of generating scheduled field length data for an aircraft flight manual although they are generally applicable to a wider range of calculations – such as project work. Other approaches, which make direct use of measured flight-test data to obtain civil certification, are mentioned in Section 5.

Authentic data have been used in calculating the numerical and graphical results given, but caution is required in their wider interpretation. In particular, since an example of this kind necessarily involves a particular combination of airworthiness requirements and aeroplane (with its associated design and flight-test history), some details of the calculation process are likely to be unique. It is **unlikely** that the **identical** process would be followed for another combination of aeroplane and airworthiness requirements.

Section 3 presents some implications of the airworthiness code used, *i.e.* JAR-25, Change 11, and, to a lesser extent, of operating requirements. Among these implications is the need to accept the use of precise terminology to define the various distances and so avoid confusion with more colloquial usages of the same words.

Section 4 presents aeroplane geometry, axes systems used and various sets of “given” data (in Tables 7.1 to 7.5). Particular flight cases to be considered are specified.

Section 5 discusses, in broad terms, the calculation methods used.

Section 6 introduces and discusses the numerical results given in Table 7.6 and Figures 1 to 7. The calculations required to obtain these results have been made using a computerised performance system made available by an airframe manufacturer. Details of the calculation processes required are given as Appendices A to C.

Appendix A identifies the beginning and end points, and associated “input” data, for each of the segments used in the take-off and accelerate-stop calculations.

Appendix B gives a comprehensive account of the sequencing for a single step within a step-by-step calculation of the type required here.

Appendix C gives details of how the given data are synthesised to obtain the required forces, moments, displacements, angles, speeds and atmospheric data for use in the calculations. Considerable reliance is placed on other Data Items and Appendix C contains the list of References for the whole Item. Reference 5, in particular, provides an introduction to all aspects of ground performance while Reference 6 gives the general forms of many of the equations and relationships used.

3. AIRWORTHINESS REQUIREMENTS

Airworthiness Requirements affect many aspects of take-off calculations.

The methods and calculations in this Data Item presume compliance with the Airworthiness Requirements of Reference 8 without recourse to any National Variant allowable under those Requirements. When using the Item, it is important to comply with the appropriate Airworthiness Requirements (*i.e.* depending on Certifying Authority and class of aeroplane and date) so the user must check for differences between the applicable Airworthiness Requirements and those assumed here. The major implications of these airworthiness requirements as regards the present Data Item are referred to in Sections 3.2 to 3.6. In cases of uncertainty, consult Reference 8 or its equivalent.

Operations Requirements do not directly affect the calculations presented in this Item but do influence the use made of the results of such calculations. The primary such requirement is that the **Distance(s) Available** (see Section 3.1) **be not less than the Distances Required** by the aeroplane (see Section 3.2).

3.1 Take-Off Distance Available and Accelerate-Stop Distance Available

For any runway, the airport authority must declare a value of runway length and may also declare values of clearway length and/or stopway length (see Sections 3.1.1 and 3.1.2). Then, for that runway,

Take-Off Distance Available is equal to the runway length plus the clearway length,

Accelerate-Stop Distance Available is equal to the runway length plus stopway length.

3.1.1 Clearway

Clearway is a region immediately beyond a runway which can be used for completing the airborne part of a take-off. For the purpose of establishing take-off distances and take-off runs, a clearway surface is defined which is considered to be part of the take-off surface from which the take-off screen height is measured, see Section 3.6. A clearway must be free from objects which might be a hazard to aeroplanes in transit between lift-off and the screen and must be under the control of the airport authority.

Although a clearway must be located symmetrically about the extended runway centre line, it need not have significant bearing strength, for example it may contain level or descending terrain or water. Detailed definitions vary but usually include limits on minimum width, maximum length, maximum uphill slope and maximum height of any minor surface irregularities or objects. See, for example, References 9 and 10.

3.1.2 Stopway

Stopway is an area immediately beyond the runway which can be used in the event of an accelerate-stop to bring the aeroplane to a stop without injury to the occupants or damage to the aeroplane. A stopway must not be less in width than the runway and must be located symmetrically about the extended runway centre line. Since a stopway is for occasional use only, it need not have the same bearing properties as the runway but must be capable of supporting the aeroplane loads without inducing airframe structural damage. Friction characteristics should not be inferior to those of the runway. See, for example, References 9 and 10.

3.2 Take-Off Distance Required and Accelerate-Stop Distance Required

The Airworthiness Requirements of Reference 8 define the following four* horizontal distances which are illustrated in Sketch 3.1 as distances A, B, C, D:

- A. The all-engines-operating take-off distance from rest to the take-off screen,
- B. The all-engines-operating accelerate-stop distance from rest to rest,
- C. The one-engine-failed take-off distance from rest to the take-off screen,
- D. The one-engine-failed accelerate-stop distance from rest to rest.

All four of these distances must be calculated for any take-off and, from them,

The Take-Off Distance Required is the greater of $[1.15 \times A]$ and C,

The Accelerate-Stop Distance Required is the greater* of B and D.

The further usages of these distances to determine a limiting case or “field length” for a particular take-off is considered in Section 3.4.

3.3 Take-Off Run Required

For take-offs where a “clearway” exists at the end of the runway, Reference 8 defines two further horizontal distances, shown in Sketch 3.1 as distances E and F:

- E. The distance, with all engines operating, from the start of take-off to a point equidistant between the point at which V_{LOF} is reached and the take-off screen,
- F. The distance, in a one-engine-failed take-off, from the start of take-off to a point equidistant between the point at which V_{LOF} is reached and the take-off screen.

These distances must be calculated for any take-off with a clearway and, from them,

The Take-Off Run Required is the greater of $[1.15 \times E]$ or F.

The further usages of these distances to determine a limiting case or “field length” for a particular take-off is considered in Section 3.4.

3.4 Take-Off Field Lengths

“Take-off field length” is a catch-all phrase customarily used to identify, for any particular take-off, the most limiting of the distances defined in Sections 3.2 and 3.3.

* The number of distances increases by two if wet runway conditions are to be considered for the accelerate-stop (JAR-25 National Variant).

3.4.1 General case of take-off with clearway and stopway

For take-off with both clearway and stopway at the end of the runway, the field length is determined by considering each of the following criteria:

The Take-Off Distance Required (Section 3.2) must be not greater than the Take-Off Distance Available (Section 3.1),

The Accelerate-Stop Distance Required (Section 3.2) must be not greater than the Accelerate-Stop Distance Available (Section 3.1),

The Take-Off Run Required (Section 3.3) must be not greater than the Runway Length Available (Section 3.1).

Such an evaluation can only be made for a specified combination of runway, clearway and stopway so that in practice, *i.e.* operationally, the distance calculations of Sections 3.1 and 3.2 are used to determine the maximum aeroplane weight at which all of the above criteria are satisfied. This weight is termed the “field-length limited take-off weight”.

3.4.2 Take-off field length with no clearway or stopway

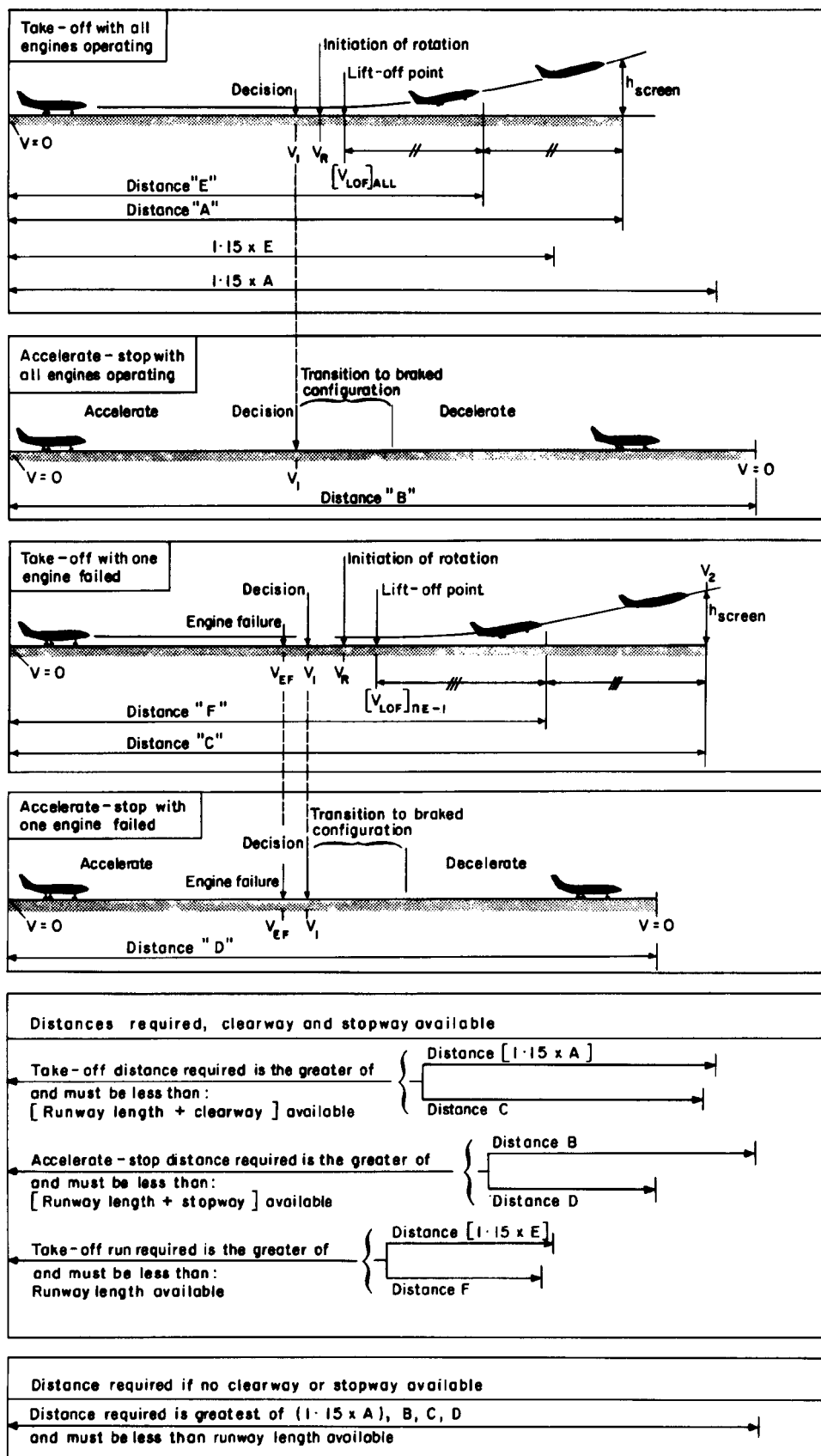
For take-off from a runway with no clearway or stopway, only the distances defined in Section 3.2 are considered and the take-off field length is always the greatest of $[1.15 \times A]$ or B or C or D. Consequently, for a given runway length, the field-length-limited take-off weight is the aeroplane maximum weight at which all four of these distances are less than, or equal to, the runway length.

In the absence of clearway or stopway it becomes possible to define a take-off field length (as the greatest of $[1.15 \times A]$, B, C, D) for any aeroplane weight. This is of particular value when take-off performance is to be investigated in a general sense and is the way in which the numerical results in this Item are presented in Section 6 and Figures 1 to 7.

3.5 Take-Off Techniques and Associated Conditions

Distances A to F in Sections 3.2 and 3.3 are calculated on a common basis of aeroplane weight and configuration, ambient and runway conditions. In each case the techniques assumed must not require exceptional piloting skill or alertness and the speeds attained must be consistent with the defined speeds in Reference 8 – see Table 7.1 of Reference 5 for summary. For take-offs a 35 ft (10.67 m) screen is used which must be cleared by the lowest point on the aeroplane. For accelerate-stops the runway is assumed dry in this Data Item. However for some Airworthiness Codes wet-runway conditions must be considered in the accelerate-stop although a reduction is then usually allowed in the screen height in the corresponding engine-failed take-off. No consideration is given here to take-off from “contaminated runways” (see Reference 8 and Section 3.8.3 of Reference 5).

In this Data Item all horizontal and vertical distances travelled by the aeroplane are those defined by the motion of the centre of gravity (so no account is taken of the length of the aeroplane). This may not be an acceptable practice in some cases, for example, if the lowest point on the aeroplane follows an airborne path which is of significantly different shape from that followed by the centre of gravity.

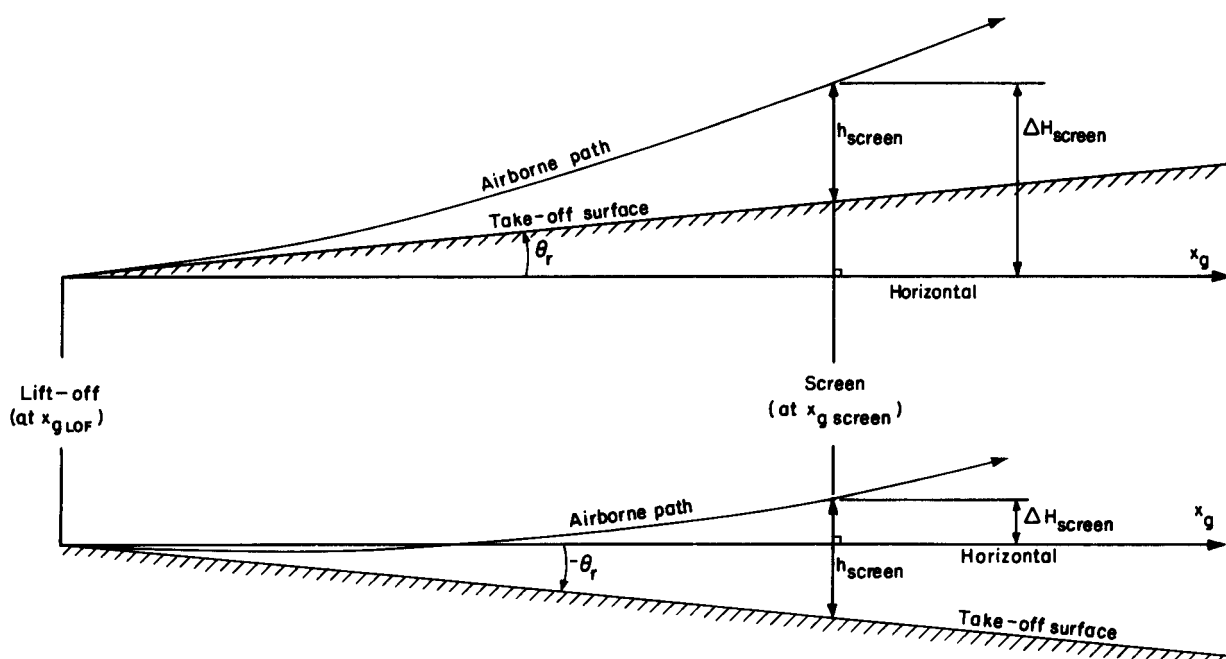


Sketch 3.1 Distances required

3.6 Location of Take-Off Screen*

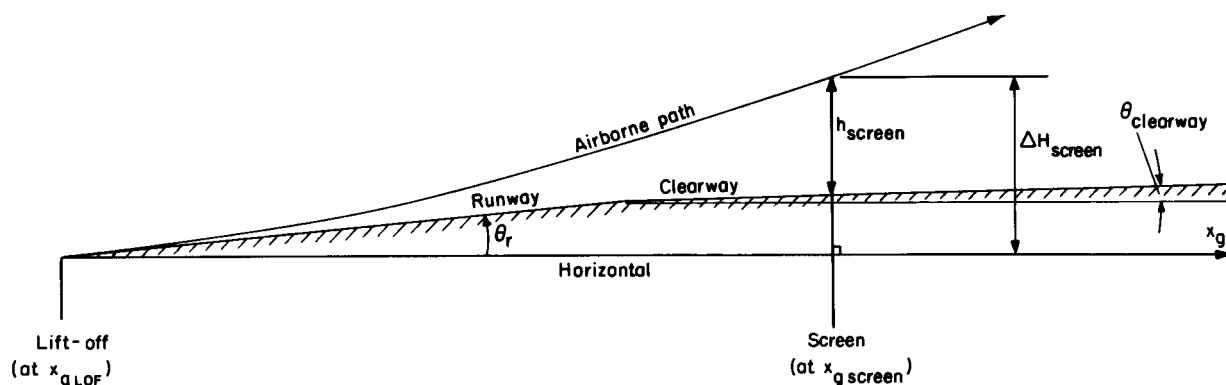
Reference 8 requires consideration of, “the horizontal distance along the take-off path from the start of the take-off to the point at which the aeroplane is 35 ft above the take-off surface”. Sketch 3.2 illustrates the take-off surface, the screen height, h_{screen} , and the vertical distance, ΔH_{screen} , between lift-off and the screen, for the most usual case where the take-off surface comprises a runway (or runway plus clearway) having a single mean slope Θ_r . For this case the value of ΔH_{screen} for any point on the aeroplane is related to the local Earth-axis coordinates defined in Sketch 1.1 by,

$$\Delta H_{\text{screen}} = -[z_{g_{\text{screen}}} - z_{g_{\text{LOF}}}] = h_{\text{screen}} + [x_{g_{\text{screen}}} - x_{g_{\text{LOF}}}] \tan \Theta_r. \quad (3.1)$$



Sketch 3.2 Take-off screen for take-off surface with single mean slope

Sketch 3.3 illustrates the application of this concept to a hypothetical case where the take-off surface includes different runway and clearway slopes. For such cases the right hand side of Equation (3.1) must be modified to account for the particular surface geometry when evaluating ΔH_{screen} .



Sketch 3.3 Take-off screen for take-off surface with multiple slopes

* The definitions given in Section 3.6 and Sketches 3.2 and 3.3 are those used for civil transport operations. For some military applications the take-off screen may be defined as a fixed distance above the lift-off point (in which case $\Delta H_{\text{screen}} = h_{\text{screen}}$).

4. AEROPLANE GIVEN DATA AND CASES USED IN CALCULATIONS

4.1 Aeroplane Given Data

The aeroplane is a twin turbo-fan engined transport having the general arrangement shown in Sketch 4.1. Dimensions are defined in terms of the 0xyz body-axis system with particular distances denoted by a subscript to the relevant co-ordinate, for example (x_{intake} , y_{intake} , z_{intake}) define the location of the centre of an engine intake.

Axis systems* are presented in Sketch 1.1 together with the angles which relate pairs of systems. Sketch 1.1 also indicates lines of action of the various force components and associated geometric information.

Numerical data for use in the field-length calculations for this aeroplane are given as follows: Table 7.1, Geometric data; Table 7.2, Aerodynamic data; Table 7.3, Powerplant data; Table 7.4, Ground forces data; Table 7.5, Manoeuvres, operational data and constants.

4.2 Cases for Calculation of Take-Off and Accelerate-Stop Distances

For the aerodynamic configuration specified in Table 7.2, calculations are performed for each of the distances A to F defined in Sections 3.2 and 3.3 for the following cases.

Aeroplane weight: $W = 270\,000$ lbf; *c.g.* at 18 per cent aerodynamic mean chord

Engine settings: maximum take-off power; no offtakes

Ambient conditions: $H_p = 2000$ ft, $\Delta T_{H_p} = 20$ K, $T = 304.2$ K, $V_W = 0$

Runway: $\Theta_r = 0$, dry

Initial condition: zero speed, maximum take-off power, brakes released instantaneously

Screen: $h_{screen} = 35$ ft, see Sketch 3.2

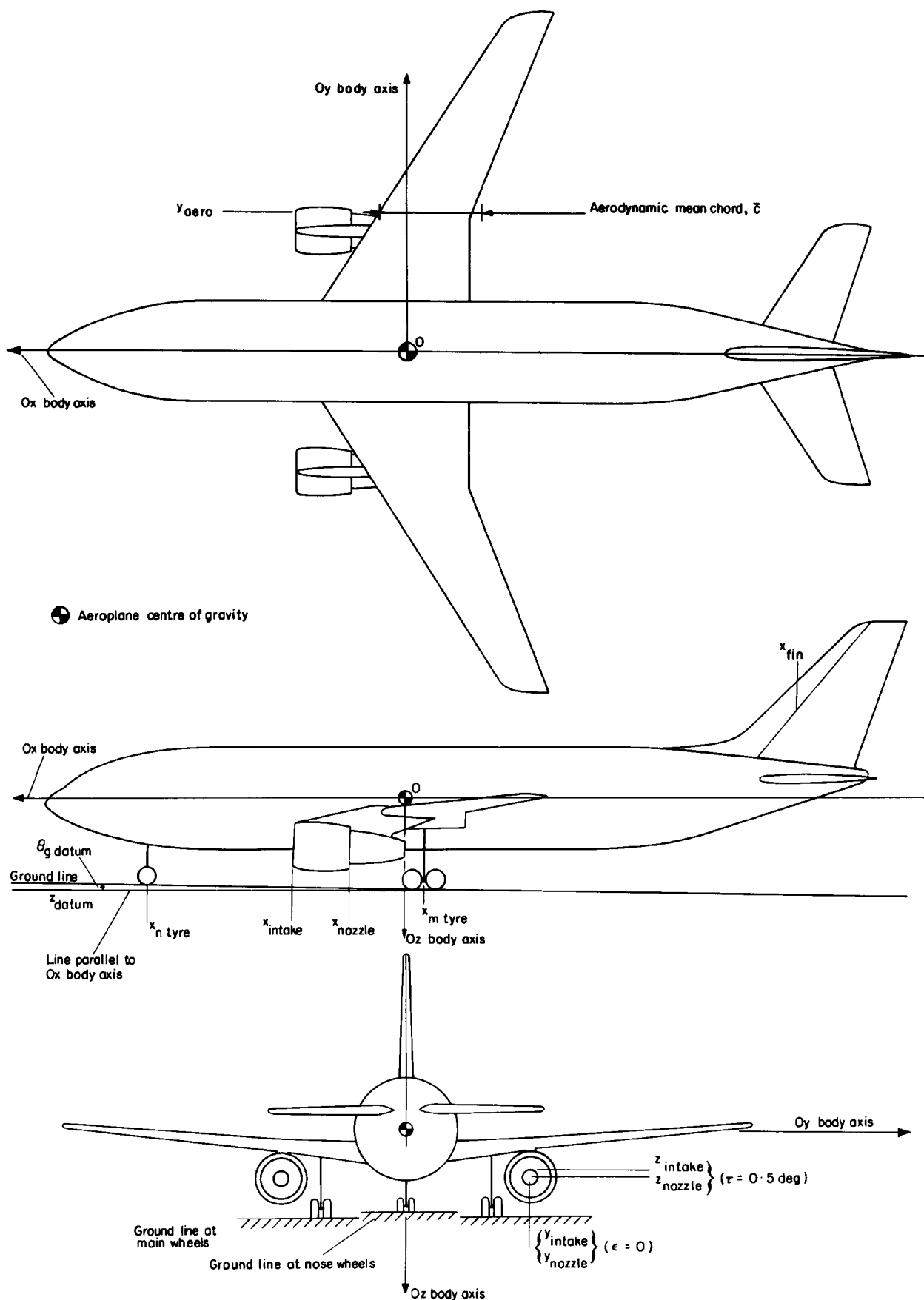
Number of engines failing: $n_{EF} = 1$

Values of decision speed ratio, $[V_1/V_R]$, to be used: 0.85, 0.9, 0.95, 1.0

Initial approximation to $V_R = 1.1 \times V_S$, see Appendix A, Section A4.1

Detailed geometric data are given in Table 7.1.

* The axis systems used here are relatively simple. This results from the assumption, made in Section 5, that all motion takes place in a plane containing the runway centre line and the aeroplane plane of symmetry. Full definitions of the various systems are given in Table 7.1 of Item No. 85030 (Reference 6); axis transformation formulae are given in Table 7.2 of that Item.



Sketch 4.1 Aeroplane geometry in terms of body-axis co-ordinates

5. CALCULATION METHODS

The broad basis of scheduled take-off performance is a combination of measurement and calculation, with the former used to validate the latter from which a wide range of operational data may be generated. The method of calculation is to solve the aircraft equation(s) of motion to give distance travelled and time taken in each of several segments which, taken together, comprise the take-off or accelerate-stop. To ensure that the calculated results comply in every respect with the airworthiness requirements, particular features such as minimum speeds, and acceptable handling techniques during rotation and initial climb are demonstrated by designated flight tests. The overall calculation method must also be validated – usually by the satisfactory correlation of measured and calculated distances for individual segments and for complete take-offs and accelerate-stops.

In this Data Item the method used to calculate distances is the step-by-step solution of the equations of motion for the aeroplane centre of gravity – with the assumption that all significant forces act in, and motion takes place in, a single plane which contains the runway centre line and the aeroplane plane of symmetry. Appendices A, B, C give detailed accounts of the calculation processes (and include some options which were not used in the calculations for the example given here). The precise form of calculations for the aeroplane considered here are most easily defined by reference to the various packages of given data, *i.e.* geometry in Sketch 4.1 and Table 7.1, aerodynamic, propulsive and ground forces in Tables 7.2, 7.3, and 7.4 and speeds and manoeuvre definitions in Table 7.5.

The numerical example given here is just one application of a particular (step-by-step) calculation procedure and any number of variants of this procedure are possible. For example, in Table 7.2, aerodynamic forces are defined by in-trim relationships between C_L , α , C_D while in Table 7.5, aircraft attitude during the ground and rotation segments is defined by pre-determined schedules and the airborne segment by limits on normal load factor and speed. A more complex aerodynamic model would require separate treatments of tail, wing-body and downwash with an associated need to evaluate the full rotational equation of motion throughout the ground, rotation and airborne segments.

Other approaches exist to the measurement, validation and prediction of take-off performance besides that which is implicit here. For example, a simpler, more direct approach utilises measured values of the mean acceleration achieved in each segment. The mean accelerations are correlated against aeroplane thrust to weight ratio and these correlations provide the basis for the subsequent synthesis of distances and times, without examination of the individual forces acting on the aeroplane. Since this approach is based on measured flight-test data only, it is particularly relevant to certification work.

5.1 Segments Used in Calculations of Take-Offs and Accelerate-Stops

For calculation purposes the take-off (or accelerate-stop) is divided into segments, the beginnings and ends of which correspond to the major control inputs and associated speeds and/or the manual or automatic activation of retardation devices. The following table lists the segments considered and identifies those which comprise each of the four distances “A”, “B”, “C”, “D” defined in Section 3.2 and the two distances “E” and “F” defined in Section 3.3. Appendix A contains more detailed descriptions of the individual segments, identifies the quantities needed to define their beginning and end points and lists the “input” data requirements which are unique to particular segments.

SEGMENTS USED TO DEFINE TAKE-OFF AND ACCELERATE-STOP

Description of Segment (see Appendix A for greater detail)	All engines operating		One engine failed	
	take-off	accel. stop	take-off	accel. stop
	A, E	B	C, F	D
	See Sections 3.2 and 3.3.			
Ground run with all engines operating , <i>i.e.</i> from start of take-off to first event (at V_{EF} or V_1 or V_R) in otherwise steady accelerating motion along ground.	✓	✓	✓	✓
Engine failure to decision point , <i>i.e.</i> short segment between V_{EF} and V_1 , usually defined by demonstrated pilot reaction time, Δt_{rec} (see Sketch 3.1 in Ref. 5).			✓	✓
Decision point , <i>i.e.</i> this is not a segment but a reminder that the distances and times must be identified at which $V_c = V_1$ in engine-failed and all-engines-operating cases.		✓	✓	✓
Continued take-off ground run with one engine failed , <i>i.e.</i> from decision point (at V_1), following an engine failure (at V_{EF}), to the point at which rotation is initiated (at V_R).			✓	
V_R to lift-off* , <i>i.e.</i> from speed at which rotation is initiated to point at which lift-off occurs (at V_{LOF}); all-engines and one-engine-failed cases must be calculated; lift-off usually defined by zero load on main wheels.	✓		✓	
Airborne segment to screen* , <i>i.e.</i> from lift-off (at V_{LOF}) until screen height is attained (at V_2 , with one engine failed; at higher speed with all engines operating).	✓		✓	
Braked ground run , <i>i.e.</i> from decision point (at V_1), through sequence of pilot actions and powerplant, brakes, spoilers, <i>etc.</i> responses (including any arbitrary time delays imposed by airworthiness requirements) to steady braked conditions and ending when the aeroplane stops.		✓		✓

* In calculations for both all-engines and one-engine-failed take-offs, V_R is increased progressively from an approximate initial value until all the requirements of Reference 8 are met. In practice this means that it is often more economic to complete the calculations of the " V_R to lift-off" and "Airborne segment to screen" segments – so as to define V_R – before proceeding to the ground-run calculations.

5.2 Equations of Motion

For the example in this Data Item only the motion of the aeroplane centre of gravity is considered when calculating distances and all motion is assumed to take place in the plane containing both the runway centre line and the aeroplane plane of symmetry. This is an adequate representation of motion for most performance calculations and permits many simplifications in both the equations of motion and the forces and moments acting. The equations in Sections 5.1.1 to 5.1.3 have been derived (using the stated assumptions) from general expressions given in Reference 6.

5.2.1 Translational motion, body axes

For a rigid aircraft the equation of translational motion is

$$R^{\Sigma} = m \frac{d^2 s}{dt^2} = m V_{in} \frac{dV_{in}}{ds} = m \frac{dV_{in}}{dt}, \quad (5.1)$$

where R^{Σ} is the resultant external force acting on the aircraft, including any reactive forces, V_{in} is the velocity relative to an inertial frame of reference, s is the distance travelled and m is the instantaneous mass. For the purpose of this Item the flight path velocity, V_K , is taken as inertial and the only significant translational motions are assumed to take place along the 0x and 0z axes. Consequently, Equations (5.1) to (5.3) of Reference 5 can be simplified and re-arranged to give the following expressions for accelerations along the body axes.

$$\text{0x axis,} \quad \dot{u}_K = \frac{X^{\Sigma}}{m} - q w_K \quad (5.2)$$

$$\text{0z axis,} \quad \dot{w}_K = \frac{Z^{\Sigma}}{m} + q u_K. \quad (5.3)$$

5.2.2 Translational motion, local Earth axes

Body axis components of acceleration from Equations (5.2) and (5.3) are converted to Local Earth Axis components using conversion formulae from Table 6.2 of Reference 6,

$$\dot{u}_{K_g} = \dot{u}_K \cos \Theta + \dot{w}_K \sin \Theta \quad (5.4)$$

$$\dot{w}_{K_g} = -\dot{u}_K \sin \Theta + \dot{w}_K \cos \Theta. \quad (5.5)$$

5.2.3 Rotational motion, body axes

For a rigid aircraft the equation of rotational motion is

$$M^{\Sigma} = \frac{dH^{\Sigma}}{dt}, \quad (5.6)$$

where M^{Σ} is the resultant moment, including components arising from any reactive forces, and H^{Σ} is the resultant angular momentum. For this Data Item the only rotational motion of significance is that about the 0y body axis, *i.e.* Equation (5.5) in Reference 5. Since no rotational motions about the 0x and 0z axes are considered, and provided that the body axes can be assumed to be principal body axes, it follows that

$$M^{\Sigma} = I_y \dot{q}. \quad (5.7)$$

5.3 Numerical Solutions of Equations of Motion

Equations (5.2) to (5.7) are all second order linear differential equations which may be solved by numerical integration in several ways. The simplest method (Euler solution) assumes a linear relationship between the instantaneous value of, say, velocity, and the value after a short time interval. A more usual approach for use in numerical integration is to use a fourth order (Runge-Kutta) method, see Reference 7, of which a number of alternatives exists. It is suggested that within the various such methods given in Reference 7 the most efficient for the present class of problem are the Runge-Kutta methods due to Hull, Scraton and England.

In Appendix B it is suggested that all distances are calculated, initially, in the local Earth axis system. This ensures that an inertial system is used for all calculations involving the equations of motion and, in any case, provides distances in the horizontal plane – as required by Reference 8. Distances are also calculated along and normal to the runway. The former of these is required in calculations of brake energy while the latter provides one of the ways of determining if the screen height has been reached.

6. RESULTS OF CALCULATIONS

Table 7.6 and Figures 1 to 7 give results for take-off and accelerate-stop calculations in compliance with the requirements of Reference 8 for the aeroplane described in Section 4.

6.1 Take-Off Speeds

From Table 7.6 the (calibrated) airspeeds which define the take-offs are as follows.

$V_R = 119.63 \text{ kn}$ (both take-offs)					
V_2	$= 124.97 \text{ kn}$	} one engine failed take-off	V_{screen}	$= 138.7 \text{ kn}$	} all-engines- operating take-off
$[V_{LOF}]_{n_E-1}$	$= 124.37 \text{ kn}$		$[V_{LOF}]_{ALL}$	$= 131.9 \text{ kn}$	

In principle, the single value of rotation speed for both all-engines-operating and one-engine-failed take-offs is the result of the whole calculation process. An initial value, $V_R = 1.1V_S$, was given as input data while*, in successive iterations in the take-off calculations, values of V_R were increased until all the requirements of Reference 8 were met. In this particular example, V_R is determined by the need to achieve $V_{2_{min}}$ at the screen in the one-engine-failed take-off. At other weights the value of V_R might be determined by other considerations – see Reference 8 or Table 13.1 in Reference 5.

6.2 Take-Off Distance Required, Accelerate-Stop Distance Required and Take-Off Run Required

Table 7.6 gives summaries, both for individual segments and overall, for each of the distances A, B, C, D, E, F defined in Sections 3.2 and 3.3. With the exception of the all-engines-operating take-off, the distances and times are quoted for $[V_1/V_R] = 0.85, 0.90, 0.95$ and 1.0 . These results have been used to generate Figures 1 to 3 which show the effect of different values of V_1/V_R .

* See Appendix A, Section A4.1.