



Flow induced vibration in tube bundles with particular reference to shell and tube heat exchangers

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FLOW INDUCED VIBRATION IN TUBE BUNDLES WITH PARTICULAR REFERENCE TO SHELL AND TUBE HEAT EXCHANGERS

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FLOW INDUCED VIBRATION IN TUBE BUNDLES WITH PARTICULAR REFERENCE TO SHELL AND TUBE HEAT EXCHANGERS

1. NOTATION AND UNITS

		Units	
		SI	British
C_u	U-bend mode constant (Equation (3.5))	—	—
D_s	heat exchanger shell internal diameter	m	ft
d_{eq}	equivalent projected diameter of low-fin tubes (Equation (3.6))	m	ft
d_f	fin tip diameter	m	ft
d_i	tube internal diameter	m	ft
d_o	tube outside diameter	m	ft
d_r	fin root diameter	m	ft
E	modulus of elasticity of tube material	N/m ² (Pa)	lbf/ft ²
F	factor to account for effect of longitudinal tube stress on tube natural frequency (Equation (3.4))	—	—
f_{ic}	isolated cylinder wake shedding frequency (Equation (4.5))	Hz	Hz
f_t	tube natural frequency (Equation (3.1))	Hz	Hz
f_{t-u}	U-bend fundamental frequency (Equation (3.5))	Hz	Hz
f_w	wake shedding frequency (Equation (4.4))	Hz	Hz
g_c	constant of proportionality in Newton's 2nd law of motion	— (1.0)	lb ft/s ² lbf (32.18)
h	fin height = $\frac{1}{2}(d_f - d_r)$	m	ft
I	second moment of area of tube = $\frac{\pi}{64}(d_o^4 - d_i^4)$	m ⁴	ft ⁴
k	added mass factor (see Section 3.1)	—	—
L	unsupported tube span (see Section 3.1)	m	ft
L_{ip}	length of rectangular impingement plate	m	ft

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L_m	characteristic span length to be used for calculating δ (see Section 4.3.3)	m	ft
m_e	effective mass per unit length of tube (Equation (3.2))	kg/m	lb/ft
N	number of spans	—	—
N_f	fin density (number of fins in unit length)	m^{-1}	ft^{-1}
P	axial force on tube	N	lbf
p	tube pitch (Sketch 4.4)	m	ft
p_l	longitudinal tube pitch (Sketch 4.4)	m	ft
p_t	transverse tube pitch (Sketch 4.4)	m	ft
R	centreline radius of U-bend	m	ft
t	time	s	s
t_b	baffle or tube support thickness	m	ft
t_f	mean fin thickness (Sketch 3.4)	m	ft
S_{ic}	isolated cylinder Strouhal number (Equation (4.5))	—	—
S_w	wake shedding Strouhal number (Equation (4.4))	—	—
V_c	critical velocity for onset of fluidelastic instability based on transverse flow area	m/s	ft/s
V_{ic}	upstream flow velocity for isolated cylinder vortex shedding	m/s	ft/s
V_m	minimum section velocity (see Section 4.3.1)	m/s	ft/s
V_{mc}	critical velocity for onset of fluidelastic instability based on minimum flow area	m/s	ft/s
V_t	transverse velocity (see Section 4.3.1)	m/s	ft/s
V_w	critical velocity for wake shedding	m/s	ft/s
W_{ip}	width of rectangular impingement plate	m	ft
X	amplitude of vibration	m	ft
x	vapour weight fraction	—	—
β	stability constant (Equation (4.1))	—	—

δ	damping logarithmic decrement (see Sections 4.3.3, 4.3.4 and 4.3.5)	—	—
λ^2	dimensionless parameter (Equation (3.1))	—	—
ρ	density	kg/m ³	lb/ft ³
Extra subscripts			
g	gas phase		
ic	isolated cylinder or tube		
l	liquid phase		
m	tube material		
mid	midspan		
s	shellside fluid		
t	tubeside fluid		
tp	two-phase		
v	vapour phase		
1, 2	end spans		

2. PURPOSE AND SCOPE

In the design of heat exchange equipment high flow velocities with associated high heat transfer coefficients are necessary in the interests of compactness, weight limitation and minimisation of possible fouling tendencies. This is particularly the case with shell and tube exchangers. The combination of high crossflow (shell side) velocities with large, unsupported, tube spans may initiate damaging tube vibrations. The likelihood of damaging tube vibrations occurring must be considered at the design stage, together with the constraints of heat transfer and allowable pressure drop, and also when process conditions are changed.

Mechanical failure due to vibration may occur by one of several mechanisms:

- (a) baffle damage where the walls of the vibrating tubes are worn by the baffle or tube support,
- (b) tube collision damage where the amplitude of vibration is such that adjacent tubes impact with one another,
- (c) tube joint failure, at the tubesheet or header, on account of either vibration opening up expanded joints or the edge of the tube holes wearing the tubes,
- (d) fatigue, possibly enhanced by corrosion and/or erosion.

The purpose of this Data Item is threefold:

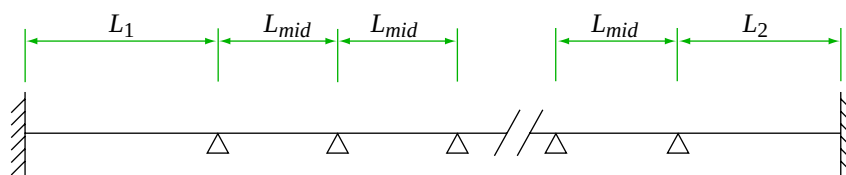
- (i) to outline the possible mechanisms which initiate flow induced vibration in tube bundles,
- (ii) to present the available data, formulate procedures for predicting the likelihood or onset of tube vibrations and to decide whether these are liable to cause damage,
- (iii) to indicate steps that may be taken at the design stage to reduce the likelihood of vibration problems.

3. PREDICTION OF TUBE FUNDAMENTAL NATURAL FREQUENCY

3.1 Calculation of the Natural Frequency of Straight, Plain Tubes

A tube has many natural frequencies. The lowest in bending is usually the most important and is called the fundamental frequency. Prediction of that frequency is essential to any analysis of tube vibration since the maximum response of the tube is generally at that frequency. Higher mode frequencies do occur but damaging tube vibration is almost invariably associated with the fundamental frequency.

A tube supported at baffles between tubesheets may be regarded as a beam clamped at its ends with intermediate simple supports, as shown in Sketch 3.1.



Sketch 3.1 Clamped-clamped multispan beam with pinned intermediate supports

In general the end spans, L_1 and L_2 , will be different from the midspans, L_{mid} , and from one another.

It should be noted that span, the distance between adjacent supports, is not necessarily the same as baffle pitch. In many shell and tube heat exchanger designs certain groups of tubes are supported at every other baffle. In natural frequency calculations, span is the correct length to use, whether or not it is coincident with baffle pitch.

If a rigorous structural vibration computer program is available for the calculation it should be used. In the absence of such a program the following procedure may be used.

The natural frequency of each span is calculated as if the spans were independent, according to the single span beam formula⁶

$$f_t = \frac{\lambda^2}{2\pi L^2} \sqrt{\frac{EI g_c}{m_e}} \cdot F \quad (3.1)$$

where

L = unsupported tube span = L_1 , L_{mid} or L_2 ,
 λ^2 = dimensionless parameter, a function of end fixing
 = 9.87 when both ends are supported by baffles
 = 15.4 when one end is in the tubesheet and the other supported by a baffle
 = 22.4 when both ends are in tubesheets and there are no intermediate supports,

m_e = effective mass per unit length of tube
 $= \frac{\pi}{4} [\rho_m (d_o^2 - d_i^2) + \rho_t d_i^2 + k \rho_s d_o^2]$, (3.2)

k = added mass factor, see Figure 1 ,

I = second moment of area of tube

$= \frac{\pi}{64} (d_o^4 - d_i^4)$, (3.3)

F = factor to accommodate the effect of any axial load on tube natural frequency

$= \sqrt{1 + \frac{PL^2}{\pi^2 EI}}$, (3.4)

P = axial force on tube; positive if load is tensile, negative if compressive.

The overall tube natural frequency is taken to be the lowest of the frequencies calculated for the individual spans. The span with the lowest individual frequency controls the overall motion of the tubes and is referred to as the dominant span. Sometimes several spans will have the same frequency. Under such circumstances all these spans will exhibit a substantial response.

For a particular heat exchanger tube one of several possibilities can arise, the four most common being:

- (i) one dominant span,
- (ii) all centre spans but not the end spans are dominant,

- (iii) either or both end spans but not the centre spans are dominant,
- (iv) all spans dominant.

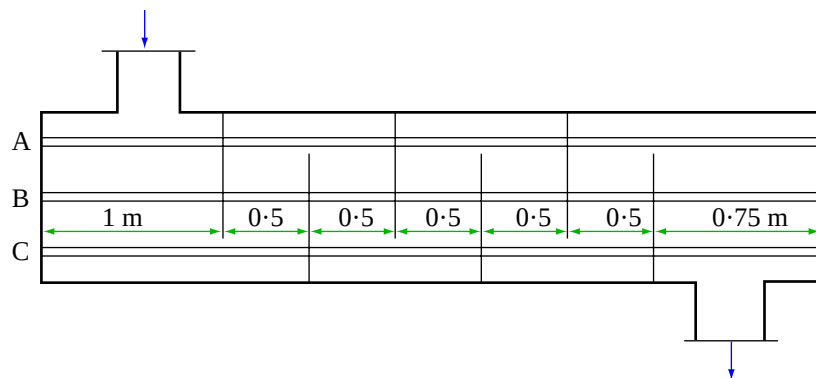
At the fundamental frequency the dominant span or spans exhibit the greatest amplitude of vibration. However, the ability to excite tubes depends on the natural distribution of amplitudes along them. For example, where the amplitude is large the flow may impart substantial energy to the tubes. In contrast, where the amplitude is small, little energy will be imparted. The four cases enumerated above exhibit widely differing amplitude distributions.

The added mass factor, k , accommodates the fact that the fluid surrounding a tube behaves as an additional mass attached to the tube, thereby lowering the natural frequency. Values of k are given in Figure 1, extracted from Reference 5. Values for the two most common tube pitch to diameter ratios are:

p/d_o	1.25	1.33
k	2.96	2.5

The mass of the shellside fluid will only contribute significantly to the magnitude of the natural frequency when that fluid is a liquid.

Different tubes, or groups of tubes, in a bundle may have different natural frequencies. This is illustrated in Sketch 3.2 which shows a typical shell and tube heat exchanger baffle arrangement.



Sketch 3.2 Unsupported tube length

The unsupported lengths are

Span	L_1	L_{mid}	L_2
Group A tubes	1 m	1 m	1.25 m
Group B tubes	1 m	0.5 m	0.75 m
Group C tubes	1.5 m	1 m	0.75 m

Group C tubes will have the lowest natural frequencies and will probably be the most prone to suffer damage. However, there are situations where group A tubes with their somewhat higher natural frequency may be most liable to damage. Both possibilities should normally be investigated.

Group B tubes are those supported at every baffle. Natural frequencies are much higher than for other cases and vibration damage is highly unlikely.

3.2 Axial Loads

Axial loads on tubes, either compressive or tensile, can arise on account of:

- (a) pressure differential between shellside and tubeside fluids,
- (b) differential thermal expansion between tubes and shell in the case of fixed tubesheet designs,
- (c) differential thermal expansion between tubes in different passes of a multi-tubepass exchanger,
- (d) residual stresses induced during manufacture.

Compressive loads will decrease the natural frequency, tensile loads will increase the frequency. These effects are usually small for heat exchanger tubes and it is usually safe to neglect them.

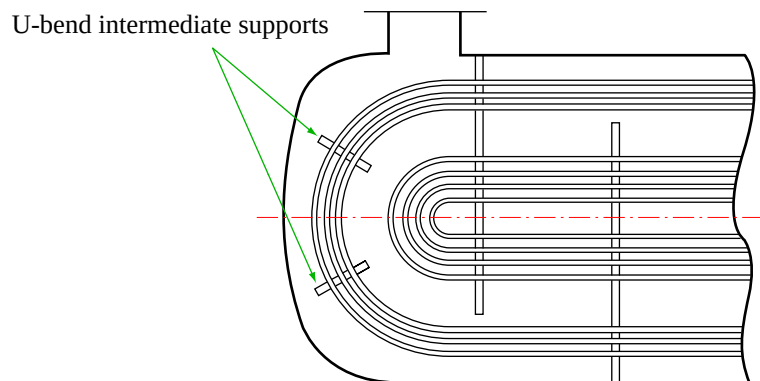
3.3 Calculation of the Natural Frequency of U-tubes

The lowest frequency of a U-bend will generally be in the out-of-plane direction and may be calculated by the method laid down in TEMA⁵ as follows:

$$f_{t-u} = \frac{0.0403 C_u}{R^2} \sqrt{\frac{EI g_c}{m_e}}, \quad (3.5)$$

the constant, C_u , being obtained from Figure 2. The centreline radius, R , should be the largest in the bundle, *i.e.* that of the outer tubes.

With large diameter bundles the tube bends may be additionally supported with intermediate supports as shown in Sketch 3.3. This requirement may control bundle geometry to the extent that square or rotated square tube layout is specified in preference to a triangular layout and is dictated by ease of manufacturing considerations. Some information on the effectiveness of U-bend supports is given in Reference 14.



Sketch 3.3 U-bends with intermediate supports

One, two, three or more intermediate supports may be introduced as appropriate. They need extend only

over those tubes most at risk, *i.e.* the outer ones.

Provided there are three or more intermediate supports each length of tube between successive supports may be treated as pseudo-straight and the natural frequency calculated from Equation (3.1).

The U-bend natural frequency should be compared with the straight length natural frequencies calculated by the method of Section 3.1 in order to ascertain the lowest value or dominant span.

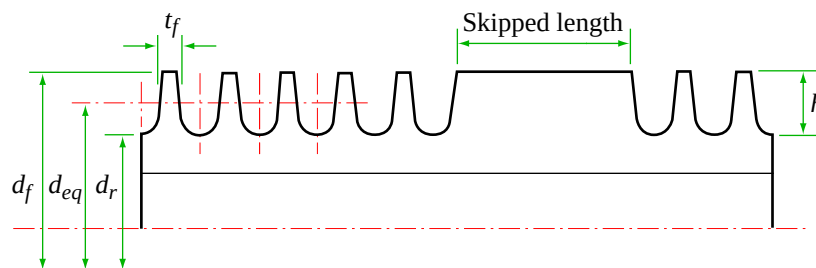
3.4 Natural Frequency of Low-fin Tubes

Finning affects the tube mass per unit length but has little effect on the tube stiffness. Manufacturers' data for mass per unit length of tube should be used if available. In the absence of such data, m_e may be estimated by substituting an equivalent projected diameter, d_{eq} , for the tube outside diameter, d_o , in Equation (3.2). The equivalent projected diameter is defined by Taborek¹² as

$$d_{eq} = d_r + 2N_f h t_f \quad (3.6)$$

and should also be used for all hydrodynamic calculations.

Fin tube notation is illustrated in Sketch 3.4.



Sketch 3.4 Fin tube notation

The relevant tube outside diameter to use with finned tubes in the expression for the second moment of area, I , is the root diameter, d_r .

4. TUBE VIBRATION PHENOMENA

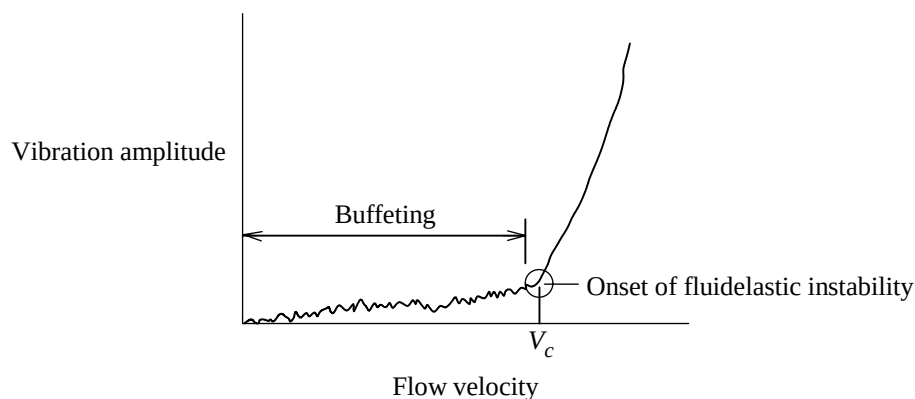
4.1 Mechanisms

The following mechanisms causing tube vibration have been identified.

- (i) Fluidelastic or dynamic instability.
- (ii) Periodic wake shedding.
- (iii) Buffeting.
- (iv) Acoustic resonance.
- (v) Parallel flow instability.
- (vi) Vibration from external sources.

4.2 Bundle Response to Increasing Fluid Flow Rates

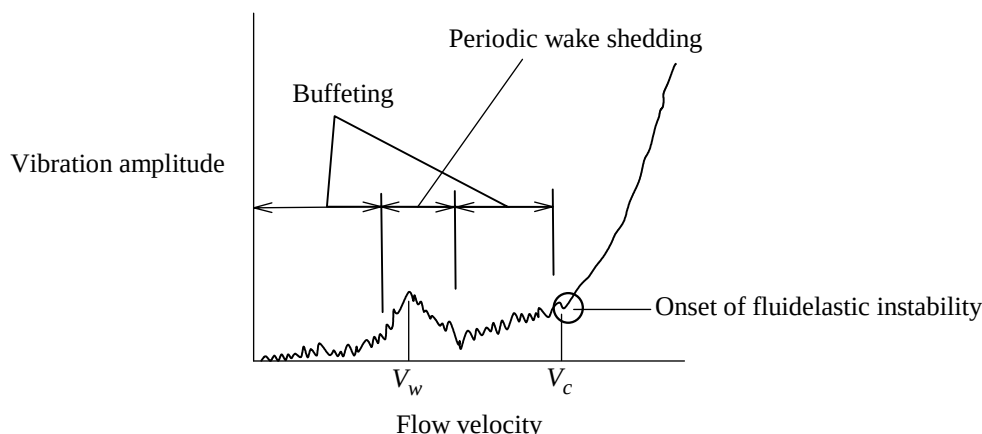
The response of a typical tube bundle to increasing liquid flow rates is shown in Sketch 4.1.



Sketch 4.1 Idealised tube amplitude response. Periodic wake shedding absent

As the flow velocity is increased from zero the r.m.s. amplitude of vibration steadily increases from zero but amplitudes are low overall. This is the zone of buffeting. Above a certain velocity amplitudes increase rapidly and damage occurs. This is the domain of fluidelastic instability. The amplitude may be sufficient for tube to tube contact to occur or non-linear effects due to the tube supports may limit the final amplitude. Operation in this zone, even for a brief duration, is totally unacceptable.

Sometimes bundle response more closely resembles that of Sketch 4.2. The resonance peak standing out of the buffeting background is the region of periodic wake shedding. Separation between the wake shedding peak and the onset of fluidelastic instability can vary considerably, depending on fluid characteristics and construction details. Frequently the resonance peak is not apparent because it lies within, or immediately adjacent to, the domain of fluidelastic instability.



Sketch 4.2 Idealised tube amplitude response with periodic wake shedding

With gas flows the response is of similar form but the characteristics are more clearly defined. In particular, the onset of fluidelastic instability may be abrupt. Wake shedding response may be imperceptible.

There is widespread agreement that fluidelastic instability is both the most usual and most dangerous mechanism. Serious tube damage can occur after only a few hours operation. Periodic wake shedding is a less vigorous mechanism than fluidelastic instability and consequently causes damage at a slower rate. Buffeting induces very small amplitudes of vibration and consequently the damage rate is the smallest of the three mechanisms.

4.3 Fluidelastic Instability

4.3.1 Prediction of the onset of fluidelastic instability

Early work by Connors³ established an empirical relationship between the mass damping parameter, $m_e \delta / (\rho_s d_o^2)$ and reduced critical velocity, $V_c / (f_t d_o)$ of the form

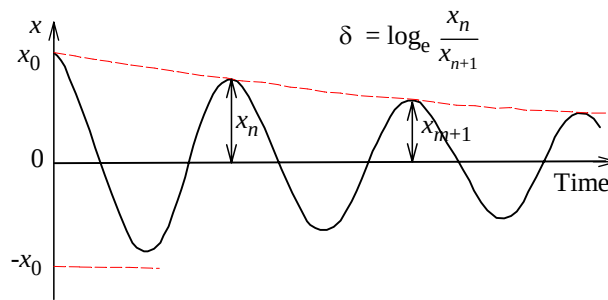
$$\frac{V_c}{f_t d_p} = \beta \sqrt{\frac{m_e \delta}{\rho_s d_o^2}} \quad (4.1)$$

based on gas flow over a single row of tubes and V_c , the critical velocity for the onset of fluidelastic instability, assumed uniform over the face of the bundle.

The logarithmic decrement, δ , is a measure of the system damping^{*} and is the most uncertain parameter in Equation (4.1). It is defined as the natural logarithm of the ratio of successive amplitude peaks as a vibration decays and is illustrated in Sketch 4.3. High values of δ imply rapid decay and the system is said to be heavily damped. Heat exchanger tube bundles are usually lightly damped with δ values typically in the range 0.02 to 0.2.

* Other definitions of damping sometimes used are:

- (i) damping ratio, $\zeta = \delta / 2\pi$,
- (ii) dynamic magnification factor, $Q = \pi / \delta$.



Sketch 4.3 Definition of logarithmic decrement

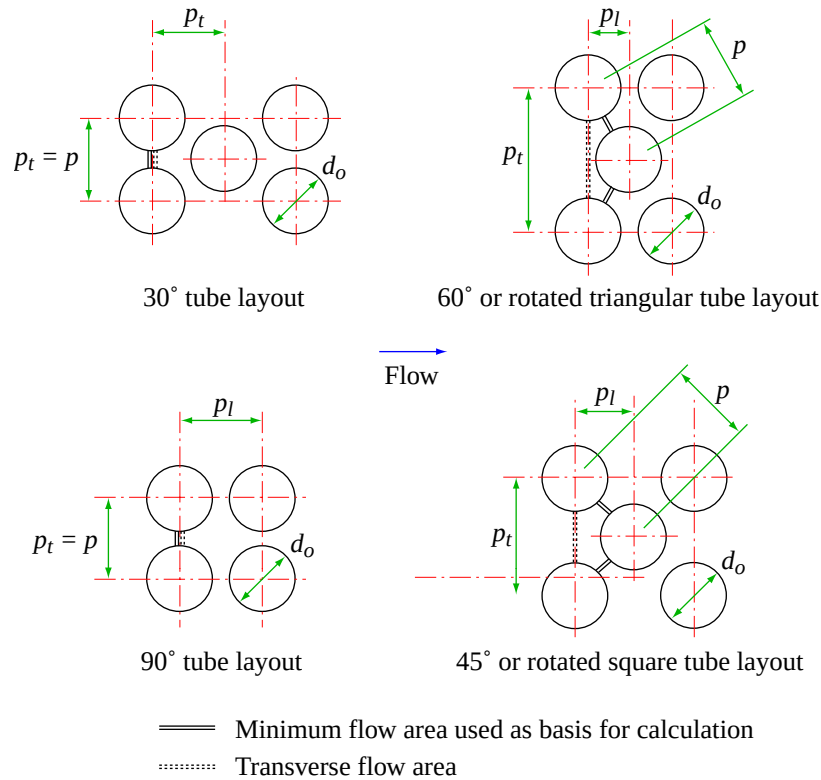
In practice, reliable δ values cannot be obtained by vibrating tubes in a bundle and measuring successive amplitudes because, as the motion decays, tube support conditions vary and response curves are difficult to interpret. Experimental methods that do not rely upon observing the decaying vibration of tube response must be used.

Chen⁹, starting from the equations of motion of a group of cylinders in a fluid stream, generalised the Connors relationship, and, using force coefficients measured by Tanaka and Takahara⁷, has suggested that there are two types of fluidelastic instability. At high values of the mass damping parameter, $m_e \delta / (\rho_s d_o^2)$, an interaction between neighbouring vibrating tubes occurs with fluid forces proportional to the displacement of the tubes. At low values of the parameter, vibration is independent of the motion of neighbouring tubes and the dominant fluid force is proportional to the velocity of the vibrating tubes.

Analytical methods, based on the work of Chen⁹, are not sufficiently developed to be able to predict critical flow velocities to an acceptable degree of accuracy. Measured force coefficients for particular tube layout geometries and pitch to diameter ratios are not generally available. In the absence of well-developed analytical methods resort must be made to the correlation of experimental data.

Figures 3, 4, 5 and 6 are plots of the available data from air or water tunnel tests in the form proposed by Weaver and Fitzpatrick¹⁹. They give the relationship between $V_c / (f_t d_o)$ and $m_e \delta / (\rho_s d_o^2)$ for the layouts of particular interest shown in Sketch 4.4. Data sources are Derivations 20 to 27.

Equations for the least squares fit to the mean values are given in Table 4.1.



Sketch 4.4 Arrangements of in-line and staggered tube layouts

TABLE 4.1 Mean Value Mass Damping Parameter-reduced Critical Velocity Relationships (Velocities based on transverse flow area)

Tube lay-out	$\frac{m_e \delta}{\rho_s d_o^2} \leq$		$\frac{m_e \delta}{\rho_s d_o^2} >$	
90°	0.3	$V_c/f_t d_o = 2.1$	0.3	$V_c/f_t d_o = 3.9[m_e \delta/(\rho_s d_o^2)]^{0.48}$
45°	0.2	$V_c/f_t d_o = 3.1$	0.2	$V_c/f_t d_o = 6.7[m_e \delta/(\rho_s d_o^2)]^{0.47}$
30°	0.5	$V_c/f_t d_o = 3.3$	0.5	$V_c/f_t d_o = 4.7[m_e \delta/(\rho_s d_o^2)]^{0.50}$
60°	0.3	$V_c/f_t d_o = 2.1$	0.3	$V_c/f_t d_o = 6.9[m_e \delta/(\rho_s d_o^2)]^{0.32}$

In order to establish the expressions in Table 4.1, the δ values used in all cases (*i.e.* for both water and air flows) were the still air values in the absence of intermediate supports. These closely approximate the in vacuo values which are the required parameter.

For application to a tube bundle with intermediate supports the damping due to the support system must be used. This damping includes the effect of structural damping, squeeze film damping and material or internal damping which is often negligible compared with other sources.

Since the data of Figures 3, 4, 5 and 6 were obtained from wind and water tunnel experiments, the question arises as to their applicability to shell and tube heat exchanger geometries. Limited data from work undertaken at the Argonne National Laboratory, USA^{8, 11} and confined to water runs have been obtained on industrial size 30° and 90° tube bundles.

Data points have been superimposed on Figures 3 and 5. They are based on measured tube frequencies with δ values reworked in accordance with Equation (4.3) and added mass factor, k , obtained from Figure 1 of this Data Item. Proximity to the mean curve is reasonable and on the 'safe' side, the actual instability threshold being higher than that predicted.

In using Table 4.1 all velocities must be based on the transverse flow area. This velocity is V_t . Often flow velocity is calculated using minimum flow areas shown in Sketch 4.4. This velocity is V_m .

It is often convenient to calculate flow velocities based on minimum flow areas from the tube layout drawing. Therefore, after calculating critical velocities, V_c , from Table 4.1, they must be converted to critical velocities based on the minimum flow area, V_{mc} . Relevant conversion factors are:

Tube layout	Minimum section velocity
30° and 90°	$V_{mc} = V_c$
45°	$p/d_o \leq 1.707,$ $V_{mc} = \frac{(\sqrt{2}p/d_o - 1)}{2(p/d_o - 1)} V_c$
	$p/d_o > 1.707,$ $V_{mc} = V_c$
	$p/d_o = 1.25,$ $V_{mc} = 1.536 V_c$
	$p/d_o = 1.33,$ $V_{mc} = 1.330 V_c$
60°	$p/d_o \leq 3.732,$ $V_{mc} = \frac{(\sqrt{3}p/d_o - 1)}{2(p/d_o - 1)} V_c$
	$p/d_o > 3.732,$ $V_{mc} = V_c$
	$p/d_o = 1.25,$ $V_{mc} = 2.331 V_c$
	$p/d_o = 1.33$ $V_{mc} = 1.965 V_c$

In shell and tube units precise flow directions are often unknown. This is because of the complex geometry, particularly in the inlet and outlet zones, and in the mid-zones when small baffle cuts are employed. Consequently the critical velocity, V_c , should be calculated for the normal and rotated layouts and the lower value used.

In some cases (e.g. bundles in rectangular ducts), the flow direction will be known. Then the appropriate direction should be used.

4.3.2 Significance of the mass damping parameter

In Figures 3, 4, 5 and 6 gases will normally lie on the sloping portion of the curves and liquids on the horizontal section with two-phase fluids in the vicinity of the knee. This is because of the strong influence of density on the mass damping parameter. However this is not invariably the case and the location on the mean instability boundary must always be ascertained.

In order to evaluate V_c , the critical velocity, the damping, δ (Sections 4.3.3, 4.3.4 or 4.3.5), the effective mass of the tube per unit length, m_e (Equation (3.2)), and the tube natural frequency, f_t (Equation (3.1)), are required.

4.3.3 Damping with gas flows

Structural damping with gas flows is due to impacting and sliding between the tube and its supports. Pettigrew *et al.*¹⁶ have analysed the available data (see Figure 7) and recommend, for structural damping with gas flows, the relationship

$$\delta = \frac{\pi}{10} \left(\frac{N-1}{N} \right) \left(\frac{t_b}{L_m} \right)^{0.5} \quad \text{for } N \geq 2 \quad (4.2)$$

where N = number of spans
 t_b = baffle thickness
 L_m = characteristic span length taken as the average of the three longest spans.

Data sources are Derivations 28 to 41.

Tentative indications were that damping values were not significantly affected by tube mass, diameter or natural frequency and the effect of tube to baffle clearance was well within the scatter of the data.

Equation (4.2) represents the lower decile implying that if the fluid velocity was at V_c , then 10% of *vulnerable* tubes (e.g. those tubes just outside the baffle cut in the case of shell and tube units) were likely to fail. This decile was chosen rather than the mean on the grounds that it tends to conservative, or safe, design.

Note that values of δ derived from Equation (4.2) are often close to the values of about 0.03 that are sometimes arbitrarily assumed.

Damping values may be obtained from Reference 2 for those cases where there are no intermediate supports between tubesheets or headers.

4.3.4 Damping with liquid flows

With liquid flows energy is dissipated by squeeze film damping. Squeeze film damping arises from the liquid being pumped into and out of the clearances between the tube and its supports as the tube vibrates. Squeeze film damping is negligible with gas flows.

Pettigrew *et al.*¹⁷ have analysed the available data (see Figure 8) and recommend the relationship

$$\delta = \frac{44\pi}{f_t} \left(\frac{N-1}{N} \right) \left(\frac{\rho_s d_o^2}{m_e} \right) \left(\frac{t_b}{L_m} \right)^{0.6} \quad \text{for } N \geq 2. \quad (4.3)$$

Data sources are Derivations 42 to 53.

When Equation (4.3) predicts damping values lower than 0.038, a minimum value of 0.038 should be taken. Equation (4.3) represents the lower decile and is justified on the same grounds as was the equation for damping with gas flows.