

# ELASTIC STRESSES AND DEFLECTIONS FOR SQUARE PLATES WITH SMALL INITIAL CURVATURE UNDER UNIFORM PRESSURE ON THE CONCAVE OR CONVEX FACE

## 1. NOTATION

Both SI and British units are quoted but any coherent system of units may be used.

|                    |                                                                                                                                                                                        |                  |                     |
|--------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------|---------------------|
| $b$                | length of edges of plate (see Sketch 2.1)                                                                                                                                              | m                | in                  |
| $E$                | modulus of elasticity of plate material                                                                                                                                                | N/m <sup>2</sup> | lbf/in <sup>2</sup> |
| $f_C$              | total stress in surface of unloaded face at centre, same in all directions                                                                                                             | N/m <sup>2</sup> | lbf/in <sup>2</sup> |
| $f_c$              | membrane stress in plate at centre, same in all directions                                                                                                                             | N/m <sup>2</sup> | lbf/in <sup>2</sup> |
| $f_E$              | greater total stress at mid-point of, and perpendicular to, edge of plate                                                                                                              | N/m <sup>2</sup> | lbf/in <sup>2</sup> |
| $f_{max}$          | maximum total stress in plate                                                                                                                                                          | N/m <sup>2</sup> | lbf/in <sup>2</sup> |
| $p$                | normal pressure on plate (see Sketch 2.1)                                                                                                                                              | N/m <sup>2</sup> | lbf/in <sup>2</sup> |
| $r$                | ratio $(1 - \nu^2)/0.91$                                                                                                                                                               |                  |                     |
| $t$                | plate thickness (see Sketch 2.1)                                                                                                                                                       | m                | in                  |
| $\delta$           | deflection at centre of plate from unloaded form                                                                                                                                       | m                | in                  |
| $\delta_0$         | initial bow at centre of plate (see Sketch 2.1)                                                                                                                                        | m                | in                  |
| $\nu$              | Poisson's ratio for plate material                                                                                                                                                     |                  |                     |
| <i>Convention:</i> | Tensile stresses are positive. Compressive stresses are negative.<br>Concave initial bow is positive. Convex initial bow is negative.<br>Deflection positive in direction of pressure. |                  |                     |

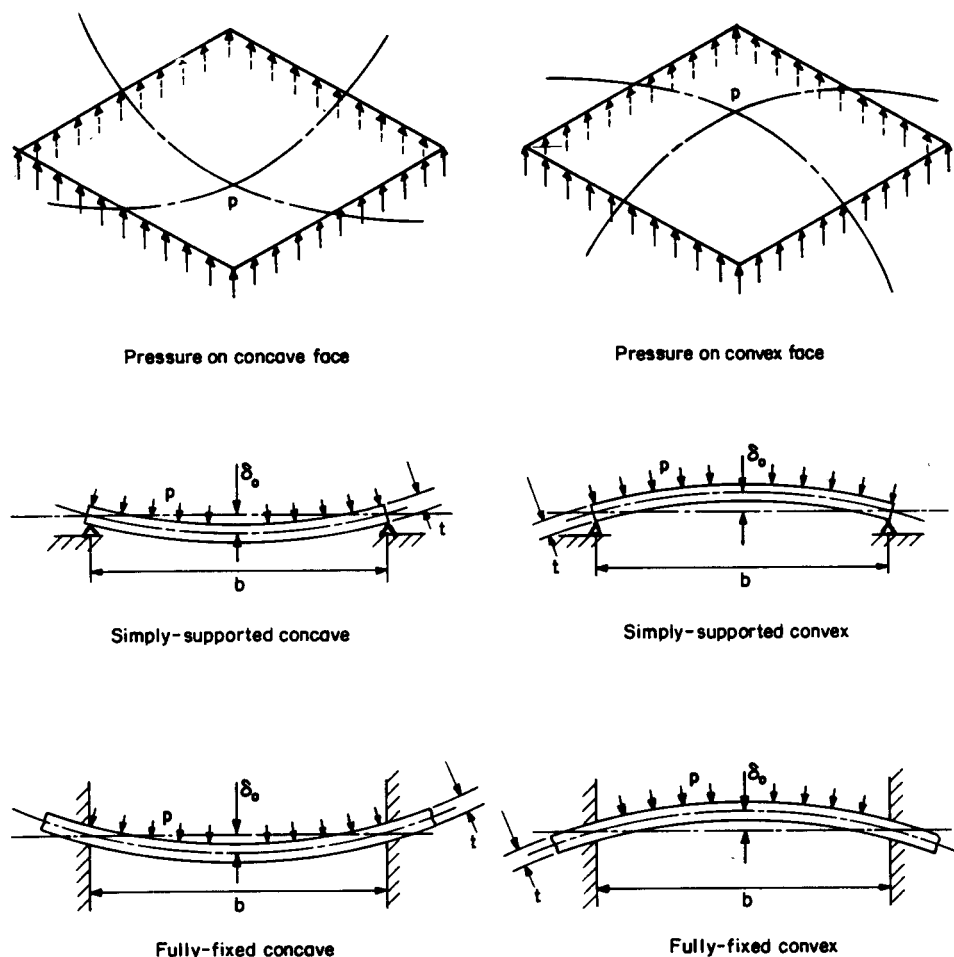
## 2. NOTES

### 2.1 General Notes

Curves are given that enable the deflection and stresses to be determined for square plates of uniform thickness with identical initial curvature across the span in both directions, under uniform pressure on the concave or convex face. The curves are given for two edge conditions, simply-supported and fully-fixed, on the assumption that the edges remain straight in their original plane.

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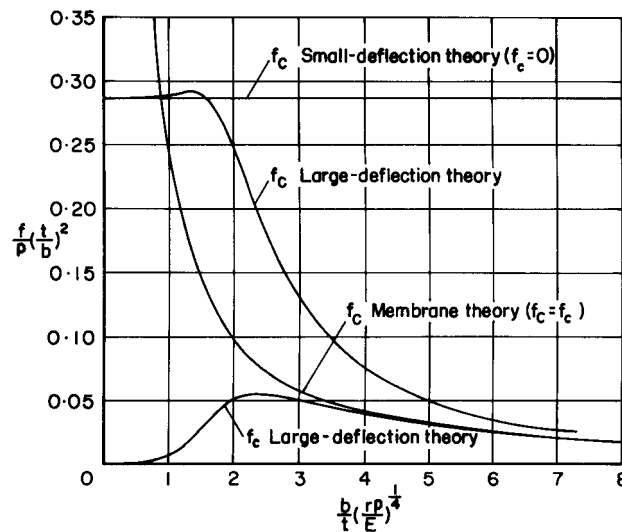
In the figures,  $\delta/t$  and  $(f/p)(t/b)^2$  are plotted against  $(b/t)(rp/E)^{1/4}$  for various values of  $\delta_0/t$ . The parameters chosen for the stress ordinate and the abscissae were selected because they encompass the whole range of elastic plate behaviour and enable the results of the non-linear analysis used to be compared to simple linear small deflection and membrane theories which are applicable at the extremities of the parameters.



**Sketch 2.1 Plate geometry and loading configurations**

Elastic thin plate behaviour can be considered in three phases: (1) plates with small deflections ( $\delta/t \leq 0.5$ ) in which bending effects dominate and linear bending theory may be applied, (2) plates with large deflections in which membrane stresses interact with the bending effects for which non-linear theory should be used, as in this Item, (3) very thin plates with very large deflections in which membrane stresses dominate and membrane theory may be applied. The plate edge conditions affect the relative importance of these effects for the stresses at various points in the plate.

Sketch 2.2 shows the interaction of these phases for the stresses at the centre of a simply-supported square plate with no initial curvature ( $\delta_0/t = 0$ ). The behaviour of other stresses, for other edge conditions and for initially curved plates, is more complex but the trends are similar as can be seen in the figures. A more complete explanation of these effects is given in Derivation 6. Linear bending and membrane formulae are given in Derivations 1 and 2.



**Sketch 2.2 Comparison of centre stress for three types of theory for flat plates with simply-supported edges ( $\delta_0/t = 0$ )**

**TABLE 2.1**

| Loaded face | Type of edge restraint |                      | Number of figure giving |       |       |       |           |
|-------------|------------------------|----------------------|-------------------------|-------|-------|-------|-----------|
|             | Rotation               | In-plane translation | $\delta/t$              | $f_C$ | $f_c$ | $f_E$ | $f_{max}$ |
| Concave     | free                   | fixed                | 1                       | 2a    | 2b    | —     | 2c        |
|             | fixed                  | fixed                | 3                       | 4a    | 4b    | 4c    | —         |
| Convex      | free                   | fixed                | 5                       | 6a    | 6b    | —     | 6c        |
|             | fixed                  | fixed                | 7                       | 8a    | 8b    | 8c    | —         |

Table 2.1 shows the figures to be used for a particular stress or deflection and a given form of loading (on the concave or convex face) and edge restraint. The total stresses given in the curves result from the combined effects of bending and middle surface tension or compression. At the centre both the total stress on the unloaded face and the middle surface stress are given and therefore the bending stress may be found by subtracting  $f_c$  from  $f_C$ . For plates that are free in rotation the maximum stress may not be at the centre of the unloaded face and therefore the maximum total stress in the plate,  $f_{max}$ , is given. Where the curve for  $f_{max}$  coincides with  $f_C$  the maximum stress will be at the centre of the plate. Where  $f_{max}$  exceeds  $f_C$  the maximum stress will lie on lines between the centre and the mid-points of the plate edges. If the plate edges are also free to move in translation as well as rotation the maximum stress will be greater than the values given here and will lie on the diagonals between the centre and the corners. For flat square plates these stresses are shown in Data Item No. 70001\*. When the edges of the plate are fixed in rotation the maximum stress may be at the edge of the plate and therefore  $f_E$  is given.

\* Data Item No. 70001 "Elastic stresses and deflections for flat square plates under uniformly distributed normal pressure".

The figures relate to plates having an initial curvature of sine wave form in two orthogonal directions across the plate normal to the edges with the maximum initial bow at the centre. The curves are derived on the basis of thin plate theory which is known to be valid for  $b/t$  not less than 20. The curves may be used with reasonable accuracy for values of  $\delta_0/b$  less than 0.1. Strictly, they apply only to stresses within the elastic range of the material but may be used up to the 0.2 per cent proof stress.

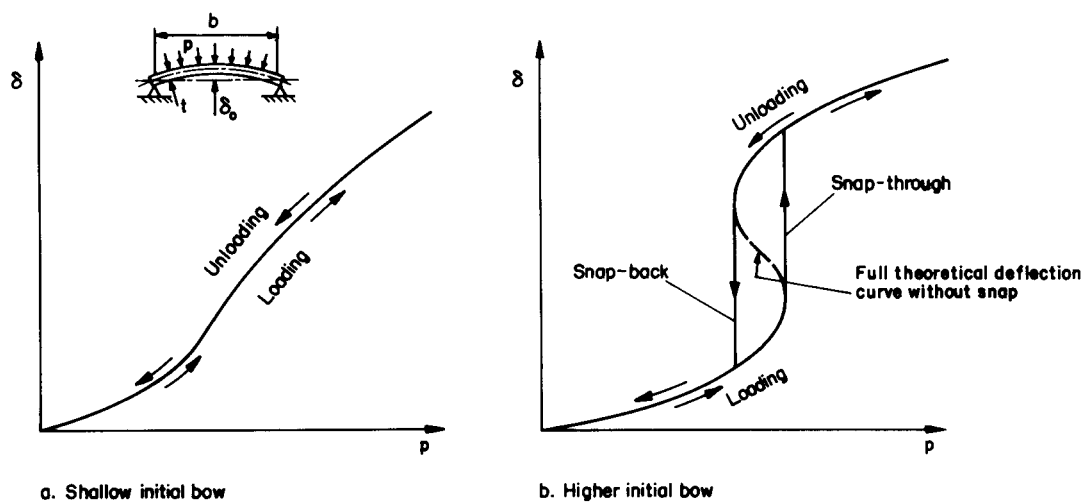
## 2.2 Form of Loading

Figures 1 to 4c inclusive are for plates with pressure applied to the concave face. For such loading the pressure has the effect of increasing the deflection of the plate in the direction of the initial curvature.

Figures 5 to 8c inclusive are for plates with pressure applied to the convex face. For such loading the pressure has the effect of initially reducing the deflection of the plate until it lies in the plane of the edges and then increasing the deflection of the plate in the direction opposite to that of the initial curvature.

For convex plates with very small initial curvature the transition of the deflection from reducing the initial curvature to opposite curvature is relatively smooth. For plates with high initial curvature, the middle surface compression stress builds up to a point where 'snap-through' of the panel occurs from the side of the initial curvature to the opposite side of the supports. When the load is removed the panel may be capable of 'snapping back' to its original form. Sketch 2.3 shows the types of deflection versus load curves relating to these forms of behaviour. For some panels a pressure in the opposite direction will be required to produce snap-back.

Studies of laterally loaded curved beams in Derivations 5 and 7 have indicated that snap-through is likely to occur if  $\delta_0/t \geq 0.6$ . Work on pressurised curved rectangular plates and square plates with beam supported edges in Derivation 3 agrees well with this value for the onset of snap-through and suggests that panels may require a definite reverse snap-back pressure if the initial bow is higher than  $\delta_0/t = 1.13$ . However, since these values have not been obtained for the square plate configurations given in the figures here they should be used only as a guide. There is further evidence that both symmetric and asymmetric forms of snap-through can occur depending on the magnitude of the initial bow, the edge conditions and the presence of imperfections.



Sketch 2.3 Forms of plate 'load-deflection' behaviour

### 3. DERIVATION

This section lists selected sources that have assisted in the preparation of this Item.

1. HENCKY, H. Die Berechnung dünner rechteckiger Platten mit verschwindender Biegungsteitigkeit, *Z. angew. Math. Mech.*, 1, 81 and 423, 1921.
2. TIMOSHENKO, S. *Theory of plates and shells*. Second edition, Chapters 4, 5, 6 and 13. WOINOWSKY-KRIEGER, S. McGraw-Hill, New York, 1959.
3. MUSHTARI, K.M. *Gibkie plastiny i paneli (Elastic plates and panels*, ESDU translation). Izdatel'stve Nauke, Moscow, 1968.
4. RUSHTON, K.R. Large deflection of plates with initial curvature. University of Birmingham, 1970.
5. RUSHTON, K.R. Buckling of laterally loaded plates having initial curvature. University of Birmingham, December 1971.
6. PENNING, R.L. Elastic stresses and deflections of flat square plates under uniformly distributed normal pressure: a critical review. *J. Strain Anal.*, Vol. 7, No. 1, pp. 23-32, 1972.
7. RHODES, J. *Unsymmetrical buckling of laterally loaded reinforced plastic shells*. MARSHAL, I.J. University of Strathclyde, 1977.

### 4. EXAMPLE

A square plate of width 10 in and thickness 0.048 in has an initial bow of 0.096 in. The plate edges are free in rotation but fixed in translation. The plate is subjected to a uniform pressure of 3.5 lbf/in<sup>2</sup> on the convex face. The modulus of elasticity for the plate material is  $30 \times 10^6$  lbf/in<sup>2</sup> and Poisson's ratio is 0.3. It is required to find the maximum deflection and stresses in the plate and to determine whether or not the plate snaps-through and if so the maximum stress when this occurs.

For this material  $\nu = 0.3$ , and thus, from the notation,  $r = 1.0$ .

The parameters required for use with the figures are:

$$\frac{b}{t} \left( \frac{rp}{E} \right)^{1/4} = \frac{10}{0.048} \left( \frac{1.0 \times 3.5}{30 \times 10^6} \right)^{1/4} = 3.85,$$

$$p \left( \frac{b}{t} \right)^2 = 15.2 \times 10^4 \text{ lbf/in}^2$$

and 
$$\frac{\delta_0}{t} = -2.0.$$

Table 2.1 shows that for this configuration Figures 5 and 6 are used to obtain the deflections and stresses. When  $(b/t)(rp/E)^{1/4} = 3.85$  and  $\delta_0/t = -2.0$ , the derived values are as given in Table 4.1.

TABLE 4.1

| Figure used | Ratio obtained from curves                              | Hence stress or deflection           |
|-------------|---------------------------------------------------------|--------------------------------------|
| 5           | $\delta/t = 4.27$                                       | $\delta = 0.205 \text{ in}$          |
| 6a          | $\frac{f_C}{p} \left( \frac{t}{b} \right)^2 = 0.14$     | $f_C = 21\,300 \text{ lbf/in}^2$     |
| 6b          | $\frac{f_c}{p} \left( \frac{t}{b} \right)^2 = 0.018$    | $f_c = 2740 \text{ lbf/in}^2$        |
| 6c          | $\frac{f_{max}}{p} \left( \frac{t}{b} \right)^2 = 0.14$ | $f_{max} = 21\,300 \text{ lbf/in}^2$ |

Inspection of the figures shows that the plate will snap-through before the operating pressure has been reached.

From the above figures when  $\delta_0/t = -2.0$  snap-through occurs at

$$\frac{b}{t} \left( \frac{rp}{E} \right)^{1/4} = 3.15.$$

Therefore the pressure on the plate at snap-through will be

$$p = 3.5 \left( \frac{3.15}{3.85} \right)^4 = 1.57 \text{ lbf/in}^2.$$

Therefore

$$p \left( \frac{b}{t} \right)^2 = 1.57 \left( \frac{10}{0.048} \right)^2$$

$$= 6.81 \times 10^4 \text{ lbf/in}^2.$$

The maximum stress is obtained from Figure 6c.

When  $(b/t)(rp/E)^{1/4} = 3.15$  and  $\delta_0/t = -2.0$

$$\frac{f_{max}}{p} \left( \frac{t}{b} \right)^2 = -0.27.$$

Hence, the maximum total stress at snap-through is

$$f_{max} = -0.27 \times 6.81 \times 10^4$$

$$= -18\,400 \text{ lbf/in}^2.$$

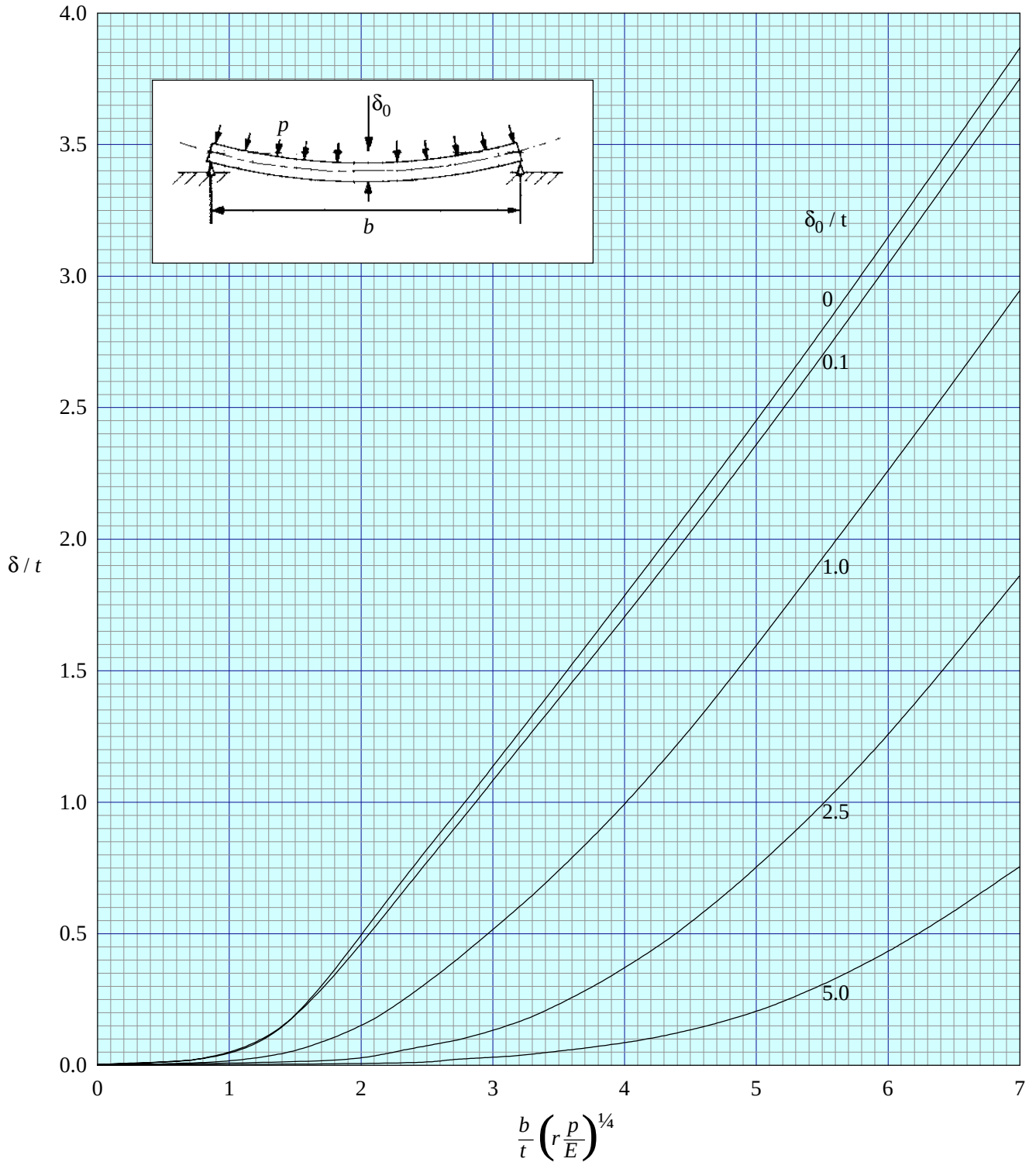


**FIGURE 1 DEFLECTION. PLATE CONCAVE. EDGES FREE IN ROTATION, FIXED IN TRANSLATION**