

CALIBRATION OF ANEMOMETRIC PRESSURE SYSTEMS FOR LAG

1. NOTATION, UNITS AND CONSTANTS

		SI	British
a	speed of sound	m/s	ft/s
F	calibration factor, see Equation (3.3)		
$f_1, f_2 \dots$	denotes functional relationship of type $f[\dots]$		
H	ratio of amplitude of output pressure to amplitude of input pressure(= $[p_i/\rho^n]_{peak}$)		
l	length of pipe from pressure-sensor port(s) to the transducer in a pressure system	m	ft
n	exponent in polytropic relationship, $p/p^n = \text{constant}$		
p	pressure* at the pressure-sensor port(s) of a pressure system	N/m ²	lbf/ft ²
p_a	ambient pressure	N/m ²	lbf/ft ²
p_i	pressure* registered at the transducer or instrument in a pressure system	N/m ²	lbf/ft ²
Q	relevant internal volume of an instrument or transducer in a pressure system	m ³	ft ³
r	mean internal radius of pipes in a pressure system	m	ft
T	temperature of air in a pressure system	K	K
t	time from instant that pressure, p , changes	s	s
v	mean velocity of air in pipes of pressure system	m/s	ft/s
Δ	denotes increment in affixed parameter		
Δp	change in input pressure from initial value, p	N/m ²	lbf/ft ²
ζ	relative damping ratio of system		
λ	viscous lag time constant for a pressure system	s	s
θ	constant related to quadrant in which ϕ lies	rad	rad
μ	coefficient of dynamic viscosity of air in pressure system	N s/m ²	lbf s/ft ²
ρ	density of air	kg/m ³	slug/ft ³

For footnote see end of the Notation.

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ϕ	phase angle between p and p_i for sinusoidal pressure variations	rad	rad
τ	acoustic lag time constant for pressure system	s	s
ω	angular frequency of sinusoidal input pressure	rad/s	rad/s
ω_n	“undamped” angular frequency of system	rad/s	rad/s

Subscripts

<i>crit</i>	denotes conditions for transition from laminar to turbulent flow
<i>flight</i>	denotes conditions in a pressure system during flight
<i>ground</i>	denotes reference (or datum) conditions to which ground (“hangar”) calibrations are corrected
<i>i</i>	denotes value of a parameter registered (indicated) by a transducer (or instrument)
<i>int</i>	denotes an intermediate stage in calculation of λ_{flight}
<i>p</i>	denotes pitot-pressure system
<i>peak</i>	denotes maximum magnitude of input or output pressure for sinusoidal input
<i>s</i>	denotes static-pressure system
<i>test</i>	denotes conditions in a pressure system during ground (“hangar”) calibrations
<i>0</i>	denotes conditions at sea-level in International Standard Atmosphere, ISA

Constants

a_0	340.294 m/s	1116.45 ft/s
p_{a_0}	101 325 N/m ²	2116.22 lbf/ft ²
T_{a_0}	288.15 K	

* Note that these pressures are absolute.

2. INTRODUCTION

The analysis of flight tests for non-steady conditions such as climbing flight involves corrections to the values of pitot and static pressures registered (indicated) by transducers, see ESDU Data Item No. 85011, Reference 6. These corrections are necessary because, in general, there is a non-trivial lag in the transmission of a pressure change at the sensor port(s) to the transducer (or instrumentation) of an anemometric pressure system.

This Item describes some of the ground (or “hangar”) calibration tests which are carried out on aircraft anemometric pressure systems. These are made so that the corrections in flight-test analyses of lags due to frictional and acoustic effects may be based on good empirical data. The ground tests are calibrations of the actual pressure systems installed in the test aircraft and are made usually for ranges of pressures appropriate to the performance capabilities of the test aircraft. One ground test method gives information on the effects of rates of change of pressure so that appropriate ranges of values of these rates must be used.

3. THEORETICAL CONSIDERATIONS

3.1 Viscous Lag of First-Order System

3.1.1 Constant rate of change of input pressure with time

For some pressure systems the response may be assumed to be that of an ideal first-order linear system so that at all times the instantaneous relationship between “input” and “output” pressures is completely defined by the time constant λ in the relationship

$$\lambda \frac{dp_i}{dt} = (p - p_i). \quad (3.1)$$

For a constant rate of change, dp/dt , of input pressure, the ratio of the change, Δp_i , in output for a change, Δp , in input pressure can be expressed as follows (see Reference 3).

$$\frac{\Delta p_i}{\Delta p} = \frac{\Delta p_i}{t(dp/dt)} = 1 - \left[\frac{\lambda}{t} \right] [1 - e^{-t/\lambda}]. \quad (3.2)$$

References 2 and 5 derive the following theoretical relationship between the lag due to viscous effects in a pressure system and the significant system parameters.

$$\lambda = F \left[\frac{8l}{\pi r^4} \right] \times \left[\frac{\mu}{np} \left\{ \frac{\pi r^2 l}{2} + Q \right\} \right] \text{ s.} \quad (3.3)$$

In Equation (3.3) the coefficient of dynamic viscosity is given by

$$\left. \begin{aligned} \mu &= \frac{1.458 \times 10^{-6} T^{3/2}}{[T + 110.4]} \text{ N s/m}^2 \\ \text{or} \quad &= \frac{30.4509 \times 10^{-9} T^{3/2}}{[T + 110.4]} \text{ lbf s/ft}^2 \end{aligned} \right\}. \quad (3.4)$$

The time constant, λ , is seen to be proportional to the gas state in the pressure system, namely, (μ/np) and to be a function of the geometry of the system, of which the most important variable is the mean internal pipe radius, r .

The factor, n , in Equation (3.3) is the exponent in the polytropic relationship, $p/\rho^n = \text{constant}$, taking the value $n = 1.0$ for isothermal conditions or $n = 1.4$ for adiabatic conditions. It is shown in Reference 4 that for the pipe-flow states consistent with Equation (3.3) the value of $n = 1.0$ should be used.

When the quantity $F > 1$ in Equation (3.3), it is regarded as a calibration factor for the pressure system. Values of F also serve to categorise the flow state in the system, as follows.

- (i) $F = 1$ for laminar (Poiseuille) pipe flow in an ideal system, that is a system having no imperfections such as pipe junctions or bends.
- (ii) $F > 1$ for laminar flow in practical systems and in such cases F , and hence λ , is invariant with $(p - p_i)$ for given conditions of temperature and pressure.
- (iii) $F > 1$ but varies with $(p - p_i)$ where the pipe flow is predominantly or fully turbulent.

Transition from laminar to turbulent flow occurs at some critical value of Reynolds number which has to be established by tests, see Section 4.2.1. A discussion of the dependence of transition behaviour on temperature at constant Reynolds number is given in Appendix A.

Equations (3.1) and (3.2) are applicable so long as the response can be represented by an ideal first-order system. However, the derivation of Equation (3.3) requires, in addition, that

- (i) the mass flow into or out of the system is small,
- (ii) the variation of pressure with time is constant and small.

3.1.2 Sinusoidal variation of input pressure

If the variation of pressure with time is sinusoidal, as may be the case for certain flight manoeuvres, it is shown in Reference 3 that p_i will lag p by a phase angle, ϕ , and, if ω is the applied frequency, the time constant, λ , is then given by

$$\lambda = -\tan\left[\frac{\phi}{\omega}\right]. \quad (3.5)$$

The amplitude of the output relative to the amplitude of the input, at frequency ω , is

$$H = \frac{p_i}{p} = \frac{1}{[(\lambda\omega)^2 + 1]^{1/2}} = \cos\phi. \quad (3.6)$$

3.1.3 Step change of input pressure

A step change of pressure from p to $(p + \Delta p)$ results in a response given by

$$\frac{(p_i - p)}{\Delta p} = 1 - e^{-t/\lambda} \quad (3.7)$$

where λ is the value of t when $(p_i - p) = 0.632\Delta p$.

3.2 Viscous Lag of Second-Order System

Second-order systems are described by the following differential equation

$$\frac{d^2 p_i}{dt^2} + 2\zeta\omega_n \frac{dp_i}{dt} + \omega_n^2 p_i = \omega_n^2 p \quad (3.8)$$

The steady-state responses to particular forms of the variation with time of input pressure, $p = f[t]$, are given in Reference 3; some of these are discussed in Sections 3.2.1 and 3.2.2.

3.2.1 Sinusoidal variation of input pressure

A sinusoidal input of frequency, ω , results in the following relationship for H , i.e. the ratio of the amplitude of the output pressure to the amplitude of the input pressure

$$H = \left[\frac{p_i}{p} \right]_{peak} = \frac{1}{\left[\left\{ 1 - \left(\frac{\omega}{\omega_n} \right)^2 \right\}^2 + 4\zeta^2 \left(\frac{\omega}{\omega_n} \right)^2 \right]^{1/2}} \quad (3.9)$$

The phase angle, ϕ , of the output pressure, p_i , relative to the input pressure, p , is given by

$$\phi = \tan^{-1} \left[\frac{2\zeta \left(\frac{\omega}{\omega_n} \right)}{\left(\frac{\omega}{\omega_n} \right)^2 - 1} \right] - \theta \quad \text{rad} \quad (3.10)$$

$$\left. \begin{array}{l} \text{where} \quad \theta = 0 \text{ for } \left(\frac{\omega}{\omega_n} \right) \leq 1.0 \\ \text{or} \quad \theta = \pi \text{ for } \left(\frac{\omega}{\omega_n} \right) > 1.0 \end{array} \right\} \text{rad.} \quad (3.11)$$

From Equation (3.10), and assuming ω is small,

$$\frac{d(\tan \phi)}{d\omega} = -\frac{2\zeta}{\omega_n} = -\lambda \quad (3.12)$$

or, to a good approximation,

$$\lambda \approx -\frac{\phi}{\omega} \quad (3.13)$$

* Note that $\phi = -\pi/2$ rad (-90 degrees) when $\omega = \omega_n$.

The value of the relative damping ratio of the system for low frequency, from Equation (3.12), is

$$\zeta = \frac{\lambda \omega_n}{2}. \quad (3.14)$$

Since no theoretical method exists for estimating values of ζ and ω_n , the characteristics of a second-order pressure system may be determined only by calibration tests. Sinusoidal input tests, described in Section 4.2.2, provide values of phase angle, ϕ , as a function of ω , from which the viscous lag time constant, λ , may be derived.

3.2.2 Constant rate of change of input pressure with time

A ramp input, *i.e.* $dp/dt = \text{constant}$, results in a lag time which is itself a function of time, t , when the product $(t \times \omega_n)$ is less than $t\omega_n \approx 9$, see Figure 1. For $t\omega_n > 9$ the lag time value is indistinguishable from its asymptotic value*. This value occurs when $d^2p_i/dt^2 = 0$ and, comparing Equation (3.8) with Equation (3.1), it can be seen that

$$\lambda = \frac{2\zeta}{\omega_n}. \quad (3.15)$$

Ramp input tests are described in Section 4.2.1 and provide the asymptotic values of total lag time, $(\lambda + \tau)$, for systems where the value of ζ is not too small.

3.3 Acoustic Lag

Acoustic lag and its associated time constant, τ , represents the time for the transmission of pressure changes at the speed of sound, a , appropriate to the system temperature

$$\tau = \frac{l}{a} = \left[\frac{288.15}{T} \right]^{1/2} \times \frac{l}{a_0} \text{ s}. \quad (3.16)$$

4. GROUND TESTING

4.1 General

Ground tests or calibrations on complete anemometric systems installed in test aircraft are recommended for all but the simplest of pressure systems on light aircraft. Calibrations are required because for first-order systems the modelling of the pressure response is inadequate (see Section 3.1.1) and for second-order systems modelling is not possible (see Section 3.2.1). Calibration tests described in Section 4.2.1 to 4.2.3 are made over a range of values of p appropriate to the performance capabilities of the test aircraft. In addition, the tests described in Section 4.2.1 are made for an appropriate range of values† of (dp_i/dt) . From these calibrations functional relationships for the viscous lag time constant, λ , may be derived having the general form†, suggested by Equations (3.1) and (3.3),

$$\lambda = f_1[p_i, \mu, dp_i/dt]_{test} = f_2[p_i, T, dp_i/dt]_{test} \text{ s}. \quad (4.1)$$

* This is true for $\zeta \geq 0.4$. For $\zeta < 0.4$, much larger times are required before the values of lag time settle down.

† The use of p_i rather than p in Equations (4.1), (4.2) and (4.3) arises from the need to present the calibrations in forms directly usable for flight-test analysis. The justification for the approximation in Equation (4.4) is that $[p/p_i]$ is approximately equal to unity in most cases.

These relationships are frequently less complex than Equation (4.1) indicates because

- (a) tests of the type described in Sections 4.2.2 and 4.2.3 provide no data for the effect of (dp_i/dt) ,
- (b) the tests described in Section 4.2.1 show no variation of λ with (dp_i/dt) when the pipe flow is laminar. (Section 4.2.1 and Appendix A discuss the problems associated with turbulent-flow conditions.)

During each calibration test, both viscous and acoustic lags are experienced so that a total lag time constant $(\lambda + \tau)_{test}$ is measured. Each of these lags is a different function of system temperature so that it is important to measure temperature during each test in order to obtain a correct value of λ_{test} using Equation (4.5). In the usual case, with the pressure in the system below ambient at the time of the test, the initial temperature in the system is also below ambient.

In analysing such calibrations, it is convenient to eliminate the variation, from test point to test point, of the temperature, T_{test} , by correcting to a reference or datum "ground temperature", for example $T_{ground} = 288$ K. It is also usual to correct the test results to convenient rounded values of p_i and, from Equation (3.3), small adjustments for pressure may be made to give arrays of test data of the form*:

$$[\lambda(T + 110.4)/T^{3/2}]_{ground} = [\lambda(T + 110.4)/T^{3/2}]_{flight} = f_3[p_i] \quad (4.2)$$

where
$$[\lambda p_i(T + 110.4)/T^{3/2}]_{ground} = [\lambda p_i(T + 110.4)/T^{3/2}]_{test} . \quad (4.3)$$

Equation (4.3) follows from Equations (3.3) and (3.4) since, from Equation (3.3), for any given system*,

$$\frac{\lambda p}{\mu} = \left[\frac{\lambda p_i}{\mu} \right] \left[\frac{p}{p_i} \right] \approx \frac{\lambda p_i}{\mu} = \text{constant} . \quad (4.4)$$

Although the value of acoustic lag, τ , is small in most cases compared with λ , it is usual to deduct it from the measured value of total lag, $(\lambda + \tau)_{test}$, to give

$$\lambda_{test} = (\lambda + \tau)_{test} - \tau_{test} . \quad (4.5)$$

The application of these calibrations to flight tests is the subject of Reference 6. In that Data Item it is recommended that an estimated value of τ , appropriate to the pressure system temperature in flight, be added to the value of λ obtained using the data array from Equation (4.2).

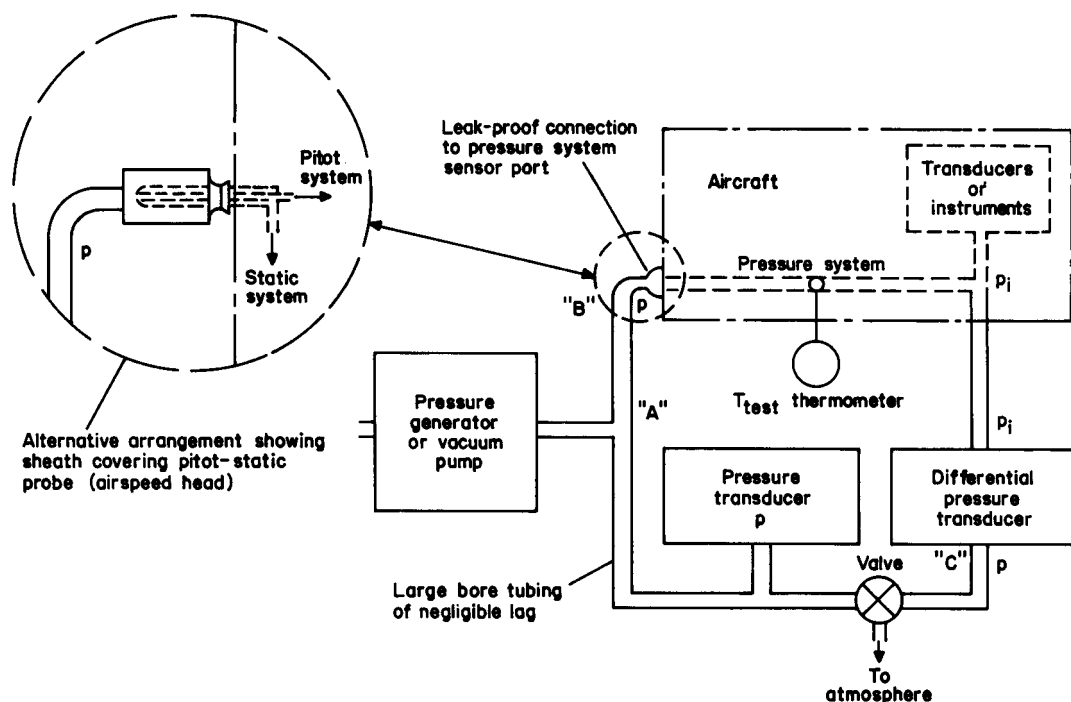
4.2 Test Apparatus for Lag Calibration Tests

The schematic arrangements shown in Sketches 4.1, 4.3 and 4.4 represent three among a number that give satisfactory results for the experimental calibration of pressure systems, provided that the sensitivities of the transducers are adequate.

* The use of p_i rather than p in Equations (4.1), (4.2) and (4.3) arises from the need to present the calibrations in forms directly usable for flight-test analysis. The justification for the approximation in Equation (4.4) is that $[p/p_i]$ is approximately equal to unity in most cases.

4.2.1 Tests with constant rate of change of input pressure

The recommended procedure is to evacuate the system under test (either the pitot or static pressure system) to a desired value of pressure, p . The valve to atmosphere is opened slowly and set to give a desired value of $(p - p_i)$. When the $(p - p_i)$ reading is steady, values of p and time are noted so that dp/dt may be computed. Because $(dp_i/dt) = (dp/dt)$, a value of total lag, $(\lambda + \tau)$, may be calculated using Equation (3.1) for the pair of values $(p - p_i)$ and dp_i/dt . This process is repeated for ranges of values of $(p - p_i)$ and of p so that, with a value of T_{test} for each test point, the functional relationship of Equation (4.1) may be derived. As explained in Section 4.1, λ_{test} is obtained using Equations (3.16) and (4.5)



Notes:

- (1) Pressure transducer may be replaced by sensitive altimeter
- (2) Differential pressure transducer may be replaced by U-tube manometer
- (3) Tubing between "A" and "B" would be bifurcated at "A" for systems (for example static systems) having two interconnected sensor ports at "B"

Sketch 4.1 Schematic arrangement of test equipment for tests with constant rate of change of input pressure

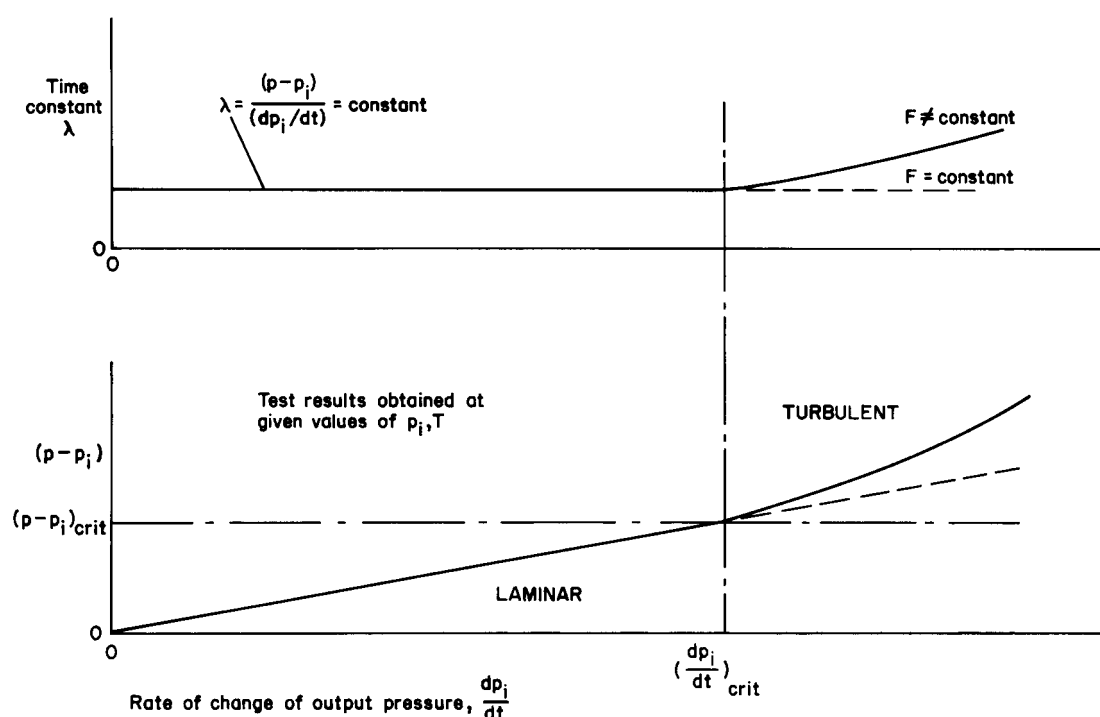
When $(p - p_i)$ and (dp_i/dt) are linearly related, λ is constant and is then not a function of (dp_i/dt) in Equations (4.1) and (4.2).

If there is a non-linear relationship between $(p - p_i)$ and (dp_i/dt) above a certain critical value, $(dp_i/dt)_{crit}$, so that λ is no longer constant, it is assumed that the pipe flow is then turbulent, see Sketch 4.2. The condition for transition from laminar to turbulent flow is that the Reynolds number of the flow in the pressure system exceed some critical value. For a given temperature, this requires that the values of (dp_i/dt) and of p_i exceed corresponding critical values. Appendix A considers how the value of $(dp_i/dt)_{crit}$ changes with temperature. The Appendix shows that a non-constant value of λ will not be relevant, provided that

- (a) $(dp_i/dt)_{crit}$ is greater than likely flight-test values,
- (b) the temperature, T_{test} , is not greater than expected flight values of temperature, T_{flight} , in the pressure system.

It is usual for condition (b) to be met because system temperatures during ground tests, where $(p_{test}/p_{a0}) < 1.0$, are such that $T_{test}/T_{a0} < 1.0$. In contrast, it is shown in Reference 6 that, because the pressure system is often located within a heated cabin, T_{flight} is greater than* 290 K in most cases. Hence, in general, $T_{test}/T_{flight} < 1.0$. If these conditions are not met, then the procedures given in Appendix A should be followed.

This calibration method has the advantage of showing whether flow transition occurs and, in that eventuality, in providing $\lambda = f(T, dp/dt)$ for each value of p tested (see Appendix A). A further advantage is that both first- and second-order systems may be calibrated (see Sections 3.1.1 and 3.2.2).



Sketch 4.2 Illustration of lag calibration test results that include transition from laminar to turbulent flow in pressure system

4.2.2 Test with sinusoidal variation of input pressure

Sketch 4.3 shows a schematic arrangement of test equipment for tests with sinusoidal variation of input pressure. The input pressure generator should be capable of controlling a pre-set pressure level, p . Sinusoidal (or other) variations of pressure with time are provided by electrical signalling in the feed-back loop from the Transfer Function Analyser. The magnitude of the peak input pressure, p , is a function of the impressed frequency and of the geometry of the pressure system under test.

The relationship between the transducer signals for pressures p and p_i is a measure of the frequency transfer function of the pressure system and this function is provided by the Transfer Function Analyser giving amplitude ratio, H , and phase angle, ϕ .

Having set the pressure in the system to a desired level, p , the procedure is to initiate sinusoidal variations around this level over a range of frequencies. A relatively large number of test readings should be made at low frequencies and at a sufficient number of higher frequencies so that the system characteristics may be

* The value of 290 K assumes that all, or the greater part of, the pressure system is within a heated cabin. For doubtful cases an estimate may be made of the system temperature, T_{flight} , using the method of Section 2.1.1 of Reference 6.