

ELASTIC WRINKLING OF SANDWICH PANELS WITH LAMINATED FIBRE REINFORCED FACE PLATES

(Face plates of AsBoDs, AsBIDs and AsBsDs types)

1. FACE PLATE WRINKLING BEHAVIOUR AND LIMITATIONS

1.1 Introduction

This Data Item, together with ESDUpac A8815, provides a Fortran computer program for calculating the theoretical wrinkling loads of balanced laminated plates and of two classes of unbalanced laminated plates which form the face plates of sandwich panels and that are subjected to bi-axial in-plane loads. The seven possible classes of laminated plates are described in detail in Data Item No. 94003*. The solution and program provided deal with the classes coded as AsBoDs, AsBIDs and AsBsDs in Data Item No. 75002. These classes are described in detail in Section 1.4.

This Data Item consists of three Sections. Section 1 describes the theoretical solution to the title problem and its limitations in application to practical panels. Section 2 describes the input and output for the program of ESDUpac A8815 and gives example calculations. Section 3 gives notes on the method of solution used, the resulting accuracy and program testing.

A possible mode of failure of a sandwich panel with thin face plates that is subjected to in-plane loads is the localised buckling (wrinkling) of the faces as illustrated in Sketches 1.2 and 1.3. For fibre reinforced face plates made up from a number of thin layers, it is possible that the face plates will each form an unbalanced lay-up about their local mid-planes. This lack of balance is further accentuated by the layer of glue that is required to bond each face plate to the core material and that can have a significant effect on the wrinkling loads. The following sections detail the forms of face plate stiffness sub-matrices dealt with by this Data Item and illustrate the influence of glue layer, core thickness and face plate thickness on the wrinkling loads of panels.

1.2 Notation

Both SI and British units are quoted but any coherent system of units may be used.

1.2.1 Face plate overall stiffnesses

A	in-plane stiffness matrix for face plate	N/m	lbf/in
A_{ij}	element of A	N/m	lbf/in
B	coupled in-plane and flexural stiffness matrix for face plate	N	lbf
B_{ij}	element of B	N	lbf
D	flexural stiffness matrix for face plate	N m	lbf in
D_{ij}	element of D	N m	lbf in

* Data Item No. 94003 "Stiffnesses of laminated flat plates".

1.2.2 Layer properties and reference axes

C	reference number for layer type (see Section 2.3.3.1 and 2.3.3.2)		
$E_{\alpha k}, E_{\beta k}$	moduli of elasticity of k th layer in α_k - and β_k -directions, respectively	N/m ²	lbf/in ²
$G_{\alpha\beta k}$	shear modulus of k th layer in $(\alpha, \beta)_k$ directions	N/m ²	lbf/in ²
t_k	thickness of k th layer	m	in
ψ_k	angle of principal axis of orthotropy, α_k , of k th layer measured from plate reference axis, $0x$, as shown in Sketch 1.1	degree	degree
$\nu_{\alpha\beta k}, \nu_{\beta\alpha k}$	Poisson's ratio for k th layer resulting from load applied in α_k -direction and in β_k -direction, respectively		
$0(\alpha, \beta)$	principal axes of orthotropy of a layer (see Sketch 1.1)		

1.2.3 Core properties

E_c	modulus of elasticity of core material in z -direction (in both tension and compression)	N/m ²	lbf/in ²
G_{cx}, G_{cy}	shear modulus of core material in (x, z) and (y, z) planes respectively	N/m ²	lbf/in ²

1.2.4 Panel loading, geometry and reference axes (see Sketch 1.1)

h	depth of core measured between mid-planes of face plates	m	in
h_t	total depth of sandwich between exterior faces of face plates	m	in
P_x, P_y	load per unit run in x - and y -directions, respectively (positive in compression)	N/m	lbf/in
t	thickness of face plate including thickness of face plate to core adhesive	m	in
t_a	effective thickness of face plate to core adhesive layer	m	in
λ	length of half wave of wrinkle	m	in
$0(x, y, z)$	panel reference axes, x and y defined by principal axes of orthotropy of face plates		

1.2.5 Displacements and stresses

u_o, v_o, w_o	displacements in x-, y- and z-directions, respectively, of core mid-plane	m	in
σ_{zo}	direct stress in z-direction at core mid-plane	N/m ²	lbf/in ²
τ_{xz}, τ_{yz}	core shear stresses in (x, z) and (y, z) planes, respectively	N/m ²	lbf/in ²

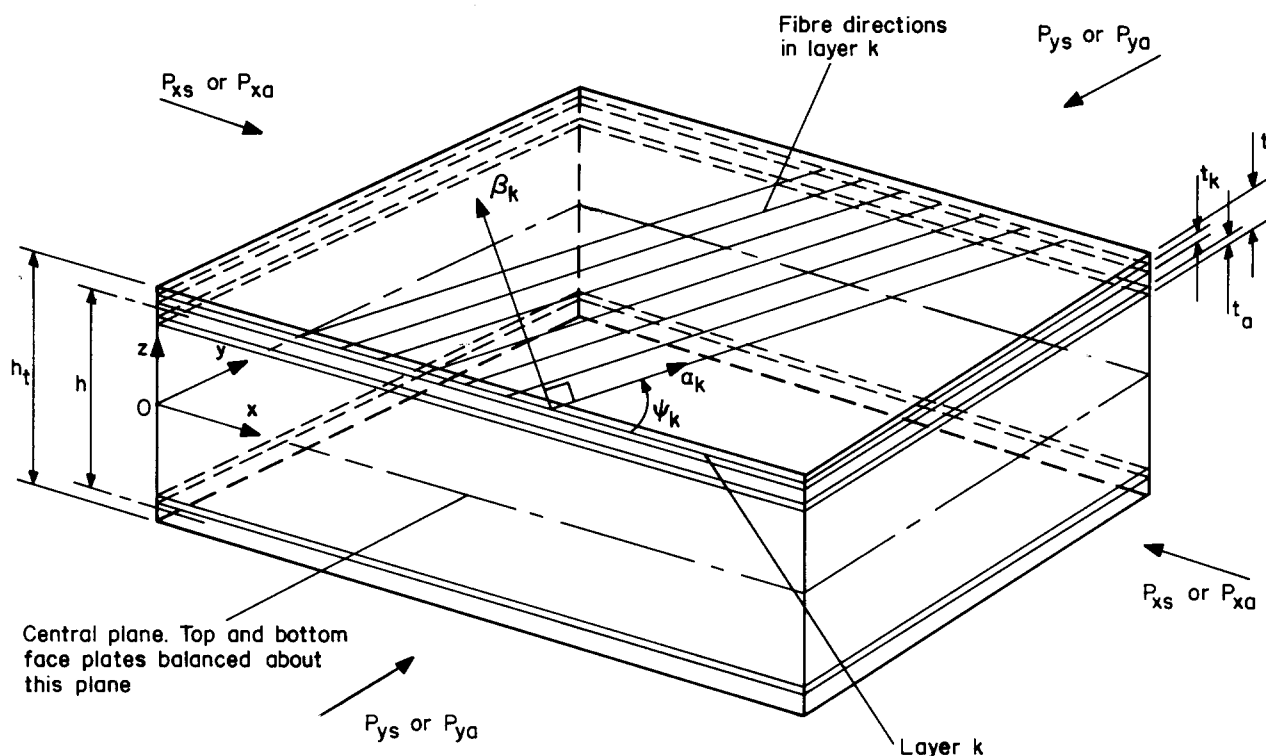
1.2.6 Suffixes

ji, j	number signifying position in row and column, respectively, of element in matrix
k	layer identification number
a	relates to antisymmetric wrinkling loads and half wavelength
s	relates to symmetric wrinkling loads and half wavelength
x, y	relates to x- and y-directions, respectively

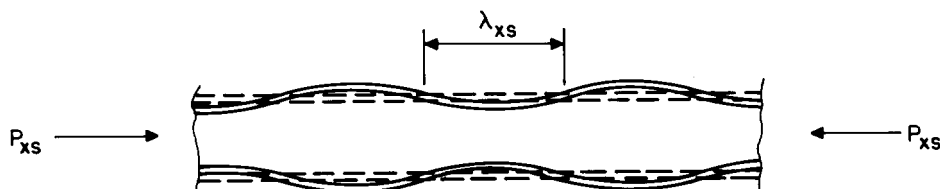
1.2.7 Mathematical notation and sign convention

sym indicates matrix is symmetric about leading diagonal

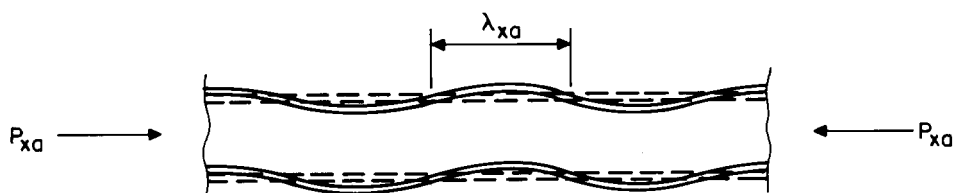
Compressive forces positive, tensile forces negative.



Sketch 1.1 Sandwich construction



Sketch 1.2 Panel face plate symmetric wrinkling in xz-plane



Sketch 1.3 Panel face plate antisymmetric wrinkling in xz-plane

1.3 The Solution Provided

The program of Section 3 calculates the total load per unit run applied to the panel in the x - and/or y -directions that causes face plate wrinkling. That is, P_{xs} and/or P_{ys} together with P_{xa} and/or P_{ya} . The x - and y -directions are defined by the principal axes of orthotropy of the face plates and are independent of their relation to the panel boundaries. The program is only valid for cases where the direction of the principal

stresses coincides with the principal axes of orthotropy of the face plates.

1.4 Class of Unbalanced Plates for Which a Solution is Given

Data Item No. 94003 shows how the stiffness matrix forms for the various types of unbalanced plate arise. This present Data Item deals only with plates in which the **A** and **D** stiffness sub-matrices are of the following forms

$$\mathbf{A} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ & A_{22} & 0 \\ (sym) & & A_{33} \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ & D_{22} & 0 \\ (sym) & & D_{33} \end{bmatrix} \quad (1.1)$$

and the **B** coupling matrix is either zero or is one of the forms,

$$\mathbf{B} = \begin{bmatrix} B_{11} & 0 & 0 \\ & B_{22} & 0 \\ (sym) & & 0 \end{bmatrix} \text{ or } \mathbf{B} = \begin{bmatrix} B_{11} & B_{12} & 0 \\ & B_{22} & 0 \\ (sym) & & B_{33} \end{bmatrix}. \quad (1.2)$$

These forms accord with the reference codes AsBoDs, AsBIDs and AsBsDs defined in Data Item No. 94003.

It should be noted that, for the panel as a whole, the pair of face plates must form a balanced lay-up about the central plane of the panel as shown in Sketch 1.1.

1.5 Statement of Basic Assumptions

Section 1.7 outlines the mathematical solution for the buckling problem provided by the program of Section 3. The solution is based upon elastic, thin plate, small deflection theory, in which the laminated face plates are assumed to behave as homogeneous orthotropic plates whose axes of orthotropy are aligned with the panel loading directions. The face plates are assumed to form a balanced lay-up about the central plane of the sandwich.

In the theory no account is taken of through-the-thickness shear deformations of the face plates. The faces are assumed to be initially flat, and to be held flat by the core until they wrinkle. The core is assumed to give continuous support to the faces and the possibility of localised buckling within honeycomb type cells (often called inter-cell buckling) is not considered. Also, the core is taken to have zero stiffness in the *x*- and *y*-directions but finite direct and shear stiffnesses in the *z*-direction. These assumptions make the analysis more applicable to honeycomb type cores than to solid foam cores, but should give conservative results for the latter. The applied load is assumed to be divided equally between the two face plates and to lie in the mid-plane of each face plate (which might include a layer of glue) and it is to that mid-plane that **B** and **D** are related.

It should be noted that the length and width of the panel do not appear in any of the equations. This is because it is convenient to work in terms of half wavelengths of the wrinkle, λ , it being assumed that the panel is large enough for the λ 's to achieve their minimum values. The program provides the values of λ for each case. However, the program requires input of approximate values for the panel dimensions. This is because a minimum may not exist for one of the λ 's and in this case the longest wave gives the lowest wrinkling load. In such cases the appropriate panel dimension is used in place of λ . This is often found to

be the case and this is discussed further in Section 1.6.

The equations and program given in this Data Item are based upon the theoretical work of Derivation 1 of Section 1.8 which employs the assumptions described above. Because of this, wrinkling loads obtained should only be used as a guide for design purposes. Wherever possible, verification of final designs should be obtained from wrinkling tests on specimens that are representative of the actual method of manufacture and configuration.

The equations and program can be applied to the simpler case of sandwiches with isotropic metallic face plates dealt with in Data Item No. 88032*. It is recommended that that Data Item should be consulted before a design is finalised since many of the factors shown to be important for sandwich panels with isotropic face plates will influence the wrinkling of panels faced with laminated composites.

1.6 Wrinkling Characteristics

Figures 1 to 4 show a set of wrinkling loads for a rectangular panel of length (in the x -direction) of 0.410 m and width (in y -direction) of 0.340 m. The face plates are of the AsBsDs type, and the geometry and material characteristics of the panel are given below.

Core:

$$\begin{aligned} h_t &= 0.030 \quad \text{m} \\ E_c &= 10 \times 10^6 \quad \text{N/m}^2 \\ G_{cx} &= 1.58 \times 10^6 \quad \text{N/m}^2 \\ G_{cy} &= 0.62 \times 10^6 \quad \text{N/m}^2 \end{aligned}$$

Face plates:

$$\begin{aligned} t &= 0.39 \quad \text{mm.} \\ \mathbf{A} &= \begin{bmatrix} 31.40 & 7.69 & 0 \\ & 11.40 & 0 \\ (sym) & & 8.45 \end{bmatrix} \times 10^6 \quad \text{N/m}, \quad \mathbf{B} = \begin{bmatrix} -2.80 & -0.685 & 0 \\ & -0.194 & 0 \\ (sym) & & -0.742 \end{bmatrix} \times 10^3 \quad \text{N}, \\ \mathbf{D} &= \begin{bmatrix} 0.357 & 0.094 & 0 \\ & 0.061 & 0 \\ (sym) & & 0.102 \end{bmatrix} \quad \text{N m.} \end{aligned}$$

The panel is subjected to biaxial compressive loads P_x and P_y and wrinkling loads are given for increasing values of the ratio P_y/P_x . Corresponding wavelengths of wrinkle, λ_x and λ_y , are also shown. Similar behaviour to that described in Sections 1.6.1 and 1.6.2 may be expected for other panels but this will vary according to the actual panel properties and thicknesses.

1.6.1 Symmetric wrinkling

Referring to Figure 1, it is seen that when P_y is zero (or $P_y/P_x = 0$) then the value of P_x required for wrinkling is given by $P_{xs} = 34\,000 \text{ N/m}$. The corresponding wavelengths are $\lambda_{xs} = 0.011 \text{ m}$ in the x -direction, and a large value for λ_{ys} , equal to the panel width, in the y -direction. Thus, the direction of the short-wave wrinkle is in the direction of the applied load P_x . This wrinkle pattern is maintained as the ratio of P_y/P_x is

* Data Item No. 88032 "Wrinkling of sandwich panel face plates (isotropic face plates on orthotropic cores, in-plane loading or bending)".

increased, and furthermore the value of P_{xs} remains constant at 34 000 N/m whilst P_y/P_x increases to a value of approximately 0.74. In fact, in this load region, the solution always corresponds to the largest value for λ_{ys} possible so that an upper limit is reached when λ_{ys} = panel width. The effect of limiting the value of λ_{ys} by setting it equal to varying panel widths is shown in Figure 3. Here P_{xs} is plotted against panel width for the special case where $P_y = 0$, and is asymptotic to 34 000 N/m for large values of panel width. Panels with small widths have the effect of increasing P_{xs} so that when panel width is equal to h_t , the value of P_{xs} is increased to 39 000 N/m. A similar behaviour is found when P_y is present, up to a value of $P_y = 0.74P_x$.

Referring back to Figure 1, when P_y/P_x is increased beyond 0.74, the wrinkling behaviour changes abruptly and it becomes dominated by a small wavelength of wrinkle in the y -direction equal to 0.0095 m with a large wavelength, equal to the panel length, in the x -direction. P_{ys} remains constant at approximately 25 000 N/m whilst P_{xs} decreases from 34 000 N/m to 8320 N/m.

In summary, the symmetric wrinkling behaviour can be described as one in which small wavelength wrinkles are developed either in the lengthwise direction or in the widthwise direction depending on the load ratio. In each case the wrinkle direction corresponds to the dominant load direction and the magnitude of the load for wrinkling remains constant over a certain range of load ratios. For this particular example, however, there is a transition region between $P_y/P_x = 0.74$ and 1.0 where P_{xs} is greater than P_{ys} even though the wrinkles are formed in the y -direction.

1.6.2 Antisymmetric wrinkling

The antisymmetric wrinkling behaviour is shown in Figure 2. The trends are similar to the symmetric wrinkling case described above with small wavelength wrinkles in the dominant load direction. Again there is a transition region; in this case between $P_y/P_x = 0.6$ and 1.0 where P_{xa} is greater than P_{ya} , with the wrinkles in the y -direction.

Comparing the magnitudes of the loads for symmetric and antisymmetric wrinkling, it is seen that, initially, the loads required for symmetric wrinkling are lower than those required for antisymmetric wrinkling. However, this situation soon changes, and at a load ratio $P_y/P_x = 0.6$ the value of P_{xa} drops off rapidly to about 32 000 N/m which compares with a value for $P_{xs} = 34 000$ N/m. The value of P_{xa} then remains well below P_{xs} for the remaining load ratio range.

Figure 4 shows the effect on the value of P_{xa} of setting λ_{ya} equal to the panel width with $P_y = 0$. For large panel widths, P_{xa} is asymptotic to 41 200 N/m but rises rapidly to 47 000 N/m at a value of panel width equal to h_t . Thus, in cases where wavelengths are constrained in one direction by a panel dimension, the wrinkling loads would be increased beyond the minimum values that would have been obtained for an infinite plate.

1.6.3 Further considerations

Other characteristics may be studied by referring to previously issued Data Items on sandwich panel wrinkling. The influence of an adhesive layer that bonds the faces to the core is discussed in Data Item No. 87013*, Section 1.5, for sandwich columns and beams. Generally, the effect of the adhesive layer is to increase all the wrinkling loads, often by a significant amount for thin faces; it is important, therefore, to include information on the adhesive layer in the data input where possible so that its effect can be taken into account in the solution.

Further information can be obtained from Data Item No. 88032 which deals with the wrinkling of sandwich

* Data Item No. 87013 "Elastic wrinkling of sandwich columns and beams with unbalanced laminated fibre reinforced face plates (Face plates of AsBoDs, AsBIDs and AsBsDs types).

panels with isotropic face plates. This is a special case of the more general laminated faces considered in this present Data Item and the design curves presented in Data Item No. 88032 have been derived using a different computer program from that used here but which is based on the same theory, see Section 3.3. Curves are given that show the effect of core direct and shear moduli, core depth, face plate thickness and modulus on symmetric and antisymmetric wrinkling face plate stresses. These can be consulted to establish general trends for panels with laminated faces.

1.7 Solution Outline and Governing Equations

The final explicit equations that govern panel wrinkling behaviour are far too lengthy to be included in this Data Item. However, certain basic equations are given below from which it is possible to obtain the final equations if required. All equations may be obtained from the fundamental theory given in Derivation 1 but it should be noted that the sign convention for the B_{ij} terms is different, with B_{ij} (present Data Item) = $-B_{ij}$ (Derivation 1). It should also be noted that h (present Data Item) = $2h$ (Derivation 1).

1.7.1 Symmetric wrinkling

The solution for symmetric wrinkling is governed by the following three partial differential equations in the variables σ_{zo} , u_o and v_o .

$$\begin{aligned}\nabla_{11}^s u_o + \nabla_{12}^s v_o &= \nabla_{13}^s \sigma_{zo} \\ \nabla_{21}^s u_o + \nabla_{22}^s v_o &= \nabla_{23}^s \sigma_{zo}\end{aligned}\tag{1.3}$$

and

$$\nabla_{31}^s u_o + \nabla_{32}^s v_o = \nabla_{33}^s \sigma_{zo},$$

where the operators are defined by

$$\begin{aligned}\nabla_{11}^s &= -B_{11} \frac{\partial^3}{\partial x^3} - (B_{12} + 2B_{33}) \frac{\partial^3}{\partial x \partial y^2}, \\ \nabla_{12}^s &= -B_{22} \frac{\partial^3}{\partial y^3} - (B_{12} + 2B_{33}) \frac{\partial^3}{\partial x \partial y^2}, \\ \nabla_{13}^s &= \frac{h}{2E_c} \left\{ \left(D_{11} - \frac{B_{11}h}{4} \right) \frac{\partial^4}{\partial x^4} + \left[2(D_{12} + 2D_{33}) - \frac{h}{2}(B_{12} + 2B_{33}) \right] \frac{\partial^4}{\partial x^2 \partial y^2} \right. \\ &\quad \left. + \left(D_{22} - B_{22} \frac{h}{4} \right) \frac{\partial^4}{\partial y^4} + \frac{P_{sx}}{2} \frac{\partial^2}{\partial x^2} + \frac{P_{sy}}{2} \frac{\partial^2}{\partial y^2} + \frac{2E_c}{h} \right\}, \\ \nabla_{21}^s &= A_{11} \frac{\partial^2}{\partial x^2} + A_{33} \frac{\partial^2}{\partial y^2}, \\ \nabla_{22}^s &= (A_{12} + A_{33}) \frac{\partial^2}{\partial x \partial y}, \\ \nabla_{23}^s &= \frac{h}{2E_c} \left\{ \left(\frac{A_{11}h}{4} - B_{11} \right) \frac{\partial^3}{\partial x^3} + \left[\frac{h}{4}(A_{12} + 2A_{33}) - B_{12} - 2B_{33} \right] \frac{\partial^3}{\partial x \partial y^2} \right\},\end{aligned}$$

$$\nabla_{31}^s = (A_{12} + A_{33}) \frac{\partial^2}{\partial x \partial y},$$

$$\nabla_{32}^s = A_{22} \frac{\partial^2}{\partial y^2} + A_{33} \frac{\partial^2}{\partial x^2}$$

and

$$\nabla_{33}^s = \frac{h}{2E_c} \left\{ \left[\frac{h}{4} (A_{12} + 2A_{33}) - B_{12} - 2B_{33} \right] \frac{\partial^3}{\partial x^2 \partial y} + \left(\frac{h}{4} A_{22} - B_{22} \right) \frac{\partial^3}{\partial y^3} \right\}.$$

By successive operations, u_o and v_o may be eliminated from Equations (1.3) leaving the following single partial differential equation in σ_{zo} .

$$\left\{ \left(\nabla_{32}^s \nabla_{21}^s - \nabla_{22}^s \nabla_{31}^s \right) \left(\nabla_{22}^s \nabla_{13}^s - \nabla_{12}^s \nabla_{23}^s \right) - \left(\nabla_{22}^s \nabla_{11}^s - \nabla_{12}^s \nabla_{21}^s \right) \left(\nabla_{32}^s \nabla_{23}^s - \nabla_{22}^s \nabla_{33}^s \right) \right\} \sigma_{zo} = 0 \quad (1.4)$$

Equation (1.4) can then be solved for the critical buckling loads with the assumption that σ_{zo} varies as $\sin(\pi x/\lambda_{sx}) \sin(\pi y/\lambda_{sy})$.

1.7.2 Antisymmetric wrinkling

The solution for antisymmetric wrinkling is governed by the following three partial differential equations in the variables w_o , τ_{xz} , and τ_{yz} .

$$\begin{aligned} \nabla_{11}^A \tau_{xz} + \nabla_{12}^A \tau_{yz} &= \nabla_{13}^A w_o, \\ \nabla_{21}^A \tau_{xz} + \nabla_{22}^A \tau_{yz} &= \nabla_{23}^A w_o \end{aligned} \quad (1.5)$$

and

$$\nabla_{31}^A \tau_{xz} + \nabla_{32}^A \tau_{yz} = \nabla_{33}^A w_o,$$

where the operators are defined by

$$\begin{aligned} \nabla_{11}^A &= \frac{h^2}{8E_c} \left(\frac{A_{11}h}{6} - B_{11} \right) \frac{\partial^4}{\partial x^4} + \left[\frac{h^3}{48E_c} (A_{12} + 2A_{33}) - \frac{h^2}{8E_c} (B_{12} + 2B_{33}) \right] \frac{\partial^4}{\partial x^2 \partial y^2} \\ &\quad + \frac{hA_{11}}{2G_{cx}} \frac{\partial^2}{\partial x^2} + \frac{hA_{33}}{2G_{cy}} \frac{\partial^2}{\partial y^2} - 1, \end{aligned}$$

$$\begin{aligned}
 \nabla_{12}^A &= \frac{h^2}{8E_c} \left(\frac{A_{11}h}{6} - B_{11} \right) \frac{\partial^4}{\partial x^3 \partial y} + \left[\frac{h^3}{48E_c} (A_{12} + 2A_{33}) - \frac{h^2}{8E_c} (B_{12} + 2B_{33}) \right] \frac{\partial^4}{\partial x \partial y^3} \\
 &\quad + \frac{h}{2G_{cy}} (A_{12} + A_{33}) \frac{\partial^2}{\partial x \partial y}, \\
 \Delta_{13}^A &= \left(\frac{A_{11}h}{2} - B_{11} \right) \frac{\partial^3}{\partial x^3} + \left[\frac{h}{2} (A_{12} + 2A_{33}) - B_{12} - 2B_{33} \right] \frac{\partial^3}{\partial x \partial y^2}, \\
 \nabla_{21}^A &= \frac{h^2}{8E_c} \left(\frac{A_{22}h}{6} - B_{22} \right) \frac{\partial^4}{\partial x \partial y^3} + \left[\frac{h^3}{48E_c} (A_{12} + 2A_{33}) - \frac{h^2}{8E_c} (B_{12} + 2B_{33}) \right] \frac{\partial^4}{\partial x^3 \partial y} \\
 &\quad + \frac{h}{2G_{cx}} (A_{12} + A_{33}) \frac{\partial^2}{\partial x \partial y}, \\
 \nabla_{22}^A &= \frac{h^2}{8E_c} \left(\frac{A_{22}h}{6} - B_{22} \right) \frac{\partial^4}{\partial y^4} + \left[\frac{h^3}{48E_c} (A_{12} + 2A_{33}) - \frac{h^2}{8E_c} (B_{12} + 2B_{33}) \right] \frac{\partial^4}{\partial x^2 \partial y^2} \\
 &\quad + \frac{hA_{22}}{2G_{cy}} \frac{\partial^2}{\partial y^2} + \frac{hA_{33}}{2G_{cy}} \frac{\partial^2}{\partial x^2} - 1, \\
 \nabla_{23}^A &= \left(\frac{A_{22}h}{2} - B_{22} \right) \frac{\partial^3}{\partial y^3} + \left[\frac{h}{2} (A_{12} + 2A_{33}) - B_{12} - 2B_{33} \right] \frac{\partial^3}{\partial x^2 \partial y}, \\
 \nabla_{31}^A &= \left(\frac{D_{11}h^2}{8E_c} - \frac{B_{11}h^3}{48E_c} \right) \frac{\partial^5}{\partial x^5} + \left[\frac{h^2}{4E_c} (D_{12} + 2D_{33}) - \frac{h^3}{24E_c} (B_{12} + 2B_{33}) \right] \frac{\partial^5}{\partial x^3 \partial y^2} \\
 &\quad + \left(\frac{D_{22}h^2}{8E_c} - \frac{B_{22}h^3}{48E_c} \right) \frac{\partial^5}{\partial x \partial y^4} + \left(\frac{h^2 P_{ax}}{16E_c} - \frac{hB_{11}}{2G_{cx}} \right) \frac{\partial^3}{\partial x^3} \\
 &\quad + \left[\left(\frac{h^2 P_{ay}}{16E_c} - \frac{h}{2G_{cx}} (B_{12} + 2B_{33}) \right) \right] \frac{\partial^3}{\partial x \partial y^2} + \frac{h}{2} \frac{\partial}{\partial x}, \\
 \nabla_{32}^A &= \left(\frac{D_{11}h^2}{8E_c} - \frac{B_{11}h^3}{48E_c} \right) \frac{\partial^5}{\partial x^4 \partial y} + \left[\frac{h^2}{4E_c} (D_{12} + 2D_{33}) - \frac{h^3}{24E_c} (B_{12} + 2B_{33}) \right] \frac{\partial^5}{\partial x^2 \partial y^3} \\
 &\quad + \left(\frac{D_{22}h^2}{8E_c} - \frac{B_{22}h^3}{48E_c} \right) \frac{\partial^5}{\partial y^5} + \left[\frac{h^2 P_{ax}}{16E_c} - \frac{h}{2G_{cy}} (B_{12} + 2B_{33}) \right] \frac{\partial^3}{\partial x^2 \partial y} \\
 &\quad + \left(\frac{h^2 P_{ay}}{16E_c} - \frac{hB_{22}}{2G_{cy}} \right) \frac{\partial^3}{\partial y^3} + \frac{h}{2} \frac{\partial}{\partial y}
 \end{aligned}$$

and

$$\begin{aligned} \nabla_{33}^A = & \left(D_{11} - \frac{hB_{11}}{2} \right) \frac{\partial^4}{\partial x^4} + [2(D_{12} + 2D_{33}) - h(B_{12} + 2B_{33})] \frac{\partial^4}{\partial x^2 \partial y^2} \\ & + \left(D_{22} - \frac{hB_{22}}{2} \right) \frac{\partial^4}{\partial y^4} + \frac{P_{ax}}{2} \frac{\partial^2}{\partial x^2} + \frac{P_{ay}}{2} \frac{\partial^2}{\partial y^2}. \end{aligned}$$

By successive operations, and with regard to reciprocal conditions with respect to the x- and y-directions, τ_{xz} and τ_{yz} may be eliminated from Equations (1.5) leaving the following single partial differential equation in w_o .

$$\left\{ \nabla_{33}^A \nabla_{11}^A \nabla_{22}^A - \nabla_{33}^A \nabla_{12}^A \nabla_{21}^A - \nabla_{11}^A \nabla_{32}^A \nabla_{23}^A - \nabla_{13}^A \nabla_{22}^A \nabla_{31}^A + \nabla_{13}^A \nabla_{32}^A \nabla_{21}^A + \nabla_{12}^A \nabla_{31}^A \nabla_{23}^A \right\} w_o = 0. \quad (1.6)$$

Equation (1.6) can then be solved for the critical buckling loads with the assumption that w_o varies as $\sin(\pi x/\lambda_{ax}) \sin(\pi y/\lambda_{ay})$.

1.8 Derivation and Reference

1.8.1 Derivation

1. PEARCE, T.R.A. Buckling of sandwich panels with laminated face plates. *The Aeronautical Quarterly*, Vol. 23, pp 148-160, May 1972.
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1.8.2 Reference

2. SULLINS, R.T. Manual for structural stability analysis of sandwich plates and shells.
SMITH, G.W. NASA CR-1457, Section 3.1, December 1969.
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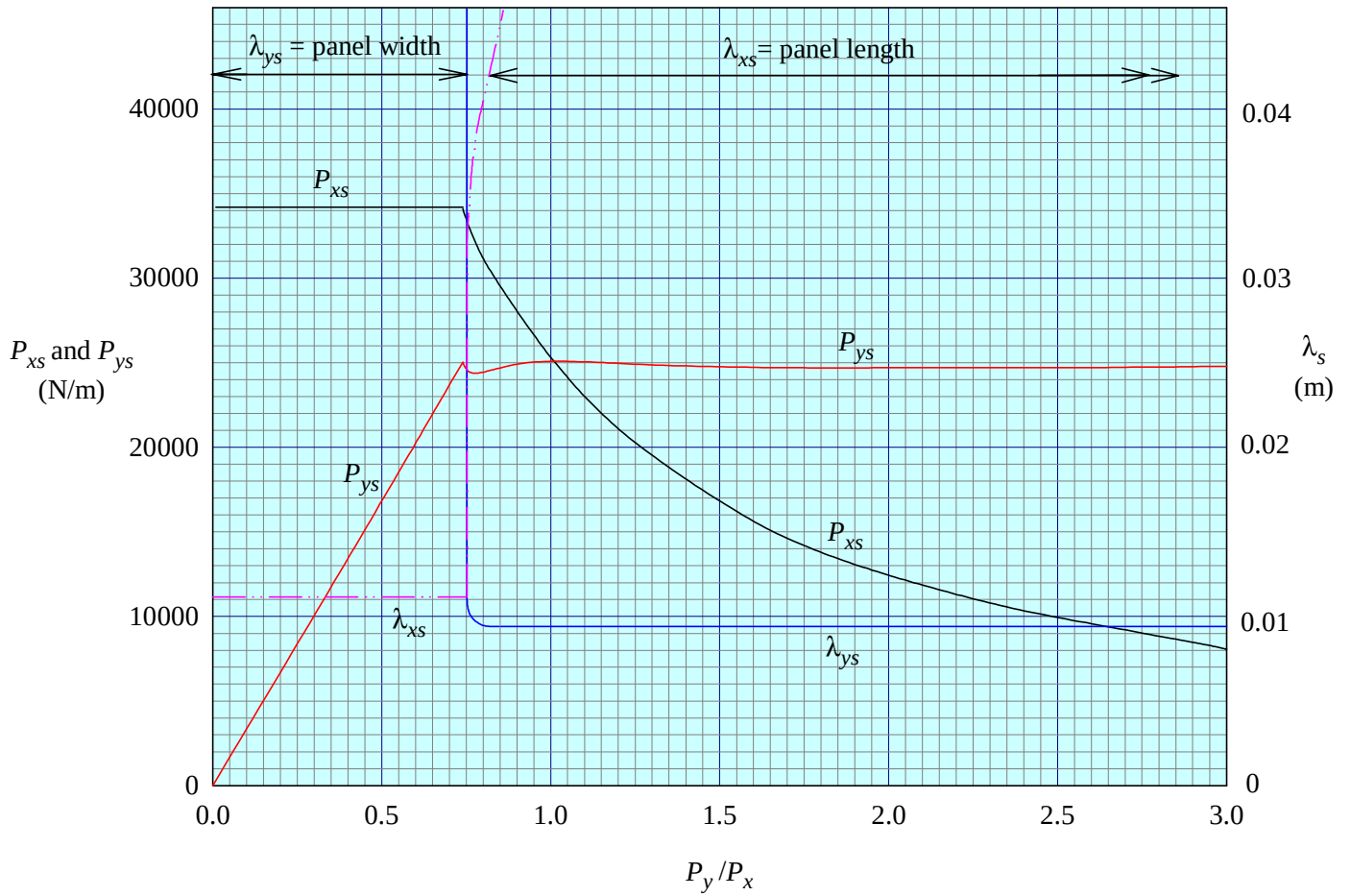


FIGURE 1 SYMMETRIC WRINKLING