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ESDU 76001

Calculation methods for along-wind loading. Part 3. Response of buildings and plate-like structures to atmospheric turbulence.

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THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item, which supersedes in part ESDU 76001, was monitored and guided by the Wind Engineering Panel. This Panel, which took over the work on wind engineering previously monitored by the Fluid Mechanics Steering Group, first met in 1979 and now has the following membership:

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CALCULATION METHODS FOR ALONG-WIND LOADING PART 3: RESPONSE OF BUILDINGS AND PLATE-LIKE STRUCTURES TO ATMOSPHERIC TURBULENCE

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CALCULATION METHODS FOR ALONG-WIND LOADING

Part 3: Response of Buildings and Plate-like Structures to Atmospheric Turbulence

1. NOTATION AND UNITS

		Units	
		SI	British
A	shadow area of structure normal to mean wind direction	m ²	ft ²
B_B, B_R^2	load velocity admittance factors (see Figures 5 and 6a - 6b and Appendix A)	—	—
B_{R1}^2, B_{R2}^2	components of B_R^2	—	—
C_{Dm}	average drag coefficient (see Section 5.6); $C_{Dm} = \text{total drag} / (\frac{1}{2} \rho V_m^2 A)$	—	—
C_{Dr}	local drag coefficient (see Section 5.6); $C_{Dr} = (\text{drag per unit area at } r) / (\frac{1}{2} \rho V_r^2)$	—	—
D	structure dimension, equal to W or H , used in evaluation of load/velocity admittance factors (see Figures 5 and 6a - 6b and Appendix A)	m	ft
d	displacement height of wind speed profile zero-plane (see Section 5.7 and Table 6.4)	m	ft
E_{mean}	mean load effect; $E_{mean} = \int_A \frac{1}{2} \rho V_r^2 C_{Dr} \beta_r dA$	*	*
E_{ref}	reference mean load effect [†] ; $E_{ref} = \int_A \frac{1}{2} \rho V_r^2 C_{Dr} \beta_r dA$	*	*
F_j	mean mode-generalised force; $F_j = \int_A \frac{1}{2} \rho V_r^2 C_{Dr} \mu_{jr} dA$	N	lbf
g_B, g_R	peak response factors for background and resonant components of response respectively	—	—
H	overall height of structure (see Sketches 1.1 and 4.1)	m	ft
H_i	vertical distance up structure at which load effect is required for cantilever structure or at which torsional axis is located (see Sketch 1.1)	m	ft
h_c	reference height (see Sketches 1.1, 4.1 and 4.2)	m	ft
h_0	height above ground of lowest point on structure (see Sketch 1.1)	m	ft
I_B	integral parameter; $I_B = \int_A m_r \mu_{1r} dA$	kg	[‡] slug

For footnotes see end of the Notation Table.

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I_{Rj}	integral parameter; $I_{Rj} = \int_A m_r \mu_{jr} \beta_r dA$	*	*
I_u	turbulence intensity of along-wind component of turbulence at height h_c $I_u = (\sigma_u/V_z)$	—	—
J_B	normalised background loading factor (see Appendix A)	—	—
J_{BH}, J_{BW}	background loading factor components corresponding to vertical and horizontal components respectively of structure influence function	—	—
J_R^2	normalised resonant loading factor (see Appendix A)	—	—
J_{RH}^2, J_{RW}^2	resonant loading factor components corresponding to vertical and horizontal components respectively of structure mode shape	—	—
K_V	mean wind speed factor; $K_V^2 = \int_0^1 (V_z/V_{hc})^2 d(z/H)$	—	—
K_ζ	aerodynamic damping factor (see Appendix A)	—	—
$K_{\zeta H}, K_{\zeta W}$	aerodynamic damping factor components corresponding to vertical and horizontal components respectively of structure mode shape	—	—
L	structure dimension; H or W as appropriate to component, vertical or horizontal, of parameter considered	m	ft
L_i	dimension H_i or W_i as appropriate	m	ft
^rL_u	lateral integral length scale, ^yL_u or ^zL_u , used in evaluation of load/velocity admittance factors (see Figures 5 and 6a - 6b and Appendix A)	m	ft
^xL_u	integral length scale of along-wind component of turbulence, measured in along-wind, x -, direction at height h_c	m	ft
$^yL_u, ^zL_u$	lateral integral length scales of along-wind component of turbulence, measured in y - and z -directions at height h_c	m	ft
l	distance along y - or z -axis as appropriate to component of mass distribution, mode shape or influence function being considered	m	ft
M	total mass of structure	kg	‡ slug
M_j	mode-generalised mass for mode j (see Section 5.2)	kg	‡ slug
m_l	total mass per unit length of structure (see Section 5.2)	kg/m	‡ slug/ft

For footnotes see end of the Notation Table.

m_r	total mass per unit face area of structure at location r (see Section 5.2)	kg/m ²	‡slug/ft ²
N_1, N_2	parameters in mode shape functions for first and second cantilever bending modes respectively (see Table 6.1)	—	—
n	frequency	Hz	Hz
n_j	natural frequency of mode j	Hz	Hz
R_j	reference mode-generalised force ^{**} ; $R_j = \int_A \frac{1}{2} \rho V_r^2 C_{Dr} \mu_{jr} dA$	N	lbf
S_{FjFj}	mode-generalised force spectral density for mode j at frequency n	N ² /Hz	lbf ² /Hz
S_{uu}	spectral density of along-wind turbulence component at frequency n	m ² /s ² Hz	ft ² /s ² Hz
V	hourly-mean wind speed	m/s	ft/s
V_z	hourly-mean wind speed at height z above velocity profile zero-plane	m/s	ft/s
V_m	averaged hourly-mean wind speed; $V_m^2 = V_{hc}^2 K_V^2$	m/s	ft/s
W	overall width of structure (see Sketches 1.1, 4.1 and 4.2)	m	ft
W_i	horizontal distance across structure at which load effect is required for cantilever structure or at which torsional axis is located (see Sketch 1.1)	m	ft
X_j	mode-generalised deflection for mode j , i.e. deflection at location where $\mu_j = 1$	m	ft
x, y, z	system of rectangular Cartesian co-ordinates with x -axis defined in mean wind direction and z -axis vertical, upwards	—	—
x_r	mean deflection at location r ; $x_r = \sum \mu_{jr} X_j$	m	ft
z	height above velocity profile zero-plane (see Section 5.7)	m	ft
z_0	terrain roughness parameter (see Section 5.7 and Table 6.4)	m	ft
β_r	overall influence function describing relationship between applied drag force and resulting load effect (see Section 5.3)	*	*
β_y, β_z	components of β_r along y - and z -axes respectively	*	*
ζ_a, ζ_s	aerodynamic and structural damping ratios, expressed as fractions of critical damping (see Appendix A and Section 5.5)	—	—

For footnotes see end of the Notation Table.

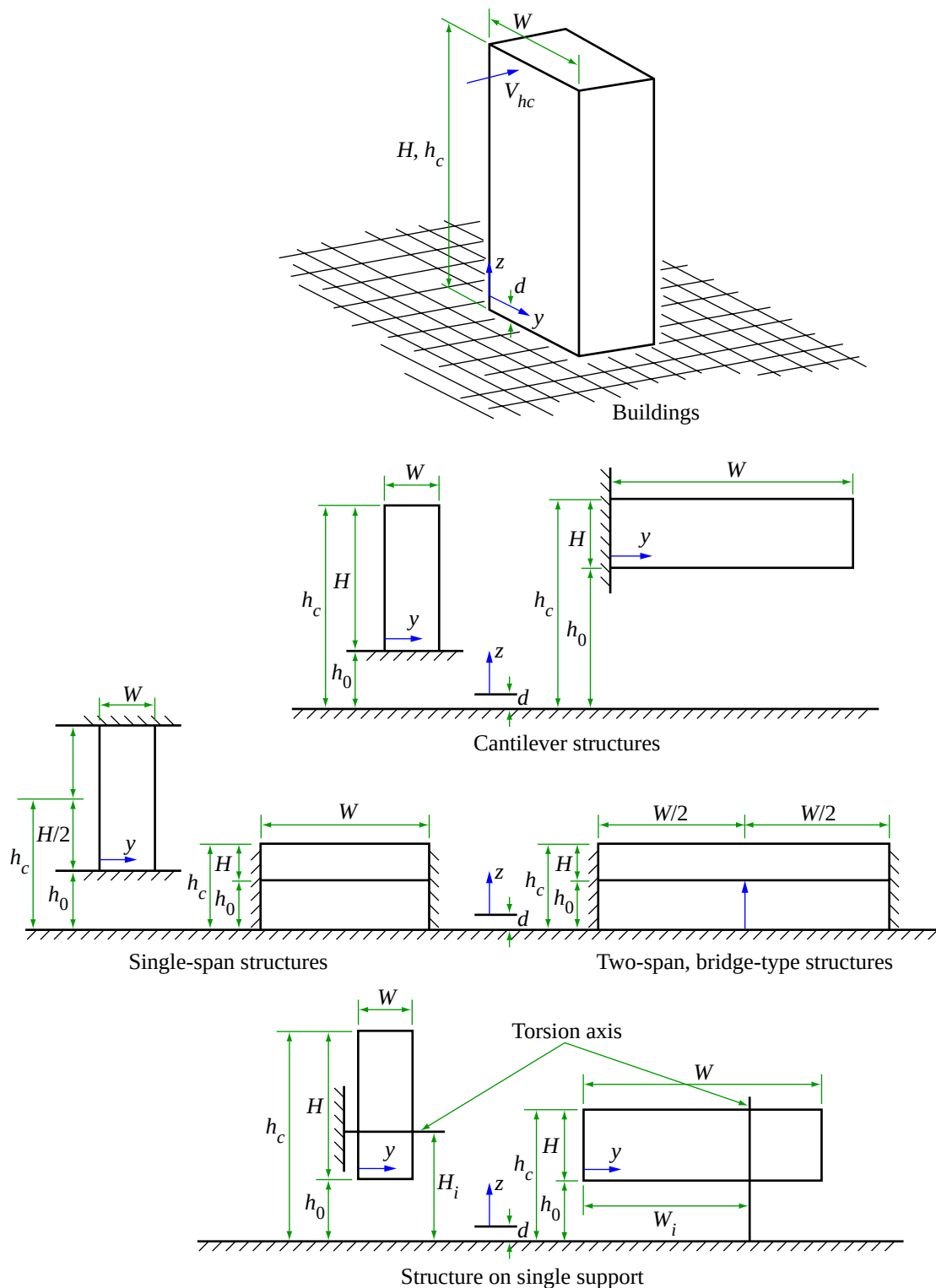
μ_{jr}	overall mode shape function describing deflected shape of structure in mode j (see Section 5.1)	—	—
μ_{jy}, μ_{jz}	components of μ_{jr} along y - and z -axes respectively	—	—
ρ	air density	kg/m ³	‡slug/ft ³
σ_E	standard deviation of fluctuating load effect	*	*
σ_{Xj}	standard deviation of mode-generalised deflection, <i>i.e.</i> deflection at location where $\mu_j = 1$	m	ft
$\sigma_{\ddot{X}}$	standard deviation of mode-generalised acceleration, <i>i.e.</i> acceleration at location where $\mu_j = 1$; $\sigma_{\ddot{X}} = (2\pi n_j)^2 \sigma_{Xj}$	m/s ²	ft/s ²
$\sigma_{x,r}$	standard deviation of deflection at location r ; $\sigma_{x,r} = \sum \mu_{jr} \sigma_{Xj}$	m	ft
σ_u	standard deviation of along-wind component of turbulence	m/s	ft/s
ϕ_{uu}	normalised spectral function of along-wind component of turbulence at frequency n and height h_c ; $\phi_{uu} = n S_{uu} / \sigma_u^2$	—	—
Suffixes			
B	denotes quantity relating to background or non-resonant component of response		
hc	denotes value at height h_c		
i	see H_i , L_i or W_i		
$j, 1, 2$	denote values relating to general mode j and first and second modes		
$mean$	denotes quantity relating to mean component of response		
max	denotes expected maximum value		
R	denotes quantity relating to resonant component of response		
y, y', z, z'	denote values at locations y and y' and z and z' along y - and z -directions on structure		
r, r'	denote values at locations (y, z) and (y', z') , respectively, on structure		
rms	denotes root mean square value		

* Units depend on load effect considered.

† The notation $|\beta_r|$ denotes the absolute value of β_r .

‡ 1 slug = 32.174 lb (pounds mass).

** The notation $|\mu_{jr}|$ denotes the absolute value of μ_{jr} .



Sketch 1.1 Geometrical parameters for regular structures
(see Sketch 4.1 for application to irregular structures)

2. INTRODUCTION

The overall response of structures in the natural wind is the result of several causes.

This Item is one of a group concerned with methods of calculating the along-wind loading and response due to atmospheric turbulence. Part 1 (Derivation¹⁸) provides a background discussion of methods of calculation and their implementation. Part 2 (ESDU 87035¹⁹) provides a calculation procedure for the along-wind loading or response of line-like structures and Part 3 (this Item) provides corresponding procedures for buildings or plate-like structures. These three parts together supersede ESDU 76001, the principal changes being to incorporate the updated wind properties given in Derivations 15, 16 and 17 and the revised load/velocity admittance factors discussed in Section A3 of Appendix A. A computer program version of the calculation methods in Parts 2 and 3 will be issued as Part 4 of the series (Reference 12) to supersede ESDU 84034.

The along-wind loading or response of a structure in the natural wind can be expressed in terms of a mean component, due to the hourly-mean wind speed, and a fluctuating component, due to the turbulent fluctuations in wind speed about that mean. A full dynamic analysis for the fluctuating component is time-consuming but for most practical cases it can be replaced by an approximate method that treats the fluctuating response as the sum of a background (quasi-static) component and separate resonant components for each mode. This method is outlined in Derivation 18 and is described in more detail in Reference 2. It forms the basis of the calculation methods in this Item where it is further simplified by provision of precalculated loading factors for a range of wind and structure properties.

An alternative, more approximate, approach is the gust method in which dynamic effects are ignored and a gust of an appropriate duration (dependent on the size of the structure) is assumed to act on the whole structure at a given time. The overall loading is then calculated on a quasi-static basis. This approach is appropriate only for structures for which the resonant component of response is negligible and is used for certain classes of structure in some national Codes of Practice. The gust method is discussed further in Derivation 18.

Section 4 of the present Item gives a step-by-step method for calculating the along-wind loading or response, in terms of a mean component and rms fluctuating components. With the assumption that the fluctuating wind speed and the response to it are stationary random processes, expected maximum values of the along-wind component of loading or response can then be estimated. The normalised loading factors required in the method are presented graphically and in equation form. The required input data are discussed in Section 5. Section 8 gives worked examples of the application of the method.

3. APPLICABILITY AND LIMITATIONS

The application of this Data Item is to single, isolated structures of generally prismatic or plate-like shape in the atmospheric wind. Such structures include buildings and towers, and some bridge and gantry-type structures.

The data apply for flow near normal to a flat face of a structure or, more generally, for flow along an axis of symmetry so that the across-flow force coefficient (usually denoted by C_L or C_C) is near zero*. Only fundamental and some second mode responses are covered by the data presented but the method could be applied more generally by integration of the appropriate equations in Appendix A.

* The essential requirement is that the fluctuating drag force due to the along-wind component of wind-speed fluctuations can be assumed uncorrelated with drag fluctuations from other sources e.g. vortex shedding or across-wind motion.

The ranges of parameters covered by the data used to generate Figures 2a - 2g and 3a - 3h are given in Appendix A. The curves in this Item reproduce the calculated background loading factor, J_B , to within about ± 10 per cent over a practical range of parameters. The resonant loading factor, J_R , is reproduced to within about ± 20 per cent. This would allow the overall predicted response to be estimated to within about ± 10 per cent in the absence of other errors. However, there are few data on which to base the admittance factors (B_B and B_R) and their uncertainty cannot be reliably assessed. The uncertainty of the calculated response is also influenced considerably by the uncertainty in the input data, which cannot be quantified here.

For heights below about $1.5d$, the wind properties, especially wind speed and direction, are subject to strong local effects. For structures that are wholly or largely below this height, results obtained from this Item should be applied with caution unless derived with wind data specific to the location considered.

Interference between structures in groups may modify the buffeting response but this effect is not considered here. Interference effects may be expected to be small if a structure is spaced more than 3 structure widths upstream or to one side of a similar structure or more than 10 structure widths downstream of a similar structure.

4. CALCULATION PROCEDURES FOR ALONG-WIND LOAD EFFECT OR RESPONSE

Graph-based calculation procedures* are given here, together with a Calculation Sheet (Section 4.3) on which the results may be recorded. The input data are discussed in Section 5 and approximate values for some parameters are given in Section 6. Worked examples of the application of the procedure are given in Section 8.

The buffeting response will usually be required for several wind speeds including the design wind speed and values corresponding to, for example, peak vortex-shedding response and serviceability criteria. The calculations in Steps (1) to (15) will be required for each wind-speed case although obviously some data will be common to all cases. It is also recommended that the effects of uncertainty in the input data, especially mass, damping and drag coefficient, be considered.

If accelerations only are required, Steps (3) to (6) may be ignored as usually only the resonant component contributes significantly.

Graphical data are presented for the mode shape and influence function components covered by Tables 6.1 and 6.2. If the required case is not covered exactly, the exact values may be used to evaluate E_{mean} or F_j , E_{ref} and R_j and M_j , I_B and I_{Rj} with the nearest approximations in the Tables used as a guide to the appropriate parts of Figures 2a - 2g, 3a - 3h and 7 to be used.

4.1 Structures of Constant Width and Cross-section

4.1.1 Determine the input data

(1) The following input data are required:

H	overall height of structure (Sketches 1.1, 4.1 and 4.2)	
W	overall width of structure (Sketches 1.1, 4.1 and 4.2)	
H_i or W_i	location at which load effect is required or at which torsion axis is located.	
M , m_l or m_r	mass or mass distribution (Section 5.2)	
n_j	natural frequency (Sections 5.1 and 6.1)	
μ_{jy} , μ_{jz}	mode shape function components (Section 5.1 and Table 6.1)	
β_y , β_z	influence function components (Section 5.3 and Table 6.2)	
ζ_s	structural damping ratio (Section 5.5 and Table 6.3)	
C_{Dm} or C_{Dr} -vs- r	drag coefficient (Section 5.6)	
h_c	† reference height (Sketches 1.1, 4.1 and 4.2)	
z_0	† terrain roughness at site and upwind (Section 5.7 and Table 6.4)	
d	† displacement height of wind-speed profile zero-plane (Section 5.7 and Table 6.4)	
V_{hc}	hourly-mean wind speed at height h_c	} for each wind-speed case (Section 5.7)
I_u	turbulence intensity at height h_c	
^xL_u , ^yL_u and ^zL_u	turbulence length scales at height h_c	
V_z -vs- z	hourly-mean wind-speed profile	

† These data are required in the calculation of the wind properties (Section 5.7).

* Equations for the curves are given in Appendix B and a computer program version (Part 4 of the series)¹² is available.

- (2) Calculate the mode-generalised mass from*

$$M_j = \int_A m_r \mu_{jr}^2 dA = \int_0^H \int_0^W m_r \mu_{jy}^2 \mu_{jz}^2 dy dz$$

or, if appropriate (see Section 5.2), from*

$$M_j = L \int_0^1 m_l \mu_{jl}^2 d\frac{l}{L} \text{ or } M_j = M \int_0^1 \mu_{jz}^2 d\frac{z}{H} \int_0^1 \mu_{jy}^2 d\frac{y}{W},$$

Repeat Step (2) for a second mode ($j = 2$) generalised mass if required.

4.1.2 Calculate the mean component of response

- (3) Calculate the mean load effect or the mean deflection as appropriate.

Calculate the mean load effect from

$$E_{mean} = \int_A \frac{1}{2} \rho V_r^2 C_{Dr} \beta_r dA = \int_0^H \int_0^W \frac{1}{2} \rho V_z^2 C_{Dr} \beta_y \beta_z dy dz$$

where C_{Dr} is a local drag coefficient at location r or, assuming a constant value of C_D over the structure, from†

$$E_{mean} = \frac{1}{2} \rho V_{hc}^2 C_{Dm} WH \int_0^1 (V_z/V_{hc})^2 \beta_z d\frac{z}{H} \int_0^1 \beta_y d\frac{y}{W}$$

where C_{Dm} is an appropriate mean drag coefficient. For cantilever structures, if the load effect is required for $L_i \neq 0$, the integration limits should be taken from L_i to L (or L_i/L to 1) in the appropriate direction only, where $L = H$ or W as appropriate.

Calculate the mean mode-generalised force from*

$$F_j = \int_A \frac{1}{2} \rho V_r^2 C_{Dr} \mu_{jr} dA = \int_0^H \int_0^W \frac{1}{2} \rho V_z^2 C_{Dr} \mu_{jy} \mu_{jz} dy dz$$

or, assuming a constant value of C_D over the structure, from*

$$F_j = \frac{1}{2} \rho V_{hc}^2 C_{Dm} WH \int_0^1 (V_z/V_{hc})^2 \mu_{jz} d\frac{z}{H} \int_0^1 \mu_{jy} d\frac{y}{W}.$$

* Values of $\int \mu_j$ and $\int \mu_j^2$ are included in Table 6.1. If integration is necessary, a Simpson's rule approximation will usually be adequate, see Section 8 for example application.

† For $\beta_z = 1$, the first integral reduces to $\int_0^1 (V_z/V_{hc})^2 d(z/H) = K_V^2$, which can be obtained for uniform upwind terrain from Figure 1. Values of $\int \beta$ are included in Table 6.2. If integration is necessary, a Simpson's rule approximation will usually be adequate, see Section 8 for example application.

The mean deflection is given by

$$X_{jmean} = \frac{F_j}{(2\pi n_j)^2 M_j}.$$

For deflections, repeat Step (3) for a second mode response ($j = 2$) if required.

4.1.3 Calculate the background or non-resonant component of response

- (4) Calculate the reference mean load effect. If β_r is always positive, as is usual except for torsional moments, $E_{ref} = E_{mean}$. Otherwise, calculate E_{ref} from

$$E_{ref} = \int_A \frac{1}{2} \rho V_r^2 C_{Dr} |\beta_r| dA = \int_0^H \int_0^W \frac{1}{2} \rho V_z^2 C_{Dr} |\beta_y \beta_z| dy dz$$

or, assuming a constant value of C_D over the structure, from*

$$E_{ref} = \frac{1}{2} \rho V_{hc}^2 C_{Dm} W H \int_0^1 (V_z/V_{hc})^2 |\beta_z| d\frac{z}{H} \int_0^1 |\beta_y| d\frac{y}{W}$$

where $|\beta|$ denotes the absolute value[†] of β .

- (5) Calculate H^2/L_u and W^2/L_u and determine the normalised background loading factor components, J_{BH} and J_{BW} , from the parts of Figures 2a - 2g corresponding to the vertical and horizontal influence function components, β_z and β_y . Use Figure 2a for all components having $\beta = 1.0$.
- (6) Determine the background load/velocity admittance factor, B_B , from Figure 5, taking $D/L_u = W^2/L_u$ if $W \leq H$ and $D/L_u = H^2/L_u$ if $H < W$.
- (7) Calculate the rms non-resonant component of load effect from

$$\sigma_{E,B} = E_{ref} \times 2B_B I_u (1 + 0.5I_u) J_B$$

where[‡] $J_B \approx 1.05(J_{BH} \times J_{BW})$.

If the deflection response is required, calculate $\sigma_{E,B}$ for $\beta_y = 1.0 = \beta_z$ and the integral parameter I_B from**

$$I_B = \int_A m_r \mu_{1r} dA$$

* For $\beta_z = 1$, the first integral reduces to $\int_0^1 (V_z/V_{hc})^2 d(z/H) = K_V^2$ which can be obtained for uniform upwind terrain from Figure 1. Values of $\int |\beta|$ are included in Table 6.2. If integration is necessary, a Simpson's rule approximation will usually be adequate, see Section 8 for example application.

† If β has both positive and negative values, the mean load effect, E_{mean} , can become very small or even zero, leading to computational difficulties in the evaluation of J_B . Use of $|\beta_r|$ in E_{ref} avoids this problem.

‡ See Section A2 of Appendix A for discussion of the factor 1.05 in this equation.

** Values of $\int \mu_j$ are included in Table 6.1. If integration is necessary, a Simpson's rule approximation will usually be adequate, see Section 8 for example application.

or, if appropriate (see Section 5.4), from*

$$I_B = L \int_0^1 m_l \mu_{1l} d\frac{l}{L} \quad \text{or} \quad I_B = M \int_0^1 \mu_{1z} d\frac{z}{H} \int_0^1 \mu_{1y} d\frac{y}{W}.$$

Hence calculate the non-resonant contribution to the deflection, evaluated¹⁸ as a mode-generalised deflection in mode 1, from

$$\sigma_{X,B} = \frac{\sigma_{E,B}}{4\pi^2 n_1^2 I_B}.$$

4.1.4 Calculate the resonant component of response

- (8) Calculate the reference mode-generalised force. If μ_{jr} is always positive, $R_j = F_j$. Otherwise, calculate R_j from*

$$R_j = \int_A \frac{1}{2} \rho V_r^2 C_{Dr} |\mu_{jr}| dA = \int_0^H \int_0^W \rho V_z^2 C_{Dr} |\mu_{jr} \mu_{jz}| dy dz$$

or, assuming a constant value of C_D over the structure, from*

$$R_j = \frac{1}{2} \rho V_{hc}^2 C_{Dm} WH \int_0^1 (V_z/V_{hc})^2 |\mu_{jz}| d\frac{z}{H} \int_0^1 |\mu_{jy}| d\frac{y}{W}$$

where $|\mu_{jr}|$ denotes the absolute value* of μ_{jr} .

- (9) Calculate $n_j H/V_{hc}$ and $n_j W/V_{hc}$ and determine the normalised resonant loading factor components, J_{RH}^2 and J_{RW}^2 , from the parts of Figures 3a - 3h corresponding to the vertical and horizontal mode shape components, μ_{jz} and μ_{jy} .
- (10) Calculate $n_j^x L_u/V_{hc}$ and determine the corresponding normalised spectral function ϕ_{uu} , from Figure 4.
- (11) Determine the resonant admittance factor components, B_{R1}^2 from Figure 6a and B_{R2}^2 from Figure 6b, taking $D = W$ and $D/L_u = W/L_u$ if $W \leq H$ and $D = H$ and $D/L_u = H/L_u$ if $H < W$. Hence calculate $B_R^2 = B_{R1}^2 + B_{R2}^2$.
- (12) Evaluate the spectral function of the resonant loading from†

$$n_j S_{FjFj} = R_j^2 \times 4 B_R^2 I_u^2 J_R^2 \phi_{uu}$$

$$\text{where}^\ddagger J_R^2 \approx (J_{RH}^2 \times J_{RW}^2)^{0.94}.$$

* If μ has both positive and negative values, the mode-generalised force based on μ_{jr} can become very small or even zero, leading to computational difficulties in the evaluation of J_R^2 . Use of $|\mu_{jr}|$ in R_j avoids this problem.

† Values of $\int \mu_j$ are included in Table 6.1. If integration is necessary, a Simpson's rule approximation will usually be adequate, see Section 8 for example application.

‡ See Section A2 of Appendix A for discussion of the exponent 0.94 in this equation.

- (13) Determine the normalised aerodynamic damping factor components $K_{\zeta H}$ and $K_{\zeta W}$ from the parts of Figure 7 corresponding to the vertical and horizontal mode shape components, μ_{jz} and μ_{jy} .
- (14) Calculate the aerodynamic damping from

$$\zeta_a = \frac{R_j \times K_{\zeta}}{2\pi n_j M_j V_{hc}}$$

where $K_{\zeta} = K_{\zeta H} \times K_{\zeta W}$.

- (15) Calculate the resonant component of the required response parameter.

For load effects calculate the integral parameter I_{Rj} from*

$$I_{Rj} = \int_A m_r \mu_{jr} \beta_r dA = \int_0^H \int_0^W m_r \mu_{jy} \beta_y \mu_{jz} \beta_z dy dz$$

or, if appropriate (see Section 5.4), from†

$$I_{Rj} = L \int_0^1 m_l \mu_{jl} \beta_l d\frac{l}{L} \text{ or } M \int_0^1 \mu_{jz} \beta_z d\frac{z}{H} \int_0^1 \mu_{jy} \beta_y d\frac{y}{W}.$$

Hence calculate $\sigma_{E, R} \{j\}$ from

$$\sigma_{E, R} \{j\} = \sqrt{\frac{\pi n_j S_{FjFj}}{4(\zeta_s + \zeta_a)}} \frac{I_{Rj}}{M_j}.$$

For deflections, calculate the mode-generalised deflection from

$$\sigma_{X, R} \{j\} = \sqrt{\frac{\pi n_j S_{FjFj}}{4(\zeta_s + \zeta_a)}} \frac{1}{(2\pi n_j)^2 M_j}.$$

For accelerations, calculate the mode-generalised acceleration from

$$\sigma_{\ddot{X}, R} \{j\} = \sqrt{\frac{\pi n_j S_{FjFj}}{4(\zeta_s + \zeta_a)}} \frac{1}{M_j}.$$

Repeat Steps (8) to (15) for a second mode response ($j = 2$) if required.

* A Simpson's rule approximation will usually be adequate for these integrals, see Section 8 for example application.

4.1.5 Estimate overall along-wind response

(16) The overall along-wind rms load effect is given by

$$\sigma_E = \sqrt{\sigma_{E,B}^2 + \sum_j (\sigma_{E,R} \{j\})^2}$$

where¹⁸ the contributions are calculated at the same wind speed.

The overall along-wind expected maximum load effect is given by

$$E_{max} = E_{mean} + \sqrt{(g_B \sigma_{E,B})^2 + \sum_j (g_R \sigma_{E,R} \{j\})^2}$$

$$\text{where } g_B \approx 3.5 \text{ and } g_R \approx \sqrt{2 \ln(n_j T_0)} + \frac{0.577}{\sqrt{2 \ln(n_j T_0)}} \quad (4.1)$$

and T_0 is taken as 3600 seconds when V_z is the hourly-mean wind speed.

The overall along-wind rms deflection response at location r is given by

$$\sigma_{x,r} = \sqrt{(\mu_{1r} \sigma_{X,B})^2 + \sum_j (\mu_{jr} \sigma_{X,R} \{j\})^2}$$

where the contributions are calculated at the same wind speed.

The overall along-wind expected maximum deflection is given by

$$x_{r,max} = \sum_j \mu_{jr} X_{jmean} + \sqrt{(g_B \mu_{1r} \sigma_{X,B})^2 + \sum_j (g_R \mu_{jr} \sigma_{X,R} \{j\})^2}$$

where $g_B \approx 3.5$ and g_R is given by Equation (4.1).

The overall along-wind rms acceleration at location r is given by

$$\sigma_{\ddot{x},r} = \sqrt{\sum_j (\mu_{jr} \sigma_{\ddot{X},R} \{j\})^2}$$

The overall along-wind expected maximum acceleration is given by

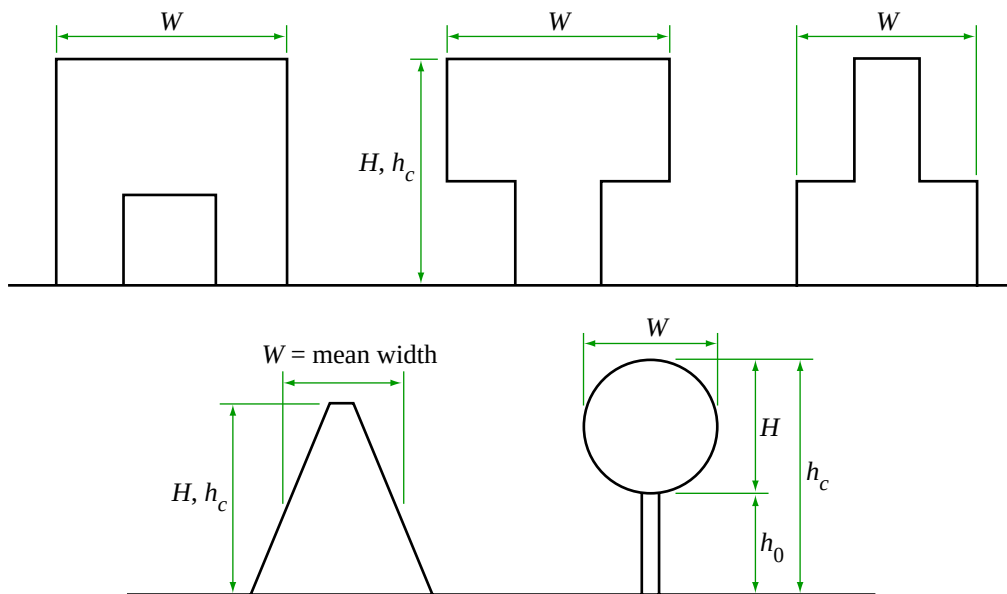
$$\ddot{x}_{r,max} = \sqrt{\sum_j (g_R \mu_{jr} \sigma_{\ddot{X},R} \{j\})^2}$$

where g_R is given by Equation (4.1).

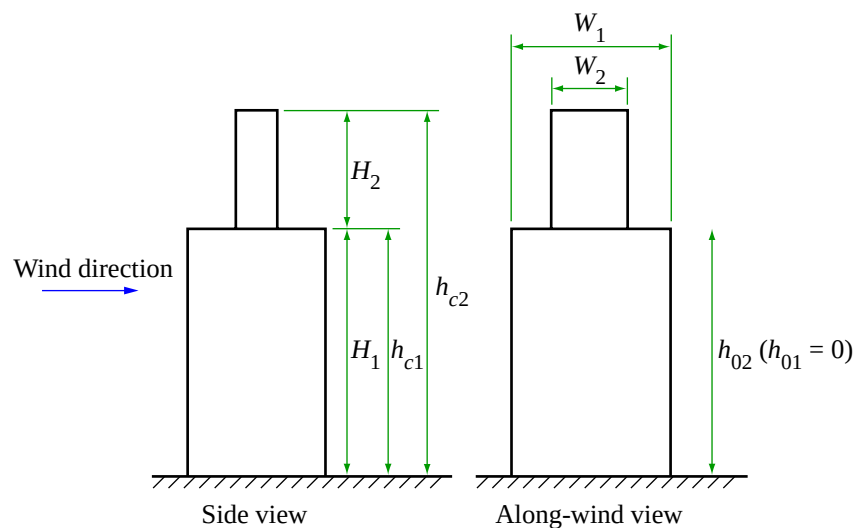
4.2 Structures of Varying Width or Cross Section

Some structures are non-uniform in width but of nearly constant streamwise depth or cross section, such as the examples shown in the along-wind views in Sketch 4.1. For such structures the use of J_B and J_R^2 factors based on the overall dimensions of the structure may be expected to overestimate the fluctuating loading or response. The mean and reference forces should, however, be calculated for the actual solid area of the structure.

For composite structures, such as shown in Sketch 4.2 for example, it is suggested tentatively that the responses of the components be evaluated separately and combined to give the overall response.



Sketch 4.1 Non-uniform plate-like (or approximately constant depth) structures



Sketch 4.2 Composite structure example

4.3 Calculation Sheet for Estimation of Along-wind Buffeting Response

TABLE 4.1

(1)	H	m	(4)	n_j	Hz	(6)	ζ_s		
(2)	W	m	(5a)	μ_{jy}		(7a)	β_y		
(3)	H_i or W_i	m	(5b)	μ_{jz}		(7b)	β_z		
(8a)*	$\int_0^1 (V_z/V_{hc})^2 \beta_z d\frac{z}{H} \int_0^1 \beta_y d\frac{y}{W}$								
(8b)*	$\int_0^1 (V_z/V_{hc})^2 \beta_z d\frac{z}{H} \int_0^1 \beta_y d\frac{y}{W}$								
(9a)*	$\int_0^1 (V_z/V_{hc})^2 \mu_{jz} d\frac{z}{H} \int_0^1 \mu_{jy} d\frac{y}{W}$								
(9b)*	$\int_0^1 (V_z/V_{hc})^2 \mu_{jz} d\frac{z}{H} \int_0^1 \mu_{jy} d\frac{y}{W}$								
(10)†	M_j (Section 5.2) kg								
(11)	V_{hc}	m/s							
(12)	I_u								
(13a)	$^x L_u$	m							
(13b)	$^y L_u$	m							
(13c)	$^z L_u$	m							
(14)	C_{Dm}								
(15)‡	$E_{mean} = \frac{1}{2} \rho \times (11)^2 \times (14) \times (2) \times (1) \times (8a)$								
(16)‡†	$F_j = \frac{1}{2} \rho \times (11)^2 \times (14) \times (2) \times (1) \times (9a)$								
(17)‡†	$X_{jmean} = (16)/[4\pi^2 \times (4)^2 \times (10)]$								
(18)	$E_{ref} = \frac{1}{2} \rho \times (11)^2 \times (14) \times (2) \times (1) \times (8b)$								
(19a)	$H/\sqrt[4]{L_u} = (1)/(13c)$								
(19b)	$W/\sqrt[4]{L_u} = (2)/(13b)$								
(20a)	J_{BH} (Figure 2a - 2g) for (19a) and β_z function								
(20b)	J_{BW} (Figure 2a - 2g) for (19b) and β_y function								
(21)	B_B (Figure 5)								
(22)	$\sigma_{E,B} = 2 \times (18) \times [1.05 \times (20a) \times (20b)] \times (12) \times (21) \times [1 + 0.5 \times (12)]$								
(23)‡	I_B (Section 5.4)								
(24)‡	$\sigma_{X,B} = (22)/[4\pi^2 \times n_1^2 \times (23)]$								

* If β is always positive, (8b) = (8a) and if μ is always positive, (9b) = (9a).

† Repeat these steps for second mode contributions if required.

‡ Select steps appropriate to required response.

TABLE 4.1 (continued)

(25)	$R_j = \frac{1}{2}\rho \times (11)^2 \times (14) \times (2) \times (1) \times (9b)$			
(26a)	$n_j H/V_{hc} = (4) \times (1)/(11)$			
(26b)	$n_j W/V_{hc} = (4) \times (2)/(11)$			
(27a)	J_{RH}^2 (Figure 3a - 3h) for (26a) and μ_{jz} function			
(27b)	J_{RW}^2 (Figure 3a - 3h) for (26b) and μ_{jy} function			
(28)	$J_R^2 = [(27a) \times (27b)]^{0.94}$			
(29)	$n_j^x L_u/V_{hc} = (4) \times (13a)/(11)$			
(30)	ϕ_{uu} (Figure 4)			
(31a)	B_{R1}^2 (Figure 6a)			
(31b)	B_{R2}^2 (Figure 6b)			
(32)	$B_R^2 = (31a) + (31b)$			
(33)*	$n_j S_{FjFj} = 4 \times (25)^2 \times (28) \times (30) \times (12)^2 \times (32)$			
(34a)	$K_{\zeta H}$ (Figure 7) for μ_{jz} function			
(34b)	$K_{\zeta W}$ (Figure 7) for μ_{jy} function			
(35)	$\zeta_a = (25) \times [(34a) \times (34b)]/[2\pi \times (4) \times (10) \times (11)]$			
(36)	$\pi \times (33)/[4[(6) + (35)]]$			
(37)*	I_{Rj} (Section 5.4)			
(38)*	$\sigma_{E,R}\{j\} = \sqrt{(36) \times (37)/(10)}$			
(39)*	$\sigma_{X,R}\{j\} = \sqrt{(36)}/[4\pi^2 \times (4)^2 \times (10)]$			
(40)*	$\sigma_{\ddot{X},R}\{j\} = \sqrt{(36)}/(10)$			
(41)*	$g_R = \sqrt{2\ln[3600 \times (4)] + 0.577/\sqrt{2\ln[3600 \times (4)]}}$			
(42)*†	$\sigma_E = \sqrt{(22)^2 + (38)^2}$			
(43)*†	$E_{max} = (15) + \sqrt{[3.5 \times (22)]^2 + [(41) \times (38)]^2}$			
(44)*†	$\sigma_{x,r} = \sqrt{[\mu_{1r} \times (24)]^2 + [\mu_{jr} \times (39)]^2}$			
(45)	$\sigma_{\ddot{x},r} = \mu_{jr} \times (17) + \sqrt{[3.5 \times \mu_{1r} \times (24)]^2 + [(41) \times \mu_{jr} \times (39)]^2}$			
(46)*†	$x_{r,max} = \mu_{jr} \times (40)$			
(47)*†	$\ddot{x}_{r,max} = \sqrt{[(41) \times \mu_{jr} \times (40)]^2}$			

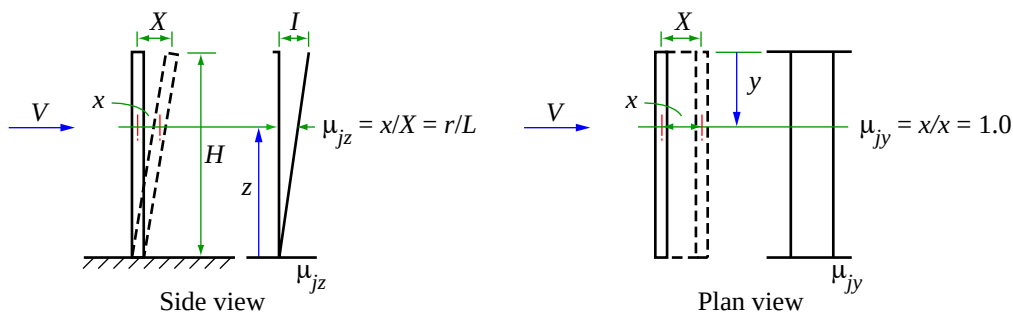
* Select steps appropriate to required response

† The mean and resonant components in these steps should be amended to include second mode responses also, if required. In that case repeat the appropriate steps from (4) to (10), (16), (17) and (25) to (41) and extend steps (42) to (47) to include the second mode responses.

5. INPUT DATA

5.1 Mode Shape Function (μ_j) and Natural Frequency (n_j)

The deflected shape of a structure oscillating in mode j is defined in terms of a non-dimensional mode shape function, μ_{jr} , with a maximum value of unity. For the prismatic or plate-like structures covered by this Item the mode shape can be expressed as the product of independent horizontal and vertical components, μ_{jy} and μ_{jz} , each with a maximum value of unity. For most simple mode shapes one of these components will be $\mu = 1.0$. For example if, as shown in Sketch 5.1, the deflected shape of a vertical cantilever structure of height H is assumed to be a linear function of the distance z up the structure, then $\mu_{jz} = z/H$ and $\mu_{jy} = 1.0$.



Sketch 5.1 Example of mode shape function

Where possible, calculated or measured natural frequencies and mode shapes should be used in evaluating the response. If sufficient information on mass and stiffness distribution is available, References 4, 5 and 6 may be used to obtain natural frequencies, mode shapes and modal parameters for cantilever structures. References 1 and 3 give extensive information on natural frequencies and mode shapes for various structures. Section 6.1 gives approximate expressions for the natural frequencies of cantilever bending modes of prismatic buildings.

Table 6.1 gives values of the component mode shape functions* covered by data in this Item. However, the normalised response factors are not sensitive to the exact form of μ assumed and may be used in conjunction with any broadly similar mode shapes.

5.2 Mass Distribution (m) and Mode-generalised Mass (M_j)

The mass distribution over the structure is required. This should include an allowance for masses, such as equipment, that are not part of the main structure but are permanently installed. If there is uncertainty as to the mass or its distribution, the effect of variations should be considered. The deflection or acceleration response is proportional approximately to $1/M$, whereas the load effects are affected only by the mass distribution. Increasing the mass without altering the stiffness will lower n_j and will also decrease the damping parameter ζ_s .

In many cases the mass distribution can be taken as uniform over a structure. If the mass distribution is non-uniform it can for most simple mode shapes be characterised adequately by the mass per unit length, m_l , along the direction in which μ varies, even when the distribution normal to that direction is also non-uniform. When considering the more complex mode shapes, however, e.g. twisting of cantilever structures, it may be necessary to use the mass per unit face area, m_r , to account for both longitudinal and lateral variations in mass distribution.

* These approximate mode shapes are not suitable for use in simple bending theory or in the determination of natural frequency because the slopes and curvatures, particularly at the base, are not necessarily correct.

The mass distribution and mode shape are used to evaluate the mode-generalised mass from

$$M_j = \int_A m_r \mu_{jr}^2 dA. \quad (5.1)$$

If the mass distribution can be described adequately by the mass per unit length and $\mu_{jr} = \mu_{jl} \times 1.0$, Equation (5.1) reduces to

$$M_j = L \int_0^1 m_l \mu_{jl}^2 d\frac{l}{L}, \quad (5.2)$$

where L is the structure dimension, H or W , as appropriate.

If the mass distribution is uniform over the structure, Equation (5.1) reduces to

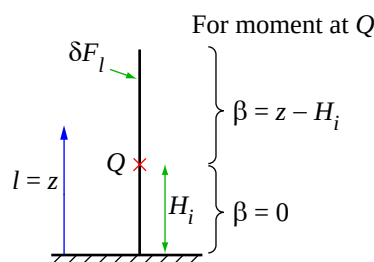
$$M_j = M \int_0^1 \mu_{jy}^2 d\frac{y}{W} \int_0^1 \mu_{jz}^2 d\frac{z}{H}, \quad (5.3)$$

where M is the total mass of the structure.

5.3 Influence Function (β)

The influence function, β_r , is defined by the ratio $\Delta E / \delta F_r$, where ΔE is the contribution to the load effect at some specified point on the structure resulting from the along-wind (drag) force, δF_r , on an element of structure at location r . It may be a function of the location of the loading point and may be discontinuous. The units of β will be N/N (or lbf/lbf) for shear forces and N m/N (or lbf ft/lbf) for moments, in the units given in the Notation section of this Item.

For the loading cases for prismatic or plate-like structures covered by this Item, the influence function can be expressed in terms of its vertical and horizontal components, β_z and β_y . Often, one of those components will be $\beta = 1.0$. For example, if E is the bending moment at a location Q along the cantilever structure in Sketch 5.2 then $\beta_z = z - H_i$ for $z \geq H_i$, $\beta_z = 0$ for $z \leq H_i$ and $\beta_y = 1.0$ for all values of y . For deflections or accelerations, $\beta_y = 1.0$ and $\beta_z = 1.0$ everywhere.



Sketch 5.2 Example of influence function

Table 6.2 gives values of the influence function components covered by data in this Item. If the precise function required is not included, values of the normalised loading parameters for the nearest approximation should be used.

5.4 Integral Parameters (I_B and I_{Rj})

It is convenient to define two integral parameters involving mass distribution, mode shape function and influence function. These are, firstly, for the estimation of background displacement from load effect,

$$I_B = \int_A m_r \mu_{1r} dA = \int_0^H \int_0^W m_r \mu_{1y} \mu_{1z} dy dz \quad (5.4)$$

and, secondly, for estimation of resonant load effect from deflection,

$$I_{Rj} = \int_A m_r \mu_{jr} \beta_r dA = \int_0^H \int_0^W m_r \mu_{jy} \beta_y \mu_{jz} \beta_z dy dz. \quad (5.5)$$

If the mass distribution can be described adequately by mass per unit length and $\mu_{jr} = \mu_{jl} \times 1.0$, Equation (5.4) reduces to

$$I_B = L \int_0^1 m_l \mu_{1l} d\frac{l}{L}. \quad (5.6)$$

If, in addition, $\beta_r = \beta_l \times 1.0$ and the distance l is the same for both μ and β , Equation (5.5) reduces to

$$I_{Rj} = L \int_0^1 m_l \mu_{jl} \beta_l d\frac{l}{L}. \quad (5.7)$$

If the mass distribution is uniform over the structure, Equations (5.4) and (5.5) reduce to

$$I_B = M \int_0^1 \mu_{1y} d\frac{y}{W} \int_0^1 \mu_{1z} d\frac{z}{H} \quad (5.8)$$

$$\text{and} \quad I_{Rj} = M \int_0^1 \mu_{jy} \beta_y d\frac{y}{W} \int_0^1 \mu_{jz} \beta_z d\frac{z}{H}. \quad (5.9)$$

5.5 Structural Damping Ratio (ζ_s)

Because of the considerable uncertainty concerning damping it is strongly recommended that the response is calculated for a range of values. Only the resonant component of response is affected and it is proportional approximately to $1/\zeta_s$. It should be noted that damping of structures and especially of buildings increases with response amplitude and may be lower at serviceability requirement conditions than at design wind speed response. Generally, steel-framed buildings exhibit lower damping than concrete-framed buildings and damping tends to decrease with increasing structure height. Section 9.1 of Reference 2 considers, briefly, some methods of increasing the damping of structures.

Typical ranges of the structural damping ratio*, ζ_s , are given in Table 6.3. ESDU 83009⁷ and References 8 and 9 give further information on damping of buildings.

5.6 Drag Coefficient (C_D)

Ideally, the variation of the local drag coefficient, C_{Dr} , over the structure should be taken into account in calculating the mean load effect, E_{mean} , reference mean load effect, E_{ref} , mode-generalised force F_j , and reference mode-generalised force, R_j . Such drag coefficient data are rarely available and appropriate mean

* The aerodynamic damping ratio, ζ_a , is calculated as part of the calculation procedure.

drag coefficients will have to be used in most cases.

For rectangular prismatic structures of aspect ratio greater than about 5 or 6, the total drag force in turbulent shear flow is given to a reasonable approximation by

$$\text{drag} \left(= \int_A \frac{1}{2} \rho V_r^2 C_{Dr} dA \right) = \frac{1}{2} \rho V_{hc}^2 C_{Du} \int_A (V_z/V_{hc})^2 dA = \frac{1}{2} \rho V_m^2 C_{Dm} A, \quad (5.10)$$

where C_{Du} is a uniform flow value. Where turbulence effects on C_{Du} can be accounted for, use of turbulence properties at the $\mu = 1$ location is recommended. From Equation (5.10) the mean drag coefficient can be defined as

$$C_{Dm} = C_{Du}. \quad (5.11)$$

Comparison of the use of mean and local values of C_D for subcritical circular cylinders suggests that for cantilever bending moments and typical cantilever mode shapes, use of C_{Dm} will not underestimate E_{mean} , E_{ref} , F_j and R_j .

If a mean drag coefficient, C_{Ds} , for a structure in a shear flow is available, the drag coefficient C_{Dm} may be calculated from

$$C_{Dm} = C_{Ds} V_s^2 / (K_V V_{hc})^2 \quad (5.12)$$

where V_s is the reference velocity used to derive C_{Ds} .

In evaluating C_{Du} it is recommended that for structures with step changes in cross section the aspect ratio be taken as (overall length)/(local width) for each section and for tapered structures as (overall length)/(mean width).

Reference 11 includes information on drag coefficients for a range of structure shapes.

5.7 Wind Speed and Turbulence Properties

The response will usually be required for several conditions including the design wind speed and conditions appropriate to serviceability criteria*. The wind speed profile V_z vs z and values, at height h_c , of the wind speed V_{hc} and turbulence properties I_u , ^xL_u , ^yL_u and ^zL_u are required as input data for each condition.

The required wind properties can be calculated conveniently using the computer program† in ESDU 92032¹⁰. The program will output directly the properties at the required height above ground, h_c . Alternatively the appropriate properties may be obtained from the tables† in ESDU 84011, 84030 and 86035 in Reference 10, using the equations in Section 3.1 of ESDU 86035 to obtain the lateral length scales. In those Items the data are tabulated for values of height, z , measured from the displaced zero-plane of the atmospheric boundary layer (Sketch 5.3). The required value of z is given by $h_c - d$.

Changes in terrain roughness upwind of the site can be allowed for in applying the computer program or ESDU 84011, 84030 and 86035.

* Design wind speeds are usually related to considerations of structural integrity whereas serviceability criteria relate to the occupant comfort and equipment operation at lower, more frequently occurring, wind speeds.

† Both the computer program and the tabulated wind property data are based on the wind model in Derivations 15, 16 and 17, which was used to derive the loading factors for this Item.