# ESDU 88022

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# Pipeline Vibrations
## Undamped natural vibration of pipelines

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# PIPELINE VIBRATIONS
# UNDAMPED NATURAL VIBRATION OF PIPELINES

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# PIPELINE VIBRATIONS
## Undamped Natural Vibration of Pipelines

## 1. NOTATION AND UNITS

| | | *SI* | *British* |
|---|---|---|---|
| $A$ | pipe flow area | $m^2$ | $ft^2$ |
| $A_b$ | cross-sectional area of pipe through bend | $m^2$ | $ft^2$ |
| $A_p$ | cross-sectional area of pipe wall | $m^2$ | $ft^2$ |
| $a$ | speed of propagation of pressure waves (wave speed) in fluid | m/s | ft/s |
| $D$ | internal diameter of pipe ( $= 4A/P_E$ for non-circular pipes) | m | ft |
| $E$ | Young's modulus of elasticity of pipe wall material | $N/m^2$ | $lbf/ft^2$ |
| $f_j$ | natural frequency of mode $j$ | Hz | Hz |
| $g$ | acceleration due to gravity (= 9.81 $m/s^2$ or 32.2 $ft/s^2$) | $m/s^2$ | $ft/s^2$ |
| $I$ | second moment of area of pipe wall cross section | $m^4$ | $ft^4$ |
| $I_b$ | second moment of area of pipe bend cross section | $m^4$ | $ft^4$ |
| $i$ | pipe index number | – | – |
| $j$ | mode number | – | – |
| $K_b$ | bend flexibility factor (defined in Section A.4.2) | – | – |
| $K_j$ | normal-mode generalised stiffness of mode $j$ | N/m | lbf/ft |
| $k$ | spring stiffness | N/m | lbf/ft |
| $L$ | pipe length | m | ft |
| $M_j$ | normal-mode generalised mass of mode $j$ | kg | *slug |
| $m$ | lumped mass | kg | *slug |
| $m_b$ | total mass of bend | kg | *slug |
| $P_E$ | internal perimeter of pipe | m | ft |
| $R$ | bend radius (see Section A.4.2) | m | ft |
| $t$ | pipe wall thickness | m | ft |

*For footnotes see end of Notation section.*

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| | | | |
|---|---|---|---|
| $x'$, $y'$ | mutually perpendicular (orthogonal) reference coordinate axes for computer model of piping system | – | – |
| $z'$ | out-of-plane axis, perpendicular to $x'$– and $y'$ – axes | – | – |
| $\lambda_j$ | non-dimensional natural frequency parameter of mode $j$ | – | – |
| $\mu$ | pipe wall mass per unit length | kg/m | *slug/ft |
| $\theta$ | orientation of pipe with respect to $x'$– (or $y'$ – ) axis | degree | degree |
| $\rho$ | fluid density | kg/m$^3$ | *slug/ft$^3$ |
| $\rho_b$ | density of pipe bend material | kg/m$^3$ | *slug/ft$^3$ |
| $\rho_p$ | density of pipe material | kg/m$^3$ | *slug/ft$^3$ |
| $\Phi$ | bend angle | degree | degree |

* 1 slug = 32.2 lb (pounds mass)

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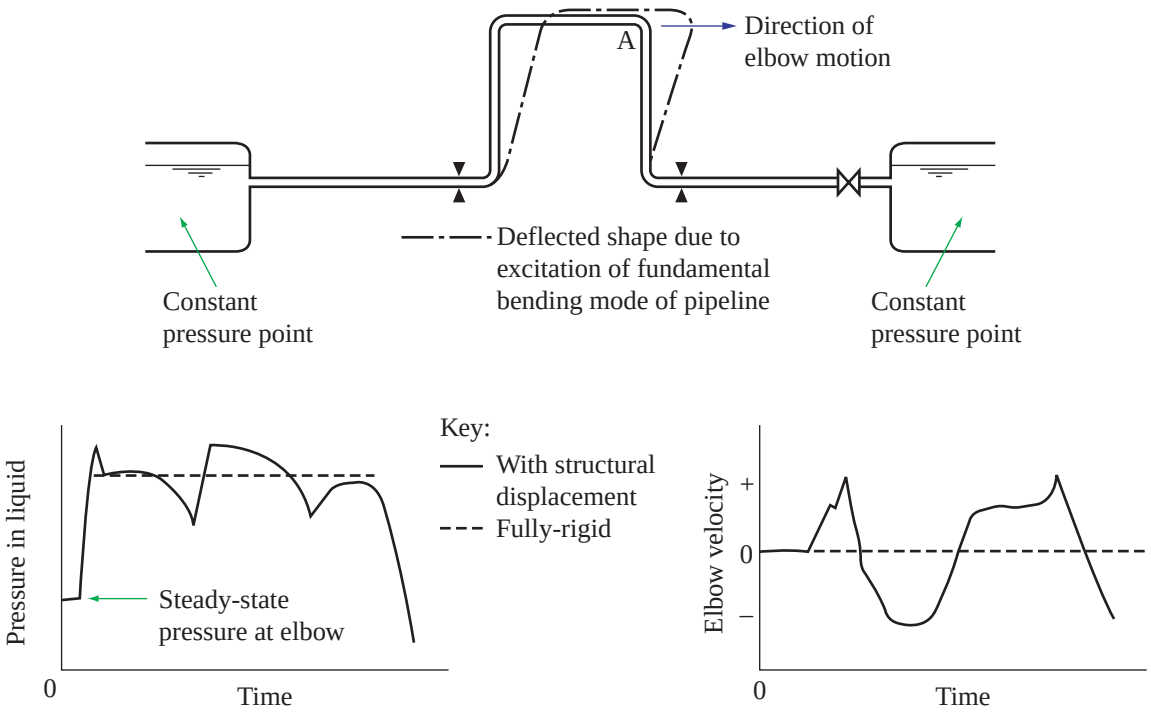
## 2. INTRODUCTION

### 2.1 Background

This Item is one of a group of three Data Items concerning the vibration of liquid-filled pipelines. Graphical and computer methods are provided for the estimation of the natural vibration parameters of planar piping systems. Natural vibration data generated by the computer program of this Item can be used, in conjunction with the methods of ESDU 89030 (Reference 33), for the prediction of the fluctuating values of the liquid pressure and flow and the structural response of planar pipelines following disturbances to the flow. The third Item in the group allows similar estimates to be made of these transient response data in more complex systems that are not necessarily planar. A method of analysis is used in the third Item that does not require the separate calculation of the natural vibration parameters.

#### 2.1.1 Pipeline vibrations




**Sketch 2.1 Fluid/structure interaction in a simple piping system**

Pipeline vibrations can occur following changes in the flow rate, due to valve closure or pump trip for example, or when the pipeline is subjected to some form of dynamic external loading, such as that due to earthquake excitation. The transient pressure changes, or fluid transients, which propagate through a fluid contained in a piping system following a disturbance in the flow, lead to pressure differences across lengths of pipework and across bends and other components. These pressure differences exert fluctuating forces on the pipework that can cause significant structural motion that in turn affects the pressure and flow in the fluid. The effects of fluid/structure interaction in a simple piping system are illustrated in Sketch 2.1.

In this sketch, the fluctuating pressure in the liquid and the structural velocity at point A in the pipe loop following rapid closure of the downstream valve are shown for two cases, where



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(i) no structural motion is allowed throughout the pipework, and

(ii) structural motion is allowed in the plane of the pipework.

It may be seen that, for this particular example, the peak pressure is greater for the case where structural displacement is allowed.

In the calculation of the transient pressure and structural motion in piping systems, it is common to neglect the mutual coupling between the fluid and structural parameters. This often leads to incorrect estimates of pressure and stress in flexible piping systems. In such uncoupled analyses, time-dependent piping forces are first estimated, based on records of fluctuating fluid pressure calculated throughout the piping system assuming there is no structural motion. These data are then used as the input to a structural model of the piping system in which the rigidity restraints are removed.

### 2.1.2 Coupled fluid/structure response calculations

In order to obtain reliable estimates of the liquid and structural parameters in non-rigid piping systems following a flow disturbance, it is necessary to use a model in which the liquid and structural parameters are coupled.

The data provided in this Item allow estimates to be made of undamped natural vibration characteristics for the bending and axial modes of planar piping systems. These data may be used in pipework vibration analysis in general, and, in particular, in the calculation of the coupled fluid and structural parameters in certain simplified liquid-filled systems using ESDU 89030 (Reference 33). In that Item, a modal model of the structure, represented by the natural vibration data calculated using this Item, is combined with a model of the pressure and flow parameters of the contained liquid based upon the method of characteristics. Estimates of the fluctuating values of pressure, flow and structural displacement may be then made throughout the piping system. The effects of stress waves in the pipe wall material are neglected, and fluid/structure coupling is assumed to occur only at 'junctions', such as pipe bends or other flow or structure discontinuities. The data in that Item are restricted to single pipe, planar systems. The third Item in the group provides a means of estimating the fluid and structural parameters in systems that are not subject to the restrictions outlined above.

It is important to note that the natural vibration parameters of a particular system are strongly dependent upon the piping layout and the type and locations of the supports. It is unlikely that systems with similar fluid performance characteristics have identical dynamic characteristics.

Further guidance on the coupled analysis used in the second Item in this group of Data Items is given in Reference 16, and general guidance on, and other forms of calculation for, fluid/structure interaction are given in References 8, 9, 12, 13, 15, 18, 19, 24, 26, 27, 29 and 30.

### 2.1.3 Typical applications

Systems for which the effects of fluid/structure interaction should be investigated include high pressure lines in processing plant, commonly suspended from the roof of the plant, or systems that have a degree of flexibility to allow for thermal expansion over the likely range of operating temperatures. Other applications include nuclear, petrochemical and power plants, and aircraft and rocket fuel systems. Fluid/structure interaction is not likely to be important in continuously supported and damped systems such as buried water supply lines or large oil pipelines. Data are provided in ESDU 83046, 84013, 85009, 85044 and 87027 (References 20, 21, 23, 25 and 32) that allow estimates to be made of the transient pressure changes in liquids contained in rigid pipelines. Similar data are provided for superheated steam and other gases that follow the perfect gas law in ESDU 86015 (Reference 28).



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## 2.2 Applicability of the Data

Common forms of dynamic excitation of fluid-filled pipelines, and comments on the parameters required for response calculations, are tabulated below.

| Type of excitation of non-rigid piping system | Comments on requirements for analysis |
|---|---|
| **Transient disturbance to flow rate**<br>*e.g.* valve closure,<br>pump trip,<br>pipe break. | Calculate maximum and minimum values of pressure. Calculate maximum structural displacements and associated pipe and support stresses. See Section 2.2.1. |
| **Resonant vibrations due to fluid or mechanical vibrations**<br>*e.g.* vibrations in pulsating flow,<br>excitation generated by pumps and other mechanical components (either externally or internally),<br>fluid turbulence. | At simplest level, ensure natural frequencies of piping do not coincide with, or lie near to, frequencies of excitation. See Section 2.2.2. |
| **External excitation**<br>*e.g.*earthquake,<br>wind,<br>wave,<br>vehicle and aircraft motion. | Calculate maximum and minimum values of pressure. Calculate maximum structural displacements and associated pipe and support stresses. If necessary perform fatigue analysis from pipe stress calculations. See Section 2.2.3. |

The natural vibration data obtained using this Item may be used in the calculation of pipeline and fluid parameters subjected to the types of excitation outlined below.

### 2.2.1 Vibrations due to transient pressure changes

Numerical values of the natural vibration parameters of piping systems are required in order to predict the fluctuating pressure and flow and structural motion, following changes in the flow rate, using the second Item in this group of Data Items (see Section 2.1.2). Fluid/structure interaction is included in the system model of that Item. Further guidance on the estimation of fluid and pipework response using that method is given in Reference 19. These natural vibration data are not required in the third Item in this group of Data Items.

### 2.2.2 Resonant vibrations in pulsating flow

Significant vibrations of piping may occur due to the close matching of the natural modes of vibration of the piping system and the frequency of mechanically induced vibrations from components such as reciprocating compressors or pumps. Vortex shedding from equipment installed in the pipeline may also lead to resonant vibration effects. The prediction of the response of piping systems to mechanically-excited vibration is complex, and excessive vibration may usually be avoided by ensuring that the frequencies of the bending modes of the piping system do not lie close to any of the frequencies of excitation. In many piping systems in which mechanical vibration has an effect on transient pressures, the fundamental mode of vibration is most likely to be excited. A means of estimating the excitation frequencies of a number of common sources of mechanical vibration in piping systems is given in Table 7.1. Further guidance on this matter is given in Reference 6.



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Another form of resonance that may occur is that due to the natural period of oscillation of the contained fluid. In a simple, unbranched single pipe system for example, the fundamental period of the fluid is $4L/a$, and is the time taken for a complete cycle of transient pressure changes to propagate from the source of the disturbance along the pipe. In this expression, $L$ is the length of the fluid line and $a$ is the speed of propagation of the pressure waves in the fluid, calculated using ESDU 83046 (Reference 20). For single pipe systems, the frequencies (in Hertz) associated with these periods of oscillation are given as

$$f_j = j\frac{a}{4L}, \tag{2.1}$$

where $j = 1, 2, 3 \ldots$ Techniques are available for the analysis of acoustic resonance in pipe networks, most notably the 'impedance method' (based upon the harmonic analysis of steady-state forced vibrations) and the free vibrations method. Guidance on these methods and on the effects of resonance in multi-pipe systems is given in References 3, 9, 10 and 16.

In general, it is necessary to ensure that the frequencies of the bending modes do not lie close to the frequencies of fluid oscillations (typically within $\pm 20$ per cent).

### 2.2.3 Vibrations due to external excitation

The estimation of coupled fluid/structure interaction effects in pipelines under loading due to external excitation is complex and rarely undertaken. In many cases, it may be adequate to consider the liquid and structure as uncoupled, and perform a dynamic analysis in which the fluctuating exciting forces are applied to a modal model of the structure. The results from such an analysis, which is often statistically based, are often used for pipe fatigue calculations and to ensure the structural integrity of the pipe supports.

## 2.3 Data Provided in this Item

Graphical data and a computer program are provided in this Item for the estimation of the natural vibration parameters of non-rigid piping systems.

For the purposes of the computer program, discussed in Appendix A, the idealisations described in Section 3.1 are imposed on the model of the piping system. The program allows the natural frequencies and corresponding mode shapes, generalised masses and generalised stiffnesses of the transverse bending and axial modes of vibration to be calculated. The pipeline may be empty or running full. The effects of large components (such as in-line valves), elastic restraints (such as hangers or snubbers) and the flexibility of pipe bends may be investigated using the program.

The graphical presentation of Section 4 allows estimates to be made of the frequency of the fundamental mode of vibration for the range of piping configurations shown in Table 7.2. A number of further idealisations are imposed on the computer model in order to facilitate the graphical presentation. These further idealisations are described in Section 4.1. The data are applicable to circular or non-circular piping systems, either empty or running full.

Throughout this Item it is assumed that the pipeline, or the vibrating components of the pipeline if an isolated portion of the pipeline is considered, lies entirely in one plane, referred to as the plane of the pipeline, and vibrates only in that plane. The vibration characteristics in a different plane are unlikely to be identical to those in the plane of the pipeline, and it may be necessary to perform a further set of calculations for the vibration parameters in the different plane (usually perpendicular to the plane of the pipeline). A portion of a piping system may be considered in isolation if the lateral deflections and rotations at the extremities of the section are negligible compared with the peak values along the portion. For example, a portion of pipeline between rigid, massive anchor blocks may be considered in isolation for the purposes of a natural vibration analysis.



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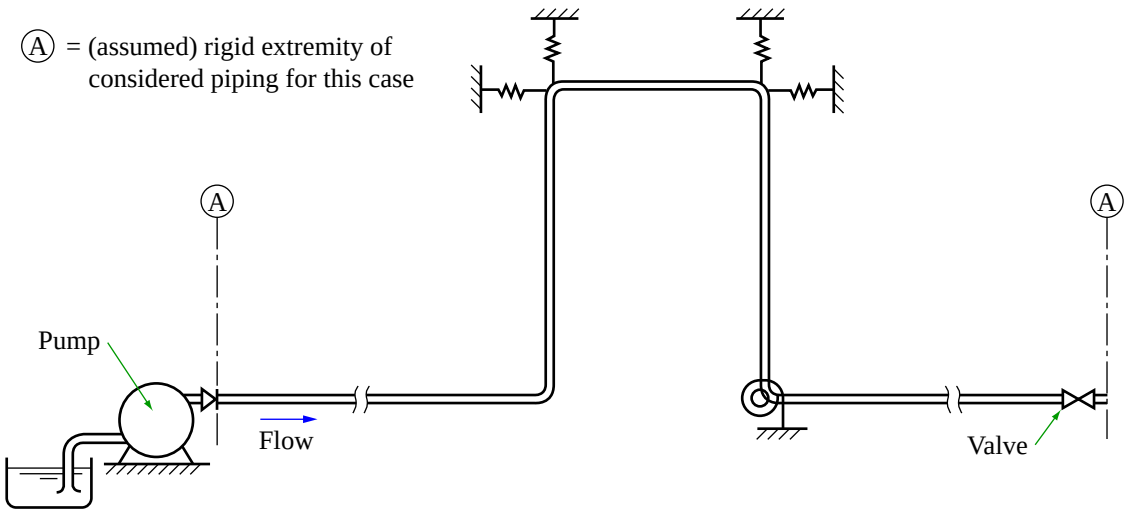
## 3. PIPEWORK MODEL

The way in which planar piping systems are idealised for the purposes of the computer model of Appendix A is described in Section 3.1. The method of solution of the model is outlined in Section 3.2.

### 3.1 Scope of Pipework Model

This Item provides a means of calculating the low frequency bending modes of planar piping systems, or systems that may be modelled as planar. The program also calculates the axial (longitudinal) modes of vibration, although the frequencies of these modes are usually much greater than those of the bending modes and are therefore less significant in a dynamic analysis. A typical system is shown in Sketch 3.1. The pipework is modelled as a series of continuous straight beams of uniform cross-section. Pipe bends and elbows are either considered fully-rigid or modelled by a number of short beam elements which have stiffnesses modified by appropriate bend flexibility factors to account for the change in cross-sectional shape in bending. Guidance on flexibility factors is given in Section A.4.2. Connections between 3 or more pipes, such as tee-junctions, are considered completely rigid.

Elastic pipeline restraints, such as hangers or other mechanical supports, are modelled as linear springs of finite stiffness. Rigid restraints are modelled by linear springs of infinite stiffness. Components that are attached to the pipeline are included in the model as lumped masses that add no bending stiffness to the pipeline. In-line bellows can be modelled by beam sections of artificially low values of Young's modulus of elasticity.




**Sketch 3.1 Simple, unbranched pipe system**

For the purposes of the computer model of the pipeline, the following idealisations are also made.

(i) The steady flow velocity is small compared to the speed of propagation of pressure waves in the fluid, and any decrease in stability of the pipeline due to an increase in flow velocity is neglected. Guidance on this matter is given in References 7 and 31.

(ii) The pipework is considered to vibrate in the plane of the piping system only.



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### 3.2 System Model

The method allows any number of modes of undamped natural vibration to be calculated for a single or multi-pipe system with pipes arranged at any orientation in the plane of the pipework. Externally-acting elastic or rigid restraints may be included at any junction between two or more lengths of pipe. Each junction has two translational, and onc rotational, degrees of freedom associated with it. The displacements in the directions of any of the rotational or translational degrees of freedom may be elastically restrained at a junction. For the translational degrees of freedom, the elastic restraints are assumed to act in mutually perpendicular (orthogonal) directions that are parallel to a set of reference coordinate axes (see Section A.4.3). Lumped masses may be included at the junctions.

Although the piping system is modelled as a system with a finite number of degrees of freedom, the method allows any natural frequency to be located, including those of order greater than the number of degrees of freedom considered in the model. In most cases, however, only the first few modes of vibration are of interest for response calculations. Unlike lumped mass models, the method does not require sections of piping to be further divided into a number of segments, and an "exact stiffness" method (Derivation 1) has been used to calculate the natural vibration parameters. The common practice of specifying an upper limit of frequency is used so that only the modes at or below that limiting frequency are calculated. If the frequency content of the dynamic excitation is known, an upper limit that exceeds the excitation frequencies by a significant margin may be specified. Typically, a value of 33 Hz is used in the power industry, although applications with smaller, stiffer members of lower mass will require a higher limit.

Further guidance on the computer model, and a full explanation of the data input requirements for system modelling, is given in Appendix A.

## 4. GRAPHICAL MODEL FOR THE FUNDAMENTAL MODE OF VIBRATION OF SIMPLIFIED PIPING CONFIGURATIONS

Data are presented in this section that allow estimates to be made of the fundamental frequency of some simplified, idealised piping systems, described in Section 4.2. The further idealisations required for the graphical method over those for the computer program are described in Section 4.1. If it is considered that resonance effects are important, this Item can be used to estimate the fundamental frequency of vibration. Further guidance on this matter is given in Section 2.2.2. For the detailed analysis of the effects of fluid/structure interaction in non-rigid piping systems using the second Item in this group of Data Items, the mode shape and generalised mass corresponding to the fundamental mode of vibration are also required (and, in addition, data for other modes are usually calculated). In these cases it is recommended that the computer program introduced in Appendix A is used.

### 4.1 Further Idealisations Imposed on the Pipework Model to Allow a Graphical Presentation

These further idealisations, which are in addition to those listed in Section 3.1, are given below.

(i) The piping system consists of lengths of pipe that are of identical specification (*i.e.* of identical cross section and mass per unit length), and are continuous and uniform.

(ii) The mass of the liquid contained in the pipes is included in the analysis by modifying the mass per unit length of the pipes.

(iii) Bends are considered to be fully-rigid. This assumption is valid for systems where the pipes adjacent to the pipe bend are of significantly greater length than the arc length of the bend (typically of the order of ten times or more). Neglecting the flexibility of the bend will lead to an overestimate of the natural frequency.



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(iv) The mass of bends and other components along the pipeline are not included. However, as an approximation, the effects of lumped masses that are distributed along the pipework may be included by increasing the mass per unit length of the pipework accordingly (noting that the mass per unit length is considered constant throughout the piping system).

(v) The pipeline is unrestrained from movement in the plane of the pipework, except at the extremities (see Table 7.2).

These data apply to vibrations in the plane of the pipework only. In pipelines where there is little or no restraint to movement out of the plane of the pipework, the fundamental mode of vibration may be out of the plane, and cannot be estimated using this Item. The computer program introduced in Appendix A should be used to calculate the natural vibration parameters in the plane of the pipework if the graphical data are not applicable.

## 4.2 Range of Graphical Data Provided

The fundamental mode of vibration in the plane of the piping may be estimated for the piping configurations shown in Table 7.2 using the equation

$$f_1 = \frac{\lambda_1^2}{2\pi L_1^2}\left(\frac{EI}{\mu + \pi\rho D^2/4}\right)^{1/2}, \tag{4.1}$$

where $E$, $I$ and $\mu$ are the values of Young's Modulus of elasticity, second moment of area and mass per unit length of the pipework, respectively. Simple expressions for the estimation of $I$ for a number of common pipe cross-sectional shapes are given in Table 7.3. The effect on the fundamental frequency of the pipework of contained fluid is included by the additional mass per unit length of the contained liquid, $\pi\rho D^2/4$, where $\rho$ is the fluid density and $D$ is the internal diameter. Appropriate adjustments must be made to this parameter for non-circular pipes. The pipe length term, $L_1$, refers to a particular pipe in the configuration as shown in Table 7.2. The non-dimensional frequency parameter, $\lambda_1$, is given in Table 7.2 for the single pipe systems (configurations 1 to 4) or in Figure 1 for configurations 5 and 6 and Figures 2 to 7 for configurations 7 to 12 respectively. These data are given for a range of values of pipe length ratios, $L_2/L_1$ and $L_3/L_1$, as appropriate (where $L_1$, $L_2$ and $L_3$ are defined in Table 7.2).

# 5. EXAMPLE

It is required to estimate the fundamental mode of vibration in the plane of the piping system shown in Sketch 5.1 in order to ensure that resonance does not occur as a result of either mechanically-induced vibration from a reciprocating pump or due to acoustic resonance. The pipes are of identical specification and are of circular cross section with internal diameter, $D$, and thickness, $t$, of 68.8 mm and 2.11 mm respectively. The density of the pipe wall material is 7800 kg/m$^3$, and Young's modulus of elasticity of the pipe wall material is $207 \times 10^9$ N/m$^2$. The pipe is full of water of density $\rho = 1000$ kg/m$^3$. The reciprocating pump has 3 cylinders, and runs at 150 rev/min. The arc length of each bend is 0.08 m.

From Table 7.3, the second moment of area, $I$, about the centroid of the cross section of the piping is estimated to be

$$\begin{aligned} I &= \pi/4[(D/2 + t)^4 - (D/t)^4] \\ &= \pi/4 \times \left[(0.0688/2 + 0.00211)^4 - (0.0688/2)^4\right] = 2.96 \times 10^{-7}\ \text{m}^4. \end{aligned}$$



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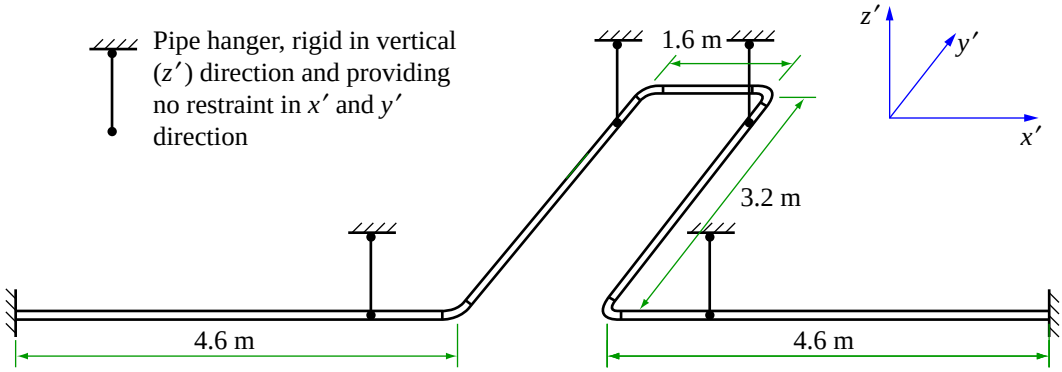
The pipe wall cross-sectional area is similarly calculated from Table 7.3 as $A_p = 4.70 \times 10^{-4}$ m$^2$. The mass per unit length of the pipe wall material, $\mu$, is given as $\mu = \rho_p A_p = 3.67$ kg/m.




**Sketch 5.1 Example piping system**

By inspection from Table 7.2, it may be seen that the pipework corresponds to piping configuration 10. The non-dimensional piping ratios, $L_2/L_1$ and $L_3/L_1$, are $L_2/L_1 = 1.9$ and $L_3/L_1 = 2.9$, respectively, for $L_1 = 1.6$ m, $L_2 = 3.1$ m and $L_3 = 4.6$ m (from Sketch 5.1).

The non-dimensional frequency parameter, $\lambda_1$, for the fundamental mode of vibration may be estimated from Figure 5 as $\lambda_1 = 0.57$.

The fundamental frequency of vibration in the plane of the piping is given by Equation (4.1) as

$$f_1 = \frac{\lambda_1^2}{2\pi L_1^2}\left(\frac{EI}{\mu + \pi\rho D^2/4}\right)^{1/2}$$

$$= \frac{0.57^2}{2 \times \pi \times 1.6^2}\left(\frac{207 \times 10^9 \times 2.96 \times 10^{-7}}{3.67 + (\pi \times 1000 \times 0.0688^2)/4}\right)^{1/2} = 1.8 \text{ Hz.}$$

From Table 7.1, the excitation frequencies due to the pump are given as $f_j = jNP/60$, where $N$ (= 150 rev/min) is the pump speed and $P$ (= 3) is the number of pump plungers. Hence, $f_1 = 7.5$ Hz, $f_2 = 15$ Hz, *etc*. It may be seen that the fundamental frequency of vibration of the piping system is significantly lower than the fundamental excitation frequency of the pump, and the pipework will not be excited in the fundamental mode.

The acoustic resonance frequencies in the liquid are estimated from Equation (2.1) as

$$f_j = j\frac{a}{4L},$$

where the speed of propagation of the pressure waves in the water is estimated to be $a = 1250$ m/s using ESDU 83046. Hence $f_1 = 18$ Hz, $f_2 = 36$ Hz, *etc*. To cater for the uncertainties in predicted values of natural frequency, a margin of uncertainty of, say, $\pm 20$ per cent should be considered when comparing natural frequencies with resonance frequencies. Hence, it is possible that the second or third modes, due to the pump excitation, of frequencies $f_1 = 15$ Hz and $f_2 = 23$ Hz, may cause resonance effects because there is relatively close matching between these pump frequencies and the fundamental frequency of acoustic resonance.



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### Derivation


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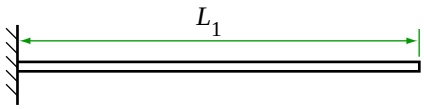
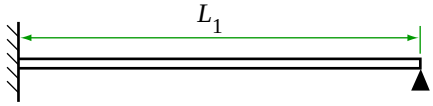
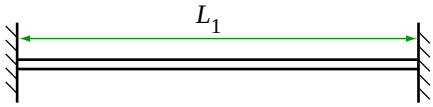
## 7. TABLES

**TABLE 7.1 Excitation Frequencies of Components**

| Component | Frequencies of excitation (Hz) | Parameters in frequency equations | Notes |
|---|---|---|---|
| Reciprocating compressors | $f_j = jN/60$ | $N$ = speed (rev/min),<br>$j$ = 1, 2, 3... | Fundamental mode ($j = 1$) of most interest. |
| Reciprocating pumps | $f_j = jNP/60$ | $N$ = speed (rev/min),<br>$j$ = 1, 2, 3...,<br>$P$ = number of pump plungers. | Fundamental mode ($j = 1$) of most interest. |
| Centrifugal pumps and compressors | $f_j = jN/60$<br><br>$f_j = jNB/60$<br><br>$f_j = jNv/60$ | $N$ = speed (rev/min),<br>$j$ = 1, 2, 3...<br><br>$N$ = speed (rev/min),<br>$j$ = 1, 2, 3...,<br>$B$ = number of blades.<br><br>$N$ = speed (rev/min),<br>$j$ = 1, 2, 3...,<br>$v$ = number of volutes or diffuser vanes. | "Whole" pump modes.<br><br>Blade passing frequency.<br><br>Staging modes. |

**TABLE 7.2 Natural Frequency Parameter of Piping Configurations**

| Piping configuration | Natural frequency parameter*, $\lambda_1$ |
|---|---|
| 1.   | $\lambda_1$ = 1.875<br>$\lambda_2$ = 4.694<br>$\lambda_3$ = 7.855<br>$\lambda_4$ = 11.00 |
| 2.   | $\lambda_1$ = 3.927<br>$\lambda_2$ = 7.069<br>$\lambda_3$ = 10.21<br>$\lambda_4$ = 13.35 |
| 3.   | $\lambda_1$ = 4.730<br>$\lambda_2$ = 7.853<br>$\lambda_3$ = 11.00<br>$\lambda_4$ = 14.14 |

*For footnotes and key see end of Table 7.2.*



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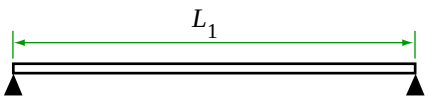
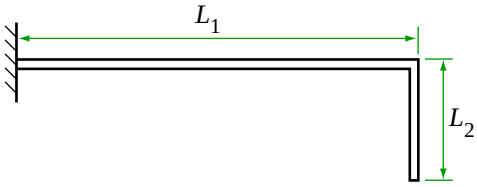
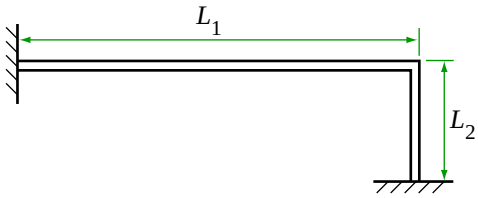
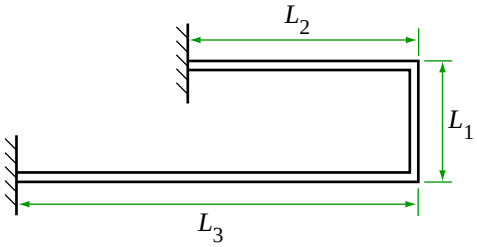
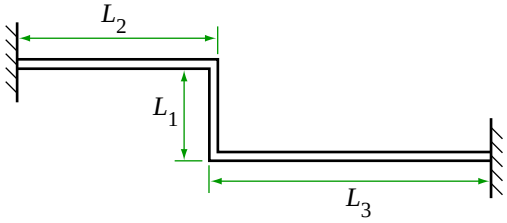
**TABLE 7.2 Natural Frequency Parameter of Piping Configurations (Continued)**

| Piping configuration | Natural frequency parameter*, $\lambda_1$ |
|---|---|
| 4.   | $\lambda_1 = \pi$<br>$\lambda_2 = 2\pi$<br>$\lambda_3 = 3\pi$<br>$\lambda_4 = 4\pi$ |
| 5.   | Data for fundamental mode, $\lambda_1$, given in Figure 1 |
| 6.   | Data for fundamental mode, $\lambda_1$, given in Figure 1 |
| 7.   | Data for fundamental mode, $\lambda_1$, given in Figure 2 |
| 8.   | Data for fundamental mode, $\lambda_1$, given in Figure 3 |

*For footnotes and key see end of Table 7.2.*

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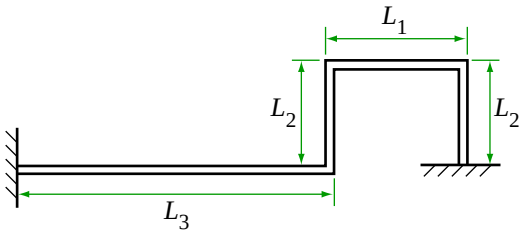
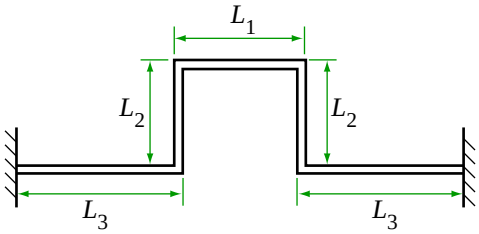
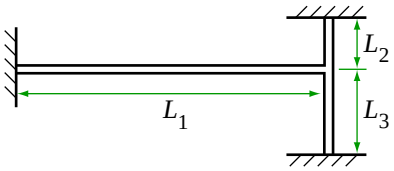
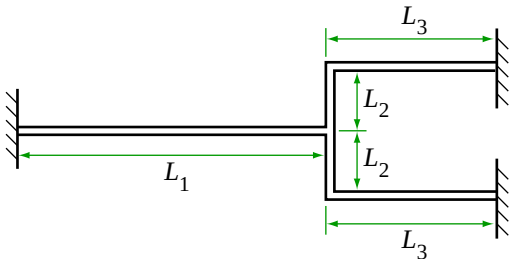
**TABLE 7.2 Natural Frequency Parameter of Piping Configurations (Continued)**

| Piping configuration | Natural frequency parameter*, $\lambda_1$ |
|---|---|
| 9.   | Data for fundamental mode, $\lambda_1$, given in Figure 4 |
| 10.   | Data for fundamental mode, $\lambda_1$, given in Figure 5 |
| 11.   | Data for fundamental mode, $\lambda_1$, given in Figure 6 |
| 12.   | Data for fundamental mode, $\lambda_1$, given in Figure 7 |

Key:

Built-in support (no rotation or translation)

Pinned support (rotation but no translation in either axis)

* For the simple one-pipe configurations considered in 1 to 4 above, values of the natural frequency parameter for the second, third and fourth modes of vibration are also given (*i.e.* $\lambda_2$, $\lambda_3$ and $\lambda_4$, respectively).



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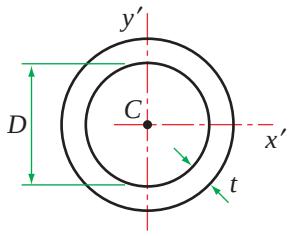
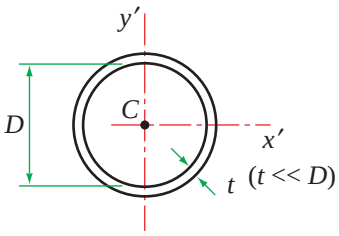
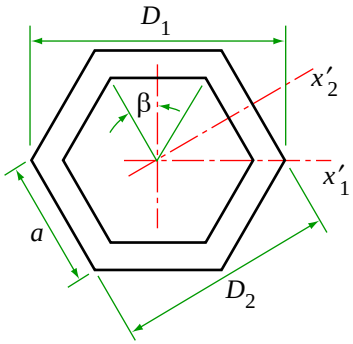
**TABLE 7.3 Section Properties for Some Common Pipe Shapes**

| Pipe cross section | $I$ | $A_p$ |
|---|---|---|
| Circular cross section   | $\pi/4\left[(D/2+t)^4-(D/2)^4\right]$ | $\pi\left[(D/2+t)^2-(D/2)^2\right]$ |
| *Thin-walled circular pipe   | $\pi(D/2+t)^3 t$ | $2\pi(D/2+t)t$ |
| Regular polygon   | About $x'_1$ axis: $A_p\left[6D_1/2^2-a^2\right]/24$ <br> About $x'_2$ axis: $A_p\left[12(D_2/2)^2+a^2\right]/48$ | $nat[1-(t/a)\tan\beta]$ <br> where <br> $n$ = number of sides of polygon |

* Note that these equations for $I$ and $A_p$ are approximations for thin-walled pipes and will calculate values that are slightly high.



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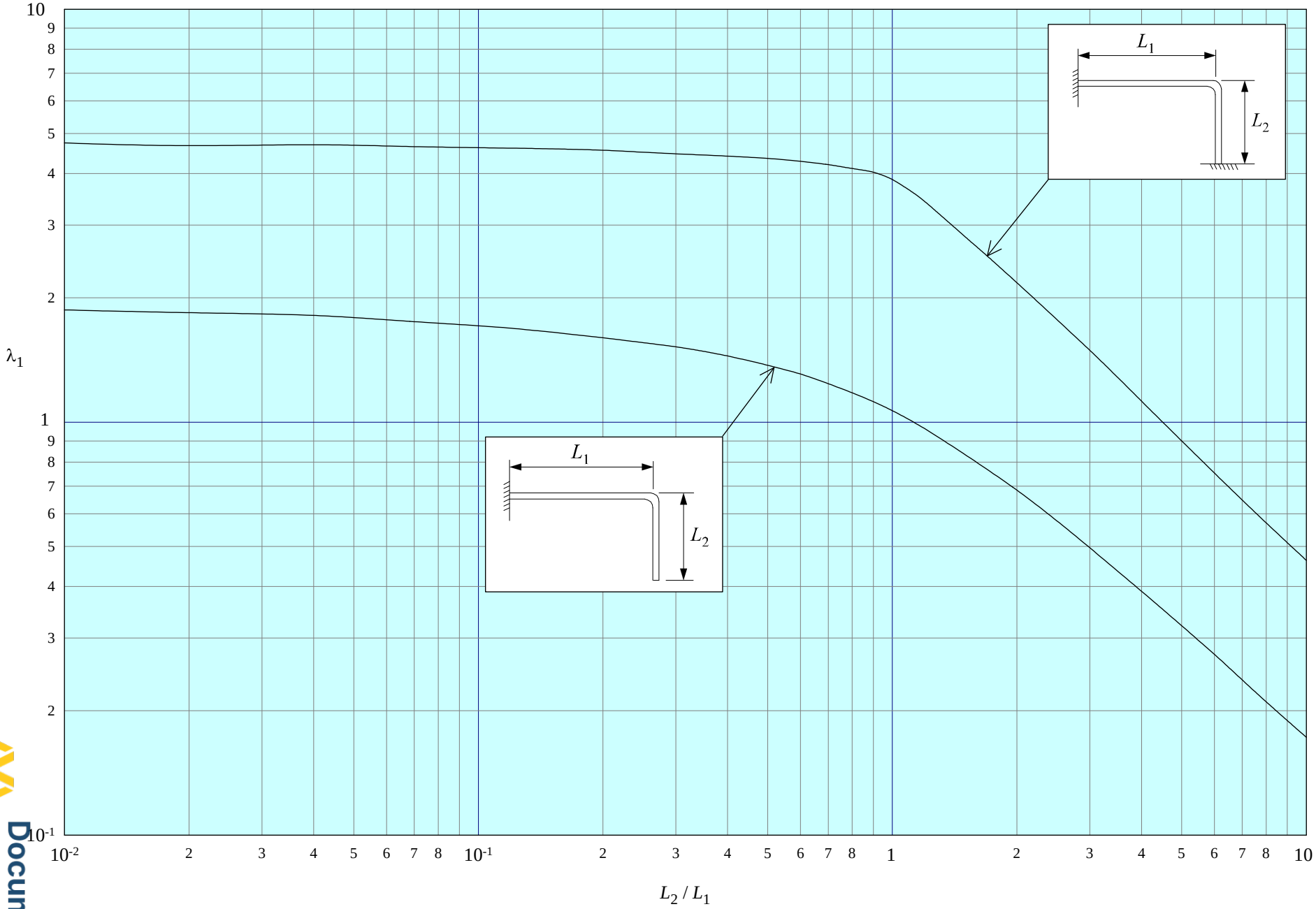


**FIGURE 1 NATURAL FREQUENCY PARAMETER FOR PIPING CONFIGURATIONS No. 5 AND No. 6**



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