



ESDU 88030

Issued November 1988

With Amendment A

January 1994

Supersedes ESDU 66033

Boundaries of linear characteristics of cambered and twisted wings at subcritical Mach numbers

Endorsed by
The Royal Aeronautical Society

AVA Documents
Document Sharing Hub

ESDU DATA ITEMS

Data Items provide validated information in engineering design and analysis for use by, or under the supervision of, professionally qualified engineers. The data are founded on an evaluation of all the relevant information, both published and unpublished, and are invariably supported by original work of ESDU staff engineers or consultants. The whole process is subject to independent review for which crucial support is provided by industrial companies, government research laboratories, universities and others from around the world through the participation of some of their leading experts on ESDU Technical Committees. This process ensures that the results of much valuable work (theoretical, experimental and operational), which may not be widely available or in a readily usable form, can be communicated concisely and accurately to the engineering community.

We are constantly striving to develop new work and review data already issued. Any comments arising out of your use of our data, or any suggestions for new topics or information that might lead to improvements, will help us to provide a better service.

THE PREPARATION OF THIS DATA ITEM

The work on this particular Item, which supersedes ESDU 66033, was monitored and guided by the Aerodynamics Committee which first met in 1942 and now has the following membership:

Chairman	
Mr H.C. Garner	— Independent
Vice-Chairman	
Mr P.K. Jones	— British Aerospace plc, Civil Aircraft Div., Woodford
Members	
Mr G.E. Bean*	— Boeing Aerospace Company, Seattle, Wash., USA
Mr E.A. Boyd	— Cranfield Institute of Technology
Mr K. Burgin	— Southampton University
Mr W.S. Chen*	— Northrop Corporation, Hawthorne, Calif., USA
Dr T.J. Cummings	— Short Brothers plc
Mr J.R.J. Dovey	— Independent
Mr L. Elmeland*	— Saab-Scania, Linköping, Sweden
Dr J.W. Flower	— Bristol University
Mr P.G.C. Herring	— Sowerby Research Centre, Bristol
Mr R. Jordan	— Aircraft Research Association
Mr J.R.C. Pedersen	— Independent
Mr N. Roberts	— Rolls-Royce plc, Derby
Mr R. Sanderson	— Messerschmitt-Bölkow-Blohm GmbH, Hamburg, W. Germany
Mr A.E. Sewell*	— McDonnell Douglas, Long Beach, Calif., USA
Mr M.R. Smith	— British Aerospace plc, Civil Aircraft Div., Bristol
Miss J. Willaume	— Aérospatiale, Toulouse, France.

* Corresponding Member

The technical work involved in the assessment of the available information and the construction and subsequent development of the Data Item was carried out under contract to ESDU by Mr J.R.J. Dovey.

BOUNDARIES OF LINEAR CHARACTERISTICS OF CAMBERED AND TWISTED WINGS AT SUBCRITICAL MACH NUMBERS

CONTENTS

	Page
1. NOTATION AND UNITS	1
2. INTRODUCTION	6
3. REQUIRED DATA ITEMS	7
4. FLOW SEPARATION - ITS CAUSES AND CONSEQUENCES	7
5. GENERAL EFFECTS OF CAMBER AND TWIST	8
6. EFFECTS OF REYNOLDS NUMBER	10
7. EFFECTS OF MACH NUMBER	10
8. PREDICTION METHOD	10
8.1 Derivation of Method	10
8.2 Estimation of Initial Separation Type	11
8.3 Symmetrical Two-dimensional Sections and Wings without Camber or Twist	12
8.3.1 Leading-edge initial separation	12
8.3.2 Trailing-edge initial separation	13
8.4 Estimation of the Increments to $\alpha_{K\infty}$ and $C_{LK\infty}$ for Cambered Two-dimensional Sections	14
8.4.1 Leading-edge initial separation	14
8.4.2 Trailing-edge initial separation	14
8.5 Estimation of $\alpha_{KC\infty}$ and $C_{LKC\infty}$ for Cambered Two-dimensional Sections	15
8.6 Estimation of the Increments to α_K and C_{LK} for a Wing with Constant Camber across the Span	15
8.7 Estimation of the Increments to α_K and C_{LK} for a Wing with Twist	17
8.8 Estimation of α_K and C_{LK} for a Wing with Twist, Camber and Varying Thickness	18
9. APPLICABILITY AND ACCURACY	19
9.1 Applicability	19
9.2 Accuracy	20
9.2.1 Effect of camber on C_{LK} and α_K	20
9.2.2 Twisted and cambered wings	20
10. DERIVATION	25
10.1 ESDU Data Items	25
10.2 Wind-tunnel Test Reports	25

11. EXAMPLE	28
FIGURES 1 TO 9	36 to 43
APPENDIX A PROCEDURE	44

BOUNDARIES OF LINEAR CHARACTERISTICS OF CAMBERED AND TWISTED WINGS AT SUBCRITICAL MACH NUMBERS

1. NOTATION AND UNITS

		SI	British
A	aspect ratio ($= 2s/\bar{c}$) ($= 4s^2/S$)		
a_∞	section lift-curve slope	degree ⁻¹	degree ⁻¹
a_1	wing lift-curve slope	degree ⁻¹	degree ⁻¹
B_i	coefficients used in estimation of zero-lift angle for cambered wing section, see Equation (8.13)	degree	degree
C_D	drag coefficient, drag/ qS		
C_L	lift coefficient, lift/ qS		
C_{LK}	wing lift coefficient at end of initial linear range of force and moment characteristics (in particular for wings without camber or twist), see Sketch 2.1		
C_{LKC}	value of C_{LK} for cambered wing		
C_{LKT}	value of C_{LK} for twisted wing		
C_{LKCT}	value of C_{LK} for cambered and twisted wing		
$C_{LK\infty}$	lift coefficient at end of initial linear range of force and moment characteristics for two-dimensional section		
C_{LL}	local lift coefficient		
C_{LLp}	peak local lift coefficient, see Sketch 8.1		
C_{L0}	lift coefficient at $\alpha = 0$ for wing		
$C_{L0\infty}$	lift coefficient at $\alpha = 0$ for two-dimensional section		
C_m	pitching moment coefficient, pitching moment/ $qS\bar{c}$		
ΔC_{LKC}	increment in C_{LK} due to camber		
ΔC_{LKT}	increment in C_{LK} due to twist		
ΔC_{LKCT}	increment in C_{LK} due to camber and twist		

Issued November 1988

With Amendment A, January 1994 – 44 pages

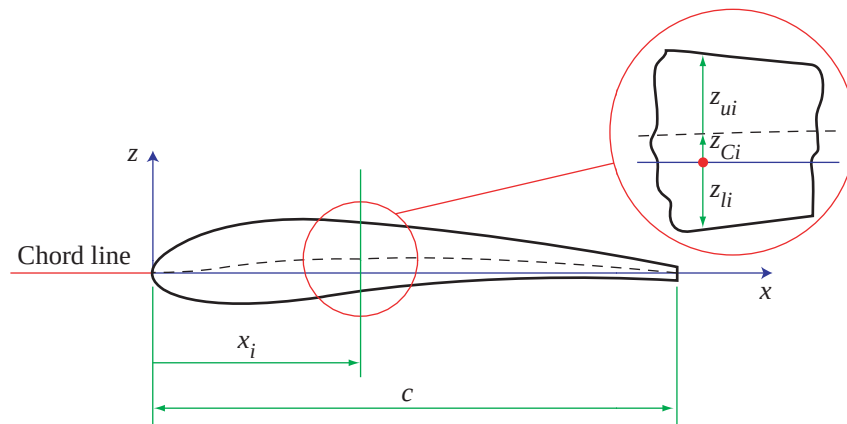
$\Delta C_{LK\infty}$	increment in $C_{LK\infty}$ due to camber		
ΔC_{L0T}	increment in C_{L0} due to twist		
c	local chord	m	ft
$\bar{c}$	geometric mean chord	m	ft
$\bar{\bar{c}}$	aerodynamic mean chord	m	ft
c_n	local chord normal to leading edge	m	ft
c_{pn}	local chord normal to leading-edge at $\eta = \eta_p$	m	ft
c_r	wing root chord	m	ft
c_t	wing tip chord	m	ft
E	quantity used to derive $\Delta\alpha_{KC\infty}$ for leading-edge initial separation, see Figure 7	degree	degree
F	Reynolds number factor used in Equation (8.9)		
q	free-stream kinetic pressure	N/m ²	lbf/ft ²
R_c	Reynolds number based on local chord and velocity, both measured in streamwise direction		
$R_{\bar{c}}$	Reynolds number based on aerodynamic mean chord and velocity, both measured in streamwise direction		
$R_{c_{pn}}$	Reynolds number based on chord and velocity component, both measured normal to leading edge at η_p		
R_ρ	Reynolds number based on leading-edge radius and velocity, both measured in streamwise direction		
$R_{\rho_{pn}}$	Reynolds number based on leading-edge radius and velocity, both measured normal to leading edge at η_p		
S	wing area, $s(c_r + c_t)$	m ²	ft ²
s	wing semi-span	m	ft
t	maximum thickness of wing section	m	ft
x	general chordwise station measured streamwise from wing leading-edge	m	ft
x_i	chordwise co-ordinate of i 'th station measured from leading edge, see Sketch 1.1	m	ft
$x_{z_{umax}}$	value of x for maximum upper-surface ordinate z_{umax}	m	ft

z	general section ordinate	m	ft
z_{Ci}	camber ordinate at x_i , see Sketch 1.1 and Equation (8.14)	m	ft
z_{li}	lower-surface ordinate at x_i , measured from chord line, see Sketch 1.1	m	ft
z_{ui}	upper-surface ordinate at x_i , measured from chord line, see Sketch 1.1	m	ft
z_{umax}	maximum upper-surface ordinate	m	ft
$z_{u1.25}$	upper-surface ordinate at 1.25 per cent chord	m	ft
$z_{u1.25n}$	upper-surface ordinate at 1.25 per cent chord normal to leading edge	m	ft
α	wing incidence measured from reference plane through wing root chord	degree	degree
α_K	incidence at end of initial linear range of force and moment characteristics (in particular for wings without camber or twist)	degree	degree
$\alpha_{K\infty}$	incidence at end of initial linear range of force and moment characteristics for uncambered two-dimensional section	degree	degree
α_{KC}	value of α_K for cambered wing	degree	degree
$\alpha_{KC\infty}$	value of α_K for cambered two-dimensional section	degree	degree
α_{KT}	value of α_K for twisted wing	degree	degree
α_{KCT}	value of α_K for cambered and twisted wing	degree	degree
α_0	zero-lift angle of incidence for cambered wing, see Sketch 8.1	degree	degree
$\alpha_{0\infty}$	zero-lift angle of incidence for cambered two-dimensional section, see Equation (8.13)	degree	degree
$\alpha_{0\eta\infty}$	value of $\alpha_{0\infty}$ for station η	degree	degree
$\Delta\alpha_K$	increment in α_K	degree	degree
$\Delta\alpha_{KC}$	increment in α_K due to camber	degree	degree
$\Delta\alpha_{KC\infty}$	increment in $\alpha_{K\infty}$ due to camber	degree	degree

$\Delta\alpha_{KT}$	increment in α_K due to twist	degree	degree
$\Delta\alpha_{KCT}$	increment in α_K due to camber and twist	degree	degree
$\Delta\alpha_{0T}$	increment in α_0 due to twist, see Sketch 8.3	degree	degree
δ	wing twist angle relative to wing root (positive leading-edge up)	degree	degree
δ_e	effective twist angle	degree	degree
δ_{et}	effective tip twist angle	degree	degree
η	spanwise distance from root as fraction of semi-span		
η_p	value of η for C_{LLp}/C_L		
$\bar{\eta}$	spanwise centre of pressure position, $\int_0^1 \sigma \eta d\eta$		
κ	wing taper parameter, $\int_0^1 (c/\bar{c})\eta d\eta$		
Λ_0	wing leading-edge sweep angle	degree	degree
$\Lambda_{1/4}$	wing quarter-chord sweep angle	degree	degree
$\Lambda_{1/2}$	wing mid-chord sweep angle	degree	degree
Λ_1	wing trailing-edge sweep angle	degree	degree
λ	wing taper ratio, c_t/c_r		
μ	normalised local lift coefficient, C_{LL}/C_L		
μ_p	peak value of μ		
ρ	section leading-edge radius	m	ft
σ	spanwise loading due to incidence, $C_{LL}c/C_L\bar{c}$		

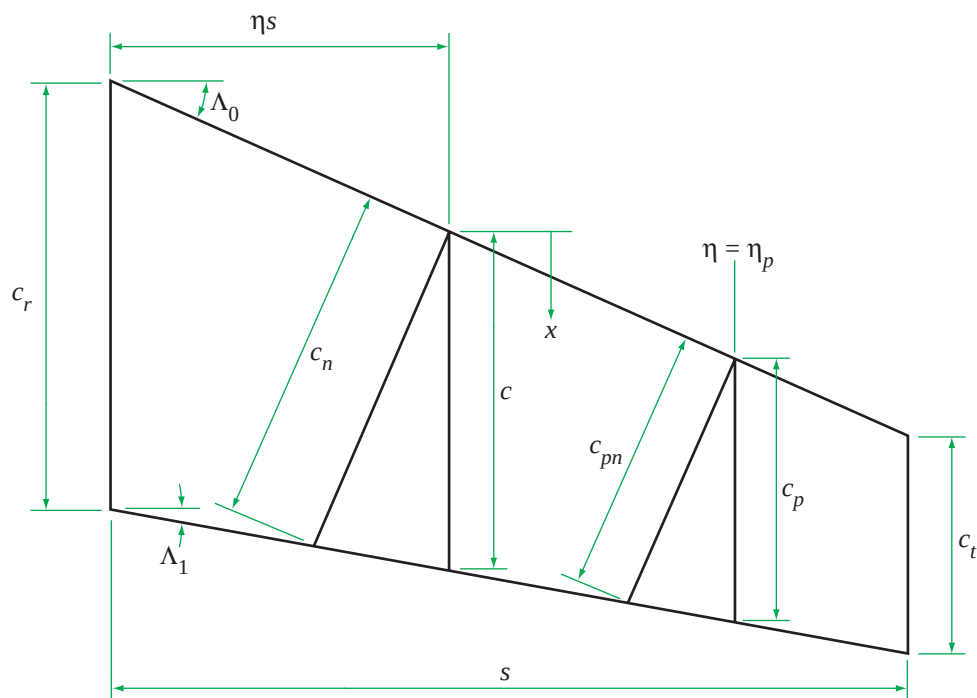
Subscripts

n	denotes quantity measured in direction normal to leading edge
p	denotes value for section at η_p
r	denotes value for section at wing root
t	denotes value for section at wing tip
∞	denotes value for two-dimensional section



Sketch 1.1 Section in streamwise direction

The chord line is defined as the straight line connecting the leading-edge and trailing-edge points. For a section with a finite base thickness, the trailing-edge point is taken as the mid-thickness point. The leading-edge point is defined as that unique point at which a circle centred at the trailing-edge point is tangential to the section.



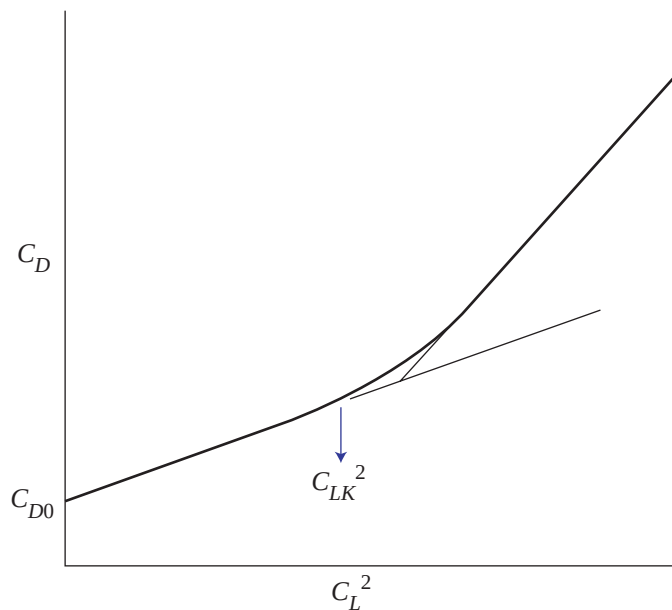
Sketch 1.2 Wing notation

2. INTRODUCTION

The boundaries of linear characteristics of wings at low speeds are dependent on the flow separation characteristics of the wing section and on the various planform parameters. This Item includes some discussion on the phenomena and causes and consequences of the flow separation which initiates non-linear characteristics and on the general effects of adding camber and twist to a plane wing.

Prediction methods available elsewhere require detailed calculations for the load distribution across the span for several incidences. In this Item a method is provided for the rapid prediction of incidence and lift coefficient for the onset of non-linear characteristics for a wing with combined twist and variation of camber and of thickness across the wing span. Separate Sections are provided to deal with the wing without camber or twist and for the increments due to camber and twist so that, if required, estimates can be made for any one of these separately.

Non-linear behaviour for wing sections normally occurs at similar values of C_L and α for drag, pitching moment and lift. The method derived in this Item has been based primarily on the onset of non-linear behaviour of C_D against C_L^2 . The value of C_{LK} is defined in Item No. 66031 (Derivation 2), as in Sketch 2.1. The methods and data of this Item are only applicable to two-dimensional sections and wings with smooth leading edges or with a narrow transition band of roughness height just sufficient to ensure transition.



Sketch 2.1 Definition of C_{LK}

The methods described in this Item are based on concepts which relate to the characteristics of wings with only small values of sweep. At large sweep angles when leading-edge vortex flows occur the essentially two-dimensional concepts assumed no longer apply and the characteristics will be dominated by the three-dimensional nature of the flow. Although the method produces results within the quoted accuracy for wings with leading-edge sweep up to 63° , caution should be exercised if the Item is used for leading-edge sweep much greater than 45° . It should be noted that for wings with a combination of high sweep (greater than about 45°) and low aspect ratio the initial departure from linearity predicted by the method of this Item occurs at a C_L very much lower than the maximum attainable value.

A guide to the calculation procedure is given in Appendix A.

3. REQUIRED DATA ITEMS

Other ESDU Data Items that may be required in the use of this Item are:

Derivation No.	Data Item No.	For Derivation of
1	W.01.01.05	Two-dimensional section lift-curve slope a_∞
3	66034	Upper-surface ordinate at 1.25 per cent chord.
4	70011	Wing lift-curve slope a_1
5	76003	Various geometrical relationships for wings
6	83040	Spanwise centre of pressure $\bar{\eta}$

4. FLOW SEPARATION - ITS CAUSES AND CONSEQUENCES

The initial departure of force and moment characteristics ($C_L \sim \alpha, C_m \sim \alpha, C_D \sim C_L^2$) from the linear is caused by the onset of regions of flow separation on the wing. The values of C_L and α at which initial departure occurs are denoted by C_{LK} and α_K . Item No. 66034 (Derivation 3) describes four types of stall and presents the relation between the lift-curve variation at high incidence and the development of the separated regions. For the purposes of identifying the onset of separation it is sufficient to consider just leading-edge and trailing-edge separation types whereby C_{LK} and α_K can be related to leading-edge radius, section thickness and Reynolds number. This applies to both symmetrical and cambered two-dimensional sections.

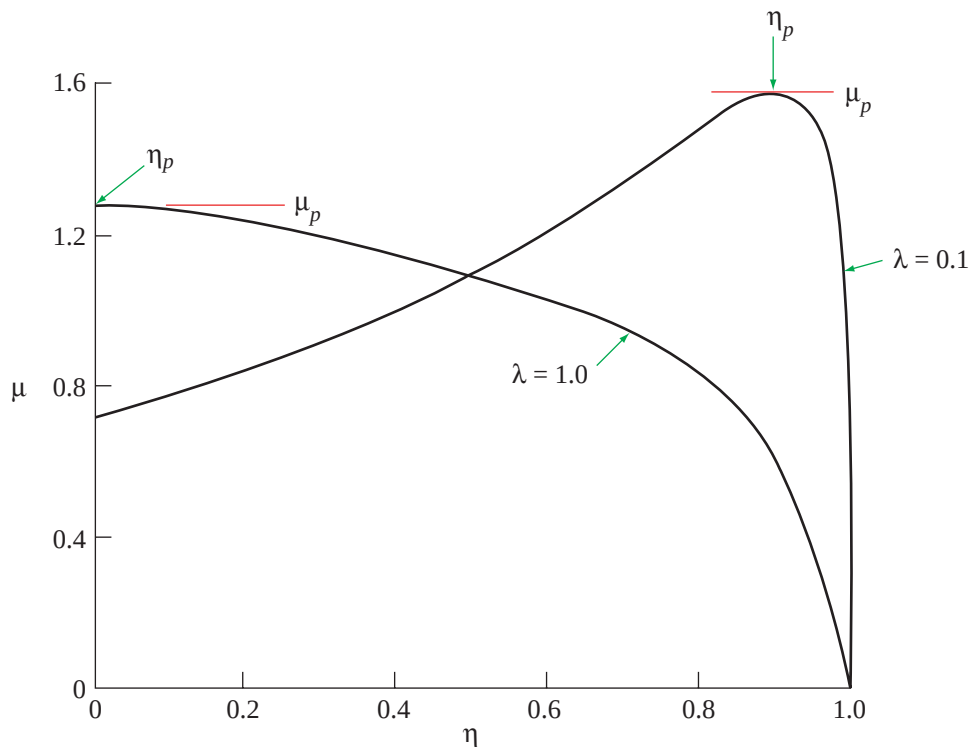
For a two-dimensional section, flow separation characteristics which mark the onset of non-linear aerodynamic characteristics are dependent on section thickness, leading edge shape, chordwise position of maximum thickness, magnitude and position of maximum camber, surface roughness, Reynolds number and Mach number. For a wing the characteristics are also dependent on the planform parameters (taper ratio, aspect ratio and sweep) and on the distribution of twist.

If the initial onset of separation occurs near the wing tip it is likely to manifest itself by producing a deterioration and probable reversal in roll damping and a marked change in rolling moment due to sideslip. The sign of this change is dependent on wing sweep. Other manifestations are pitch break and buffet which vary in severity depending on wing sweepback and aspect ratio.

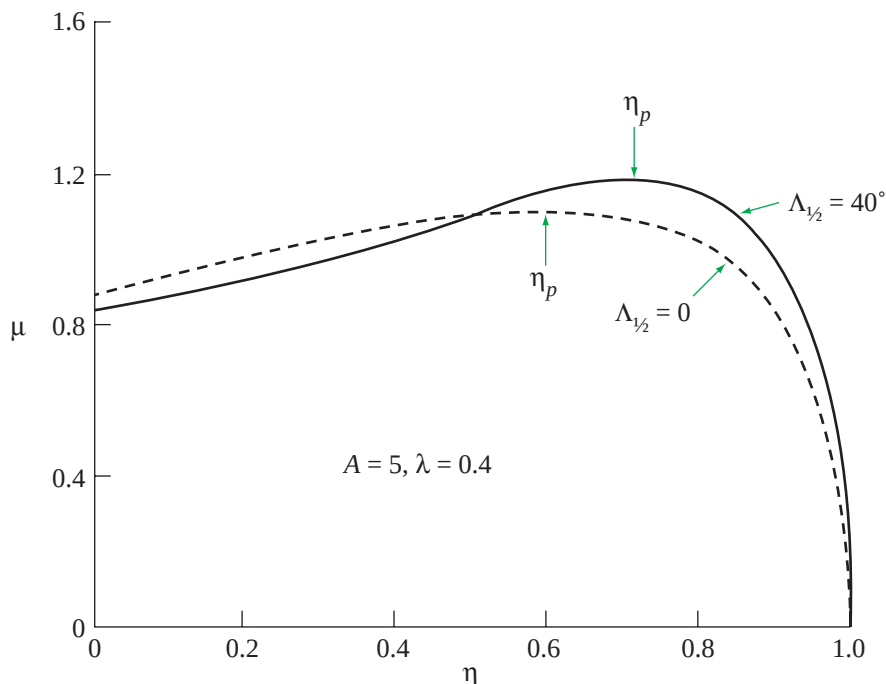
5. GENERAL EFFECTS OF CAMBER AND TWIST

Twist and combination of twist and camber may be used to alleviate the problems of tip separation by encouraging an inboard movement of the initial point of separation. A far inboard initial point of separation will result in a pitch down for a swept-back wing and this is likely also to result in some reduction in the maximum lift attainable for the wing. A good compromise to obtain satisfactory pitch characteristics is for the initial separation to occur over a broad area of the inner half of the wing semispan, see Derivation 26.

For an untwisted wing of constant profile across the span it can be deduced from Item No. 83040 (Derivation 6) that the spanwise position with the greatest local lift coefficient for the loading due to incidence is dependent on those parameters that affect $\bar{\eta}$, the spanwise centre of pressure of this basic loading. These are aspect ratio, sweep, taper ratio and Mach number, see Sketches 5.1 and 5.2, which show the effect of taper ratio and of sweep on the spanwise distribution of $\mu (= C_{LL}/C_L)$. Secondary terms involving aspect ratio, sweep and Mach number give minor changes in the spanwise distribution at constant $\bar{\eta}$, which can be neglected.



Sketch 5.1 Effect of taper ratio on spanwise variation of local lift coefficient



Sketch 5.2 Effect of sweep on spanwise variation of local lift coefficient

For wings with little or no sweep the onset of separation will occur close to η_p , the spanwise station of the streamwise section with the highest local lift coefficient, as shown in Sketch 5.1. In Derivation 26 it is concluded that it is possible to choose a spanwise variation of section characteristics to match the desired variation of C_{LL} for a cambered and twisted wing. In this way satisfactory pitching, rolling, buffet and drag characteristics can be achieved.

However, for swept back wings the attendant boundary-layer crossflow removes boundary layer from the inner part of the wing thereby delaying the inboard separation, and increases the thickness of the boundary layer on the outer part of the wing so as to induce an earlier outboard separation. For a swept back wing the effective angle of incidence at the tip is increased and negative camber is induced at the tip. These effects combine to promote stall nearer the tip.

The effect on the C_L for initial separation of adding washout (*i.e.* progressively more nose down twist across the span towards the tip) to a wing of constant profile across the span tends to be small for taper ratios in the region of 0.4 to 0.6. For taper ratios of 0.6 to 1.0 the effect of such a twist is to reduce C_{LK} since the inboard part of the wing is almost unaffected by the twist (and therefore suffers separation at a similar incidence) whilst the lift on the outboard portion has been reduced by applying washout. Conversely, for more highly tapered wings the effect of twist will be to produce a small increase in C_{LK} since the greater inboard wing chord will sustain a higher lift before separation whilst the washout will only result in a small reduction of lift on the shorter chord near the tip.

If the addition of camber to the symmetrical section does not change the type of section stall, the characteristics of a wing with camber will be essentially the same as for the wing without camber except that the initial onset of separation will occur at a lift coefficient similar to that for the cambered section.

6. EFFECTS OF REYNOLDS NUMBER

The values of α_K and C_{LK} for symmetrical section wings are dependent on Reynolds number. However, from the data of Derivation 25 it is apparent that the increments in α_K and C_{LK} due to camber for wing sections are not dependent on Reynolds number. Also, for the wings for which data were available for use in the preparation of this Item there were no clear Reynolds number effects on the increments in α_K and C_{LK} due to camber and twist for the Reynolds number range covered.

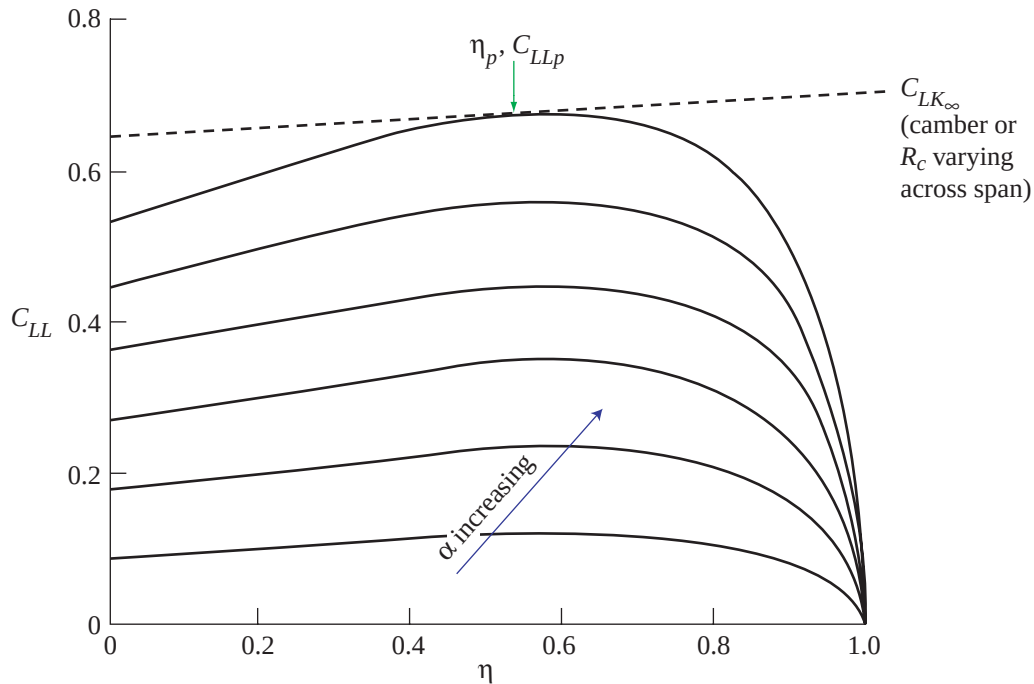
7. EFFECTS OF MACH NUMBER

From Derivations 29 and 30 it was found that the effect of Mach number on α_K , for uncambered and cambered wings respectively, was negligible up to the drag-rise Mach number. This results in a small increase in C_{LK} and C_{LKC} due to Mach number which reflects the change of wing lift-curve slope. It was also found in Derivation 29 that no change in the separation type took place with increase of Mach number up to the drag-rise. However, in view of the small amount of experimental evidence it is advised that caution be exercised if this Item is to be used for the assessment of initial separation characteristics when there is local supersonic flow.

8. PREDICTION METHOD

8.1 Derivation of Method

The method is developed from that of Derivations 9, 13 and 27. For wings with little or no sweep, where the effects of outboard flow of the boundary layer may be neglected, it is necessary to establish the spanwise variation of C_{LL} and to compare this with the spanwise variation of $C_{LK\infty}$. To obtain the spanwise lift distribution at this condition calculations may be made using (for example) Item No. 83040 for a number of incidences, including the effects of camber and twist. For the incidence at which C_{LLp} matches $C_{LK\infty}$ the distribution of $C_{LL}c/\bar{c}$ must be obtained and integrated to obtain the value of C_{LK} for the wing, see Sketch 8.1.



Sketch 8.1

To derive the method of this Item some simplifying assumptions have been made to reduce this task to the calculation for only the most highly loaded spanwise station. The spanwise position of the most highly loaded section for the loading due to incidence, η_p , and the maximum normalised lift coefficient, μ_p , have been derived from Item No. 83040 and are shown in Figures 1 and 2, in terms of taper ratio, λ , and the spanwise centre of pressure, $\bar{\eta}$, corresponding to the spanwise loading, σ , due to incidence.

For large amounts of twist or large variation of $C_{LK\infty}$ across the span due to exceptionally rapid changes of camber or a section characteristic critically dependent on Reynolds number the method of this Item might not give reliable results. However, for the usual case where these variations are more gradual the method of this Item may be used with confidence.

8.2 Estimation of Initial Separation Type

Since α_K , C_{LK} and the increments due to camber are dependent on the initial separation type it is necessary to establish which type applies.

Figure 3 gives a qualitative guide to the type of initial separation to be expected on two-dimensional sections and wings, based on the parameters R_{cpn} and $z_{u1.25n}/c_n$. The value of R_{cpn} is given by

$$R_{cpn} = R_{\bar{c}} \cos \Lambda_0 \frac{c_{pn}}{\bar{c}} \quad (8.1)$$

where

$$\frac{c_{pn}}{\bar{c}} = \frac{3}{2} \left[\frac{1 + \lambda}{1 + \lambda + \lambda^2} \right] [\cos \Lambda_0 + \sin \Lambda_0 \tan(\Lambda_0 - \Lambda_1)] (1 - \eta_p + \lambda \eta_p) \quad (8.2)$$