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Lift and longitudinal forces on propeller/nacelle/wing/flap systems

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LIFT AND LONGITUDINAL FORCES ON PROPELLER/NACELLE/WING/FLAP SYSTEMS

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LIFT AND LONGITUDINAL FORCES ON PROPELLER/NACELLE/WING/FLAP SYSTEMS

1. NOTATION AND UNITS

		<i>SI</i>	<i>British</i>
A	wing aspect ratio, b^2/S_w		
b	wing span	m	ft
$[C_D]_0$	nacelle/wing/flap drag coefficient without slipstream, (drag)/($q_0 S_w$)		
$[C_L]_0$	nacelle/wing/flap lift coefficient without slipstream, (lift)/($q_0 S_w$)		
C_T	propeller thrust coefficient, $T/(q_s S_p)$		
C'_T	propeller thrust coefficient, $NT/(q_0 S_w)$ see Equation (3.10)		
C_X, C_Z	total force coefficient on propeller/nacelle/wing/flap system in X-, Z-direction, $X/(q_s NS_p), Z/(q_s NS_p)$		
C'_X, C'_Z	total force coefficient on propeller/nacelle/wing/flap system in X-, Z-direction, $X/(q_0 NS_w), Z/(q_0 NS_w)$, see Equations (3.8) and (3.9)		
C_{Xp}, C_{Zp}	component of C_X, C_Z due to forces on propellers		
C_{Xwf}, C_{Zwf}	component of C_X, C_Z due to forces on wing/flap system, including nacelles when appropriate		
c	wing chord at propeller station, see Sketch 1.1	m	ft
c_t	flap chord, see Sketches 1.1 and 1.2	m	ft
c_{teff}	effective flap chord, see Section 5.1.2 and Equation (5.7)	m	ft
c_{t1}, c_{t2}	chord of first and second effective flap elements, see Sketches 1.2	m	ft
D	propeller disc diameter, see Sketch 1.1	m	ft
d_n	nacelle diameter, see Sketch 1.1	m	ft
f	factor in Equation (5.7) for effective flap chord		
H_{zp}	factor in Equation (5.4) allowing for effect of z_p on $[\theta_s]_{1d}^0, \alpha_p = 0$		

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I_{yc}, I_{zp}	factors in Equation (6.3) allowing for effects of y_c and z_p on flap turning effectiveness		
i_w	angle between wing chord and propeller shaft axis, $\alpha_w - \alpha_p$, see Sketch 1.1	degree	degree
J_{iw}, J_{yc}, J_{zp}	factors in Equation (6.5) allowing for effect of i_w, y_c and z_p on $\Delta[k_s]_{1d}$		
K	flap turning effectiveness		
K_d	value of K for datum condition, as function of c_{teff}/D in Figure 1, see Sections 5.1 and 5.1.2		
K_{dc}	'full-chord' value of K_d taken from mean line in Figure 1 with $c_{teff}/D = c/D$		
k_s	slipstream viscous loss parameter at thrust coefficient C_T		
$\Delta[k_s]_1$	increase in k_s due to flap deployment under static ($C_T = 1$) conditions, see Equation (6.5)		
N	total number of propellers in propeller/nacelle/wing/flap system		
q_0	free-stream kinetic pressure, $\frac{1}{2}\rho V_0^2$	N/m ²	lbf/ft ²
q_s	kinetic pressure in fully-developed slipstream, $\frac{1}{2}\rho V_s^2$	N/m ²	lbf/ft ²
r	velocity ratio, $V_0/V_s = (1 - C_T)^{1/2}$, see Equation (3.5)		
S_n	nacelle maximum cross-sectional area	m ²	ft ²
S_p	propeller disc area, $\pi D^2/4$	m ²	ft ²
S_{sp}	spinner frontal area	m ²	ft ²
S_w	wing planform area	m ²	ft ²
T	thrust per propeller	N	lbf
V_0	free-stream velocity, see Sketch 1.1	m/s	ft/s
V_s	velocity in fully-developed slipstream	m/s	ft/s
X, Z	total forces in X-, Z-direction, see Sketch 1.1	N	lbf
x_p	axial location of propeller disc plane with respect to wing leading edge, see Sketch 1.1	m	ft
y_{ol}	propeller overlap, see Sketch 8.1	m	ft
y_c	flap cut-out, see Sketch 6.1	m	ft
y_{ceff}	effective flap cut-out, see Sketch 6.1	m	ft

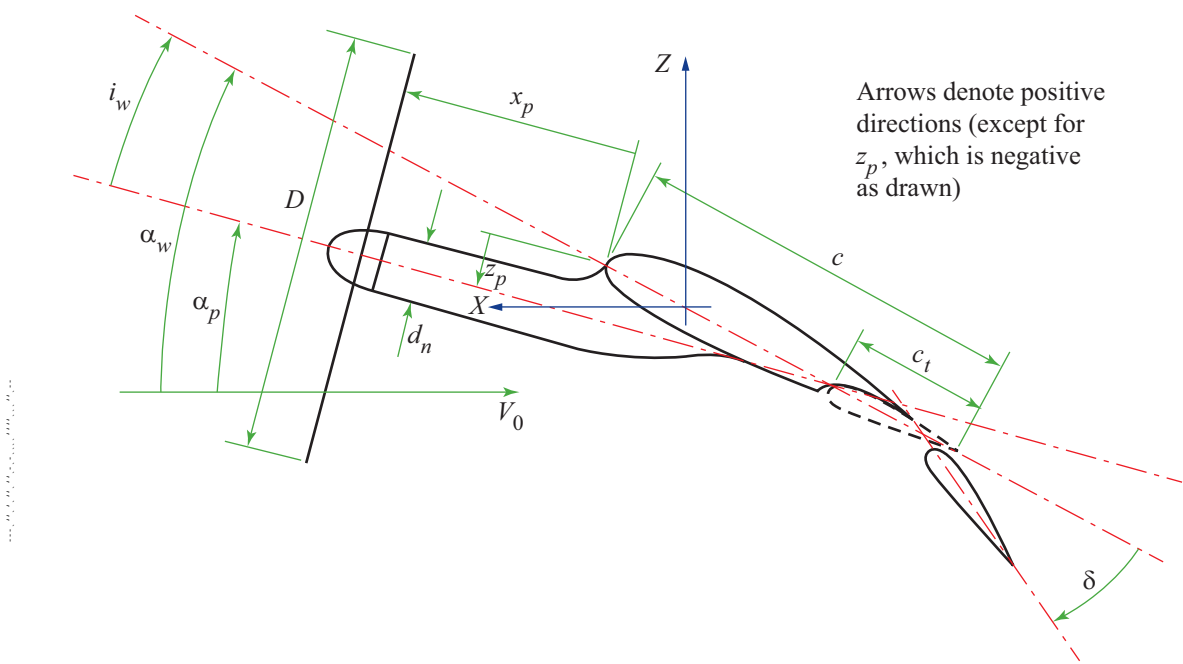
z_p	height of propeller shaft axis above wing leading edge, see Sketch 1.1	m	ft
α_p	angle of attack of propeller shaft axis, see Sketch 1.1	degree	degree
α_w	angle of attack of section chord line at propeller station, see Sketch 1.1	degree	degree
α_0	angle of attack at zero lift of wing section at propeller station	degree	degree
δ_{t1}, δ_{t2}	deflection of first and second flap elements, see Sketch 1.2	degree	degree
δ	total flap deflection, deflection of final flap element relative to chord line, see Sketch 1.2	degree	degree
θ_s	slipstream deflection angle relative to free-stream velocity	degree	degree
$\Delta[\theta_s]_1$	increase in θ_s due to flap deployment under static ($C_T = 1$) conditions	degree	degree
λ	wing taper ratio		
ρ	density of air	kg/m ³	lb/ft ³

Subscripts

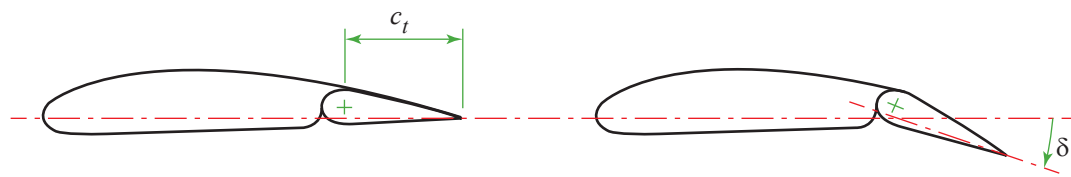
$[]_d$	denotes datum condition, $i_w = 0$ and $z_p = 0$
$[]_0$	denotes quantity in power-off ($C_T = 0$) condition
$[]_1$	denotes quantity in static ($V_0 = 0$, $C_T = 1$) condition

Superscripts

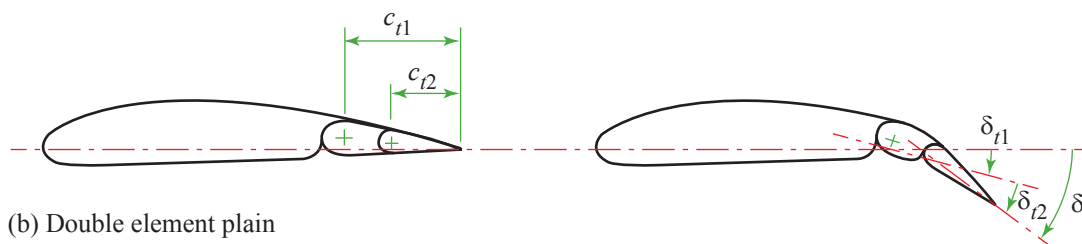
$[]^{i_w=0}$	denotes quantity relating to $i_w = 0$
$[]^{\alpha_p=0}$	denotes quantity relating to $\alpha_p = 0$
$[]^\delta$	denotes quantity relating to flaps deployed condition, δ
$[]^0$	denotes quantity relating to flaps-up condition, $\delta = 0$



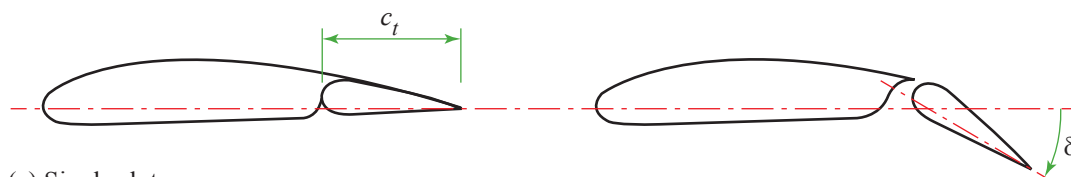
Sketch 1.1 Geometry of propeller/nacelle/wing/flap



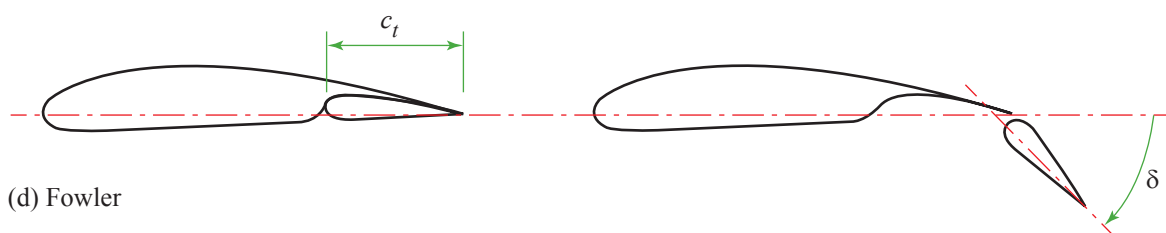
(a) Single element plain



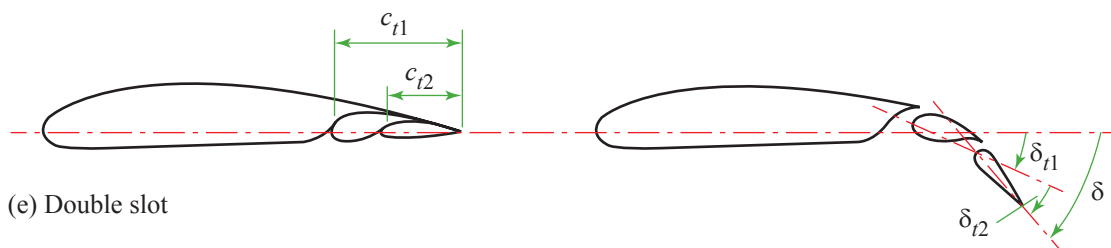
(b) Double element plain



(c) Single slot



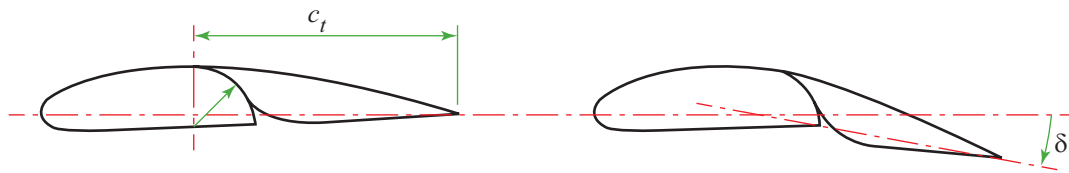
(d) Fowler



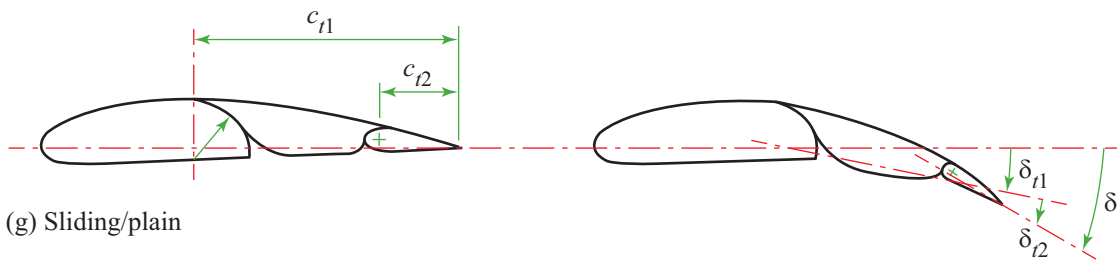
(e) Double slot

Sketch 1.2 Flap types used in the analysis

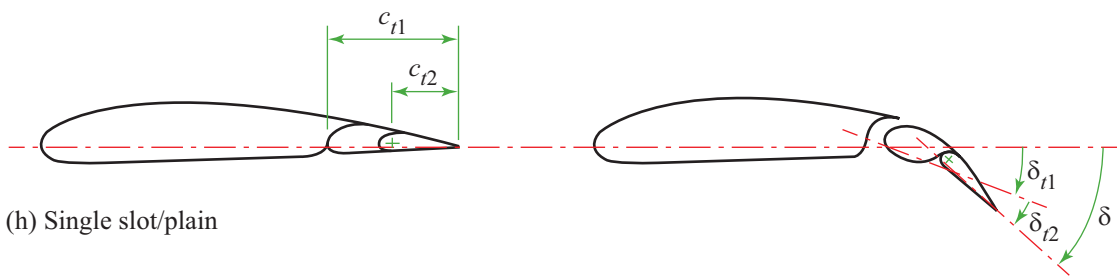
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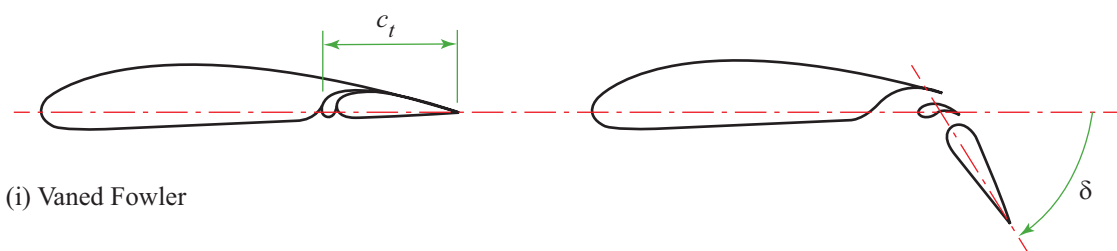
(f) Sliding



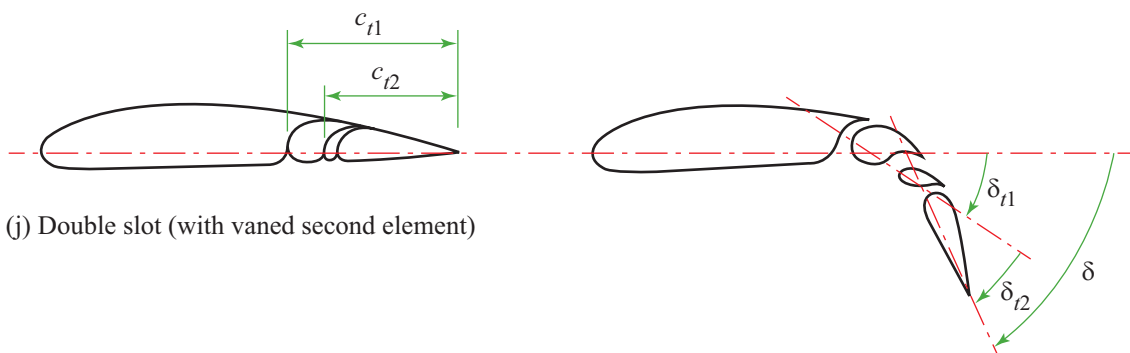
(g) Sliding/plain



(h) Single slot/plain



(i) Vaned Fowler



(j) Double slot (with vaned second element)

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2. INTRODUCTION

This Item provides a means of estimating the free-air low-speed lift and longitudinal forces on unswept wing/flap systems fitted with multiple tractor propeller/nacelle combinations. The propeller slipstreams are assumed to flow over the wing/flap system and are therefore deflected by it, thereby providing a form of thrust vectoring which augments the lifting force. The method is best suited to cases with moderately large flap deflections, as in take-off or landing, provided that the flow is fully attached. It does not however account for ground effects.

The method (Section 3), which is semi-empirical, is based on the application of simple momentum theory (see Appendix A). In this respect it is similar to the method of Derivation 10 but overcomes the acknowledged deficiencies of that method. If the power-off values of lift and longitudinal force are known the method requires a knowledge of only two additional, slipstream related, parameters (Section 4) which are dependent on thrust coefficient. The form of the variation of these parameters with thrust coefficient for attached flow conditions is determined empirically (see Appendix B) and the power-off values are determined by using the known power-off forces. Therefore the variation of the slipstream parameters with thrust coefficient can be simply determined with a knowledge of their values under static (zero forward speed) power-on conditions (Section 5).

The basic method is restricted to (i) flaps which are continuous across the slipstream, (ii) nacelles which are circular in cross-section with a diameter equal to that of the spinner and (iii) propellers which are not overlapped. Sections 6 to 8 provide some guidance on the effects of flap cut-out, nacelle size and propeller overlap, respectively. The applicability and accuracy of the method, which is applicable to a wide range of flap types, are discussed in Section 9.

Detailed worked examples illustrating typical uses of the Item are given in Section 11.

A program of the method is provided as ESDUpac A8831 and is described in Appendix C.

3. TOTAL LIFT AND LONGITUDINAL FORCES ON PROPELLER/NACELLE/WING/FLAP SYSTEMS

In free air at low speeds the total force coefficient in the Z-direction on a wing/flap system equipped with N non-overlapping tractor propellers is obtained from the application of simple momentum theory in Equations (A4.22) and (A4.14) of Appendix A as

$$C_Z = C_{Zp} + C_{Zwf} = (1+r)(1-k_s)\sin\theta_s + \left[\frac{2}{N} \left(\frac{b}{D} \right)^2 - (1+r) \right] r^2 \left[1 - [k_s]_0^\delta \right] \sin[\theta_s]_0^\delta \quad (3.1)$$

and similarly in the X-direction from Equations (A4.21) and (A4.13)

$$C_X = C_{Xp} + C_{Xwf} = (1+r) \left[(1-k_s)\cos\theta_s - r \right] + \left[\frac{2}{N} \left(\frac{b}{D} \right)^2 - (1+r) \right] r^2 \left\{ \left[1 - [k_s]_0^\delta \right] \cos[\theta_s]_0^\delta - 1 \right\}. \quad (3.2)$$

Although Equations (3.1) and (3.2) were derived for propeller/wing/flap systems, they are also assumed to apply when nacelles are present, for which the subscripts wf are retained.

If the forces on the wing/flap system, or nacelle/wing/flap system ($C_{z_{wf}}$, $C_{x_{wf}}$) only are required it may be assumed, as in Equation (A4.16) and (A4.15), that the propeller force coefficients (C_{Zp} , C_{Xp}) in Equations (3.1) and (3.2) are given by

$$C_{Zp} = C_T \sin \alpha_p \quad (3.3)$$

$$\text{and} \quad C_{Xp} = C_T \cos \alpha_p \quad (3.4)$$

for $\alpha_p \leq 15^\circ$, say.

The force coefficients C_Z and C_X are based on q_s , the kinetic pressure in the slipstream and NS_p , the total propeller disc area for the configuration. Section 3.1 gives factors for their conversion to a system based on q_0 and S_w .

The parameter r is the ratio of the free-stream velocity V_0 , to that in the fully-developed slipstream V_s , obtained from simple momentum theory as

$$r = V_0/V_s = (1 - C_T)^{1/2} \quad (3.5)$$

where C_T is the propeller thrust coefficient.* The factor b/D is the ratio of the wing span to propeller disc diameter.

The values of θ_s and k_s corresponding to the slipstream deflection angle and the viscous loss factor in the power-on condition are obtained using the empirically-based procedure described in Section 4. These two slipstream parameters are used simply as a means of evaluating Equations (3.1) and (3.2) and it is not intended that they be used for any other purpose. Their values in the power-off condition, $[\theta_s]_0^\delta$, and $[k_s]_0^\delta$, are assumed to be obtainable from known values of the lift and drag coefficients, $[C_L]_0$ and $[C_D]_0$, in the power-off condition via the equations

$$[\theta_s]_0^\delta = \tan^{-1} \left\{ \frac{2}{\pi A} [C_L]_0 / \left(1 - \frac{2}{\pi A} [C_D]_0 \right) \right\} \quad (3.6)$$

and

$$[k_s]_0^\delta = 1 - \left\{ \left(\frac{2}{\pi A} [C_L]_0 \right)^2 + \left(1 - \frac{2}{\pi A} [C_D]_0 \right)^2 \right\}^{1/2}, \quad (3.7)$$

as in Equations (A5.11) and (A5.12)

* From actuator theory, C_T can also be written conveniently as $C_T = (T/S_p)/[(T/S_p) + q_0]$, which shows that $C_T = 1$ when $q_0 = 0$, i.e. the static condition.

3.1 Conversion of Force and Thrust Coefficients

If the force and thrust coefficients in Equations (3.1) to (3.4) are to be based on the more usual kinetic pressure in the free-stream, q_0 , and wing planform area, S_w , they can be converted using the following equations

$$C'_Z = \frac{C_Z}{1 - C_T} \frac{NS_p}{S_w}, \quad (3.8)$$

$$C'_X = \frac{C_X}{1 - C_T} \frac{NS_p}{S_w} \quad (3.9)$$

and
$$C'_T = \frac{C_T}{1 - C_T} \frac{NS_p}{S_w}. \quad (3.10)$$

The inverse forms of these equations are

$$C_Z = \frac{C'_Z}{(NS_p/S_w) + C'_T}, \quad (3.11)$$

$$C_X = \frac{C'_X}{(NS_p/S_w) + C'_T} \quad (3.12)$$

and
$$C_T = \frac{C'_T}{(NS_p/S_w) + C'_T}. \quad (3.13)$$

4. EVALUATION OF θ_s AND k_s

Appendix B develops empirical prediction schemes for θ_s and k_s as functions of C_T between the power-off ($C_T = 0$) and static ($C_T = 1$) cases. The resulting equations (with the argument of sine in degrees) are

$$\theta_s = [\theta_s]_0^\delta + \left[C_T + 0.3 \sin(180 C_T^{1.36}) \right] \left[[\theta_s]_1^\delta - [\theta_s]_0^\delta \right] \quad (4.1)$$

where
$$[\theta_s]_1^\delta = [\theta_s]_1^{0, \alpha_p=0} + K\delta + \alpha_p \quad (4.2)$$

and
$$k_s = [k_s]_0^\delta + C_T \left[[k_s]_1^\delta - [k_s]_0^\delta \right] \quad (4.3)$$

where
$$[k_s]_1^\delta = [k_s]_1^0 + \Delta[k_s]_1. \quad (4.4)$$

The remaining unknown variables, $[\theta_s]_1^{0, \alpha_p=0}$ and K in Equation (4.2) and $[k_s]_1^0$ and $\Delta[k_s]_1$ in Equation (4.4), are determined from correlations of test data at zero forward speed ($C_T = 1$) as described in Sections 5.1 and 5.2, respectively.

5. DETERMINATION OF $[\theta_s]_1^\delta$ AND $[k_s]_1^\delta$

In the determination of $[\theta_s]_1^\delta$ and $[k_s]_1^\delta$ it was found that the important variables were:-

(i) For propeller

α_p , the angle of attack of the propeller shaft axis,

i_w , the angle between the wing chord and the propeller shaft axis,

x_p/D , the dimensionless location of the propeller disc forward of the wing leading-edge,

z_p/D , the dimensionless location of the propeller shaft axis, positive above the wing leading-edge,

and y_{ol}/D , the dimensionless overlap for multiple propellers.

(ii) For nacelle

S_n/S_p , the dimensionless nacelle frontal area.

(iii) For wing

α_0 , angle of attack at zero lift for the wing section at the propeller station

and c/D , dimensionless wing chord at the propeller station.

(iv) For flap system

c_{teff}/D , the dimensionless effective flap chord at the propeller station, see Equation (5.7)

δ , the deflection of the final flap element relative to the wing section chord, *i.e.* the total flap deflection at the propeller station

and y_c/D , the dimensionless flap cut-out in the slipstream.

Flap type was found to be unimportant to the correlations.

The effects of these parameters are dealt with in different ways. In the cases of i_w , z_p/D and y_c/D , it is convenient to define their effects in terms of datum values with allowance for deviations from the datum values.

Sections 5.1 and 5.2 deal with the estimation of $[\theta_s]_1^\delta$ and $[k_s]_1^\delta$ respectively, for flaps which are continuous across the slipstream, nacelles which are circular in cross-section with a diameter equal to the spinner diameter and propellers which are not overlapped. Some guidance on the effect of flap cut-out, nacelle size and propeller overlap is given in Sections 6 to 8 respectively.

5.1 Determination of $[\theta_s]_1^\delta$

Analysis of test data (Derivations 1 to 9, 11 to 18 and 20) under static conditions ($V_0 = 0$, $C_T = 1$) for a wide range of unswept wing models, having single-element or multi-element plain or slotted flaps and either one or two* propellers per semi-span, led to the following extension of Equation (4.2):

$$[\theta_s]_1^\delta = H_{zp}[\theta_s]_{1d}^{0, \alpha_p=0} + 1.15 I_{zp} K_{dc} i_w + I_{zp} K_d \delta + \alpha_p, \quad (5.1)$$

where the subscript d denotes the datum condition with $i_w = 0$ and $z_p = 0$, K_{dc} and K_d are turning effectiveness parameters and H_{zp} and I_{zp} are correction factors for wing height when $|z_p| \neq 0$. The empirical factor 1.15 is associated with K_{dc} and there is the proviso that the value of $1.15 K_{dc}$ must be capped at unity when evaluated, which means $K_{dc} \leq 0.87$ here. Comparison of Equation (4.2) with Equation (5.1) gives

$$[\theta_s]_1^{0, \alpha_p=0} = H_{zp}[\theta_s]_{1d}^{0, \alpha_p=0} + 1.15 I_{zp} K_{dc} i_w \quad (5.2)$$

$$\text{and} \quad K = I_{zp} K_d. \quad (5.3)$$

5.1.1 Flaps-up value, $[\theta_s]_1^{0, \alpha_p=0}$

The flaps-up value of $[\theta_s]_1^{0, \alpha_p=0}$ is given by Equation (5.2). Test data in Derivation 3 show that, to a reasonable approximation, for any flap deflection the value of $([\theta_s]_1^{0, \alpha_p=0} - i_w)$ is independent of i_w and this has been incorporated in Equation (5.2) in terms of the flaps-up case, $\delta = 0$. To generalise the result to geometries other than those tested in Derivation 3 ($c_{teff}/D = 0.78$ inboard, 0.68 outboard and $z_p = 0$), i_w appears in Equation (5.2) factored by $1.15 I_{zp} K_{dc}$, with $1.15 K_{dc}$ and I_{zp} both being unity for the Derivation 3 model. The application of a turning factor to i_w is in line with the original work in Derivation 10 and the presence of the height factor I_{zp} ensures a correct theoretical limit because as $|z_p|$ becomes large both H_{zp} and I_{zp} decrease from unity and become small so that $[\theta_s]_1^{0, \alpha_p=0}$ tends towards zero. This is consistent with the physical reasoning that the wing can have no influence on the slipstream turning angle when it is well above or below the propeller. The factor 1.15 is an empirical adjustment and in the use of the method an upper bound of about 0.87 must here be imposed on the value of K_{dc} , as noted above Equation (5.2). The factors H_{zp} , I_{zp} and K_{dc} are discussed further below.

There are very few reliable data on which to base a method for estimating $[\theta_s]_1^{0, \alpha_p=0}$ when $i_w = 0$. Furthermore, the majority of the models tested had either symmetrical sections, for which $[\theta_s]_1^{0, \alpha_p=0, i_w=0} = 0^\dagger$, or the NACA 4415 section, for which $[\theta_s]_1^{0, \alpha_p=0, i_w=0} > 0$ due to camber. Data are given in Derivation 8 which can be analysed for the most significant effects on $[\theta_s]_1^{0, \alpha_p=0, i_w=0}$ which are related to camber, c/D and z_p/D . In an attempt to allow a wider application of the data in Derivation 8, which concern only the NACA 4415 section, it has been assumed that camber effects can be characterised by α_0 , the section angle of attack for zero lift at the propeller station. This leads to the formula

$$[\theta_s]_1^{0, \alpha_p=0, i_w=0} = H_{zp}[\theta_s]_{1d}^{0, \alpha_p=0}, \quad (5.4)$$

* Data for overlapped propellers were included in the analysis since overlap was found to have no significant effect on $[\theta_s]_1^\delta$.

† Strictly, of course, this is only true for an untapered, untwisted wing with constant thickness/chord across the span. However, in practice the effects of such asymmetries across the slipstream width are insignificant.

$$\text{where } \left. \begin{aligned} H_{zp} &= 1 - 2.5|z_p/D| && \text{for } |z_p/D| \leq 0.4, \\ H_{zp} &= 0 && \text{for } |z_p/D| > 0.4, \end{aligned} \right\} \quad (5.5)$$

$$\text{and } [\theta_s]_{1d}^{0, \alpha_p=0} = -2K_{dc}\alpha_0, \quad (5.6)$$

where K_{dc} is the ‘full-chord’ flap turning effectiveness from the mean line in Figure 1 (see Section 5.1.2) when c_{teff}/D is replaced by c/D as hypothesised by Kuhn in Derivation 10. An estimate of α_0 may be obtained from ESDU 87031 (Derivation 22).

Needless to say, Equations (5.5) and (5.6) are very tentative and should be applied with caution. Fortunately, however, the contribution of $[\theta_s]_1^{0, \alpha_p=0, i_w=0}$ to $[\theta_s]_1^\delta$ is usually small for moderate to large values of δ so that any errors in its estimation will be relatively insignificant.

There is only a single set of data, from Derivation 25 for the Short Belfast, where i_w and z_p/D are both simultaneously of sufficient magnitude to check the modelling of the influence of i_w in Equation (5.2) when $|z_p| \neq 0$. As stated previously, for theoretical reasons a height factor that decreases from unity as $|z_p|$ increases must be present and a satisfactory pragmatic option is provided by I_{zp} , the factor applied to the flap turning effectiveness, see Section 5.1.2. This use of I_{zp} has been established as satisfactory for the test in Derivation 25, where $i_w = 4.23^\circ$ and $z_p/D = -0.26$. However, with the dearth of data for further verification, the influence of the contribution from the second term of Equation (5.2), like that from the first, should be reviewed in the overall evaluation of θ_s to confirm that uncertainty in its exact value is of minor importance. (See for instance the worked example in Section 11.1.1.)

5.1.2 Flap turning effectiveness, K

Analysis of test data in Derivations 1, 2, 3, 5, 8, 9, 12, 14, 15, 16 and 18 has shown that flap turning effectiveness can be represented by Equation (5.3), *i.e.* $K = I_{zp}K_d$, where K_d is the flap turning effectiveness for $z_p = 0$ and I_{zp} is a factor allowing for variation in z_p/D .

Figure 1 shows the variation of K_d with c_{teff}/D . The Figure was derived from test data for single-element and double-element plain and slotted flaps of the types shown in Sketches 1.2a to 1.2e. The hatched area represents the approximate spread of the test data, and its upper boundary is labelled as good design practice to indicate what might be achieved with an optimised flap system, see Section 9.1. The mean line is always used for K_{dc} (*i.e.* for the flaps undeployed case). Within this area no consistent trends in the test data, with other than c_{teff}/D , were discernible and, in particular, no consistent effect of propeller overlap was seen.

For the single-element flap c_{teff} is simply the flap chord defined in Sketches 1.2a, 1.2c and 1.2d. For the double-element flaps of Sketches 1.2b and 1.2e c_{teff} is dependent on the relative deflection of each element and is defined empirically by

$$\frac{c_{teff}}{D} = f \frac{c_{t1}}{D} + (1-f) \frac{c_{t2}}{D}, \quad (5.7)$$

where f is a function of $\delta_{t1}/(\delta_{t1} + \delta_{t2})$ shown in Figure 2, which was obtained mainly from an analysis of data in Derivations 1 and 3.

Data from Derivations 5 to 8, 13, 17, 18 and 20 show that Figure 1 also applies to the flap types shown in Sketches 1.2f to 1.2j, with flap chords and deflections defined as drawn. For this purpose the small fore-vanes in the flaps of Sketches 1.2i and 1.2j were considered as part of the next element.

It should be noted that, for the analysis, the following restrictions applied.

- (i) Only flap deflections giving substantially linear variations of $[\theta_s]_1^\delta$ with δ were used.
- (ii) For tapered wings with two propellers per semi-span the average of the c_{teff} values appropriate to the two propeller stations was used.
- (iii) Values of x_p/D varied from about 0.2 to 0.7.

The factor I_{zp} is obtained from Figure 3, which is based on the small amount of available data from Derivations 2, 8 and 11.

5.2 Determination of $[k_s]_1^\delta$

Analysis of test data (Derivations 2, 4, 5, 6, 8, 9 and 11 to 18) under static conditions ($V_0 = 0$, $C_T = 1$) for a wide range of unswept wing models having single-element or multi-element plain or slotted flaps and either one or two (non-overlapped) propellers per semi-span has led to the following extension of Equation (4.4):

$$[k_s]_1^\delta = [k_s]_1^0 + J_{iw} J_{zp} \Delta[k_s]_{1d}, \quad (5.8)$$

where again the subscript d denotes the datum condition with $i_w = 0$ and $z_p = 0$.

Comparison of Equation (4.4) with Equation (5.8) gives

$$\Delta[k_s]_1 = J_{iw} J_{zp} \Delta[k_s]_{1d}. \quad (5.9)$$

5.2.1 Flaps-up value, $[k_s]_1^0$

The value of $[k_s]_1^0$ is governed by the effects of viscosity in the slipstream flowing over the wing. In an inviscid flow, by definition, $[k_s]_1^0$ would be zero. In a viscous flow it could be anticipated that any parameter affecting the profile drag of the wing should have an effect on slipstream velocity and hence on $[k_s]_1^0$. However, analysis of data in Derivations 2, 4, 5, 6, 8, 9, and 13 to 18 indicates no significant or consistent effect of section shape, thickness/chord, Reynolds number, propeller angle of attack, number of propellers per semi-span, propeller location or direction of rotation. To within experimental accuracy then, it is perhaps best to assume

$$[k_s]_1^0 = [k_s]_0^0, \quad (5.10)$$

where $[k_s]_0^0$ is obtained from Equation (3.7) with known values of $[C_L]_0$ and $[C_D]_0$ for the flaps-up case.

5.2.2 Contribution from flap deflection, $\Delta[k_s]_1$

Analysis of test data in Derivations 2, 4, 5, 6, 8, 9, and 11 to 18 has shown that $\Delta[k_s]_1$ is independent of α_p and x_p/D over the range 0.2 to 0.7 and can be represented by Equation (5.9), where $\Delta[k_s]_{1d}$ is the increment in $[k_s]_1$ due to flap deflection for $i_w = 0$ and $z_p = 0$, and J_{iw} and J_{zp} are factors allowing for variation in i_w and z_p/D .

Figure 4 shows the variation of $\Delta[k_s]_{1d}$ with $\Delta[\theta_s]_{1d}$ for non-overlapping propellers. As with Figure 1, a hatched area shows the approximate spread of the test data, with the lower boundary here corresponding to good design practice, see Section 9. An important point to be made is that data for two propellers per semi-span (Derivations 8, 12, 13 and 18) were available only for propeller rotations in the same direction, as shown on Figure 4. There is some evidence (Derivations 1, 2, 3, 7 and 8) albeit insubstantial and for overlapped propellers only, that for propellers rotating in opposing directions the values of $\Delta[k_s]_{1d}$ could be smaller for given $\Delta[\theta_s]_{1d}$. With this in mind, the application of Figure 4 to the case of two propellers rotating in opposing directions is not recommended.

The effect of i_w is covered only in Derivation 3 for overlapping propellers. The small amount of data indicate that the effect of i_w is non-linear, but the approximation

$$J_{iw} = 1 + 0.05i_w \quad \text{for } i_w \leq 5^\circ \quad (5.11)$$

is reasonably representative. It is suggested that, lacking further evidence, this equation be used for non-overlapping propellers.

The analysis of test data in Derivations 2, 4, 8 and 11 for the effect of z_p/D showed no consistent trends but indicated apparently random changes of up to ± 0.06 in $\Delta[k_s]_1$ for $-0.3 \leq z_p/D \leq 0.3$. In the absence of further evidence it is recommended that

$$J_{zp} = 1. \quad (5.12)$$

This is expected to be satisfactory for $|z_p/D| \leq 0.1$ but is likely to be increasingly suspect for greater values of $|z_p/D|$.

6. EFFECT OF FLAP CUT-OUT

The methods of estimating $[\theta_s]_1^\delta$ and $[k_s]_1^\delta$ given in Section 5 apply to flaps which are continuous across the slipstream width. Rearward extension of the engine nacelle, often dictated by undercarriage housing requirements, will sometimes necessitate a flap cut-out. For maximum flap efficiency in such cases the flap segments should abut the nacelle sides with minimum gap throughout their operating range as shown in Sketch 6.1. Slab-sided nacelles help in this respect.

In well-sealed cases such as that depicted in Sketch 6.1 an effective flap carryover from each segment of about $0.25y_c$ may be assumed for the purposes of this Item, giving an effective flap cut-out

$$y_{ceff} = 0.5y_c. \quad (6.1)$$

In all other cases it should be assumed that

$$y_{ceff} = y_c. \quad (6.2)$$