

WRINKLING OF SANDWICH PANEL FACE PLATES

(ISOTROPIC FACE PLATES ON ORTHOTROPIC CORES, IN-PLANE LOADING OR BENDING)

1. INTRODUCTION

The application of in-plane compressive forces to sandwich panels can cause localised buckling (wrinkling) of the face plates. Two modes of buckling can occur (symmetric and antisymmetric) and both are dealt with in this Data Item. Under the application of an out-of-plane bending moment the compressed face of a sandwich can wrinkle and this case is also included. In addition to the basic geometry of the panel the influences of the face-plate modulus of elasticity, the core compressive (z-direction) modulus and the two core shear moduli in the orthogonal loading directions are treated. A method of allowing for the effects of face-plate plasticity is also included.

Sandwich panels can buckle in modes other than those dealt with in this Data Item all of which should be checked*.

2. NOTATION

Both SI and British units are quoted but any coherent system of units may be used.

E	compressive modulus of elasticity of face-plate material	N/m ²	lbf/in ²
E_c	modulus of elasticity of core material in z-direction in both tension and compression (see Section 5.1)	N/m ²	lbf/in ²
E_t	tangent modulus of face-plate material	N/m ²	lbf/in ²
f_{cf}	minimum failing stress of core or core to face plate bond under tension or compression in z-direction	N/m ²	lbf/in ²
f_n	stress at which $E_t = \frac{1}{2}E$ (see Data Item No. 76016*)	N/m ²	lbf/in ²
f_x, f_y	applied direct stresses aligned with axes of orthotropy of core material (see Sketch 2.1)	N/m ²	lbf/in ²
f_η	larger principal compressive stress in face plate (see Sketch 2.1)	N/m ²	lbf/in ²
$f_{\eta cf}$	value of f_η at which initial irregularities are such that z-direction stress in core equals f_{cf}	N/m ²	lbf/in ²
f_ζ	principal stress corresponding to f_η (see Sketch 2.1)	N/m ²	lbf/in ²
G_{cx}, G_{cy}	principal shear moduli of core material	N/m ²	lbf/in ²
$G_{c\eta}, G_{c\zeta}$	shear moduli of core material in ηz - and ζz -planes respectively (see Section 3.2 and Equation (3.2))	N/m ²	lbf/in ²

For Footnotes see end of Notation Section

* Shear crimping is a limiting case of overall buckling and as such is dealt with in the Data Items on overall buckling of sandwich panels.

h	depth of sandwich measured between mid-planes of face plates	m	in
m	material characteristic (see Data Item No. 76016)		
q_{xy}	applied shear stress aligned with axes of orthotropy of core material (see Sketch 2.1)	N/m ²	lbf/in ²
t	thickness of each face plate	m	in
δ	amplitude of initial waviness of a face plate in z-direction (see Section 5.4)	m	in
$\lambda_{\eta}, \lambda_{\zeta}$	half wavelength of wrinkle in η - and ζ -directions respectively	m	in
ν	Poisson's ratio for face-plate material		
γ	angle between core axis of orthotropy and f_{η} (see Sketch 2.1)	degrees	degrees

* Data Item No. 76016 "Generalisation of smooth continuous stress-strain curves for metallic materials".

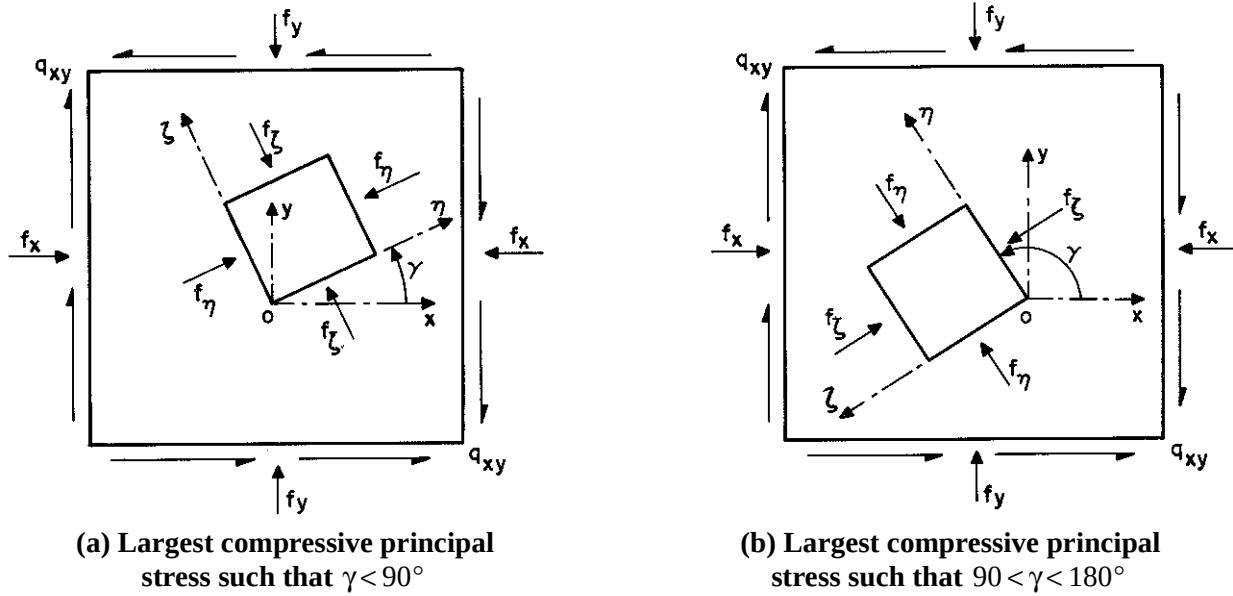
2.1 Suffixes

a	refers to antisymmetric wrinkling (see Sketch 2.3)
b	refers to wrinkling under bending (see Sketch 2.4)
s	refers to symmetric wrinkling (see Sketch 2.2)

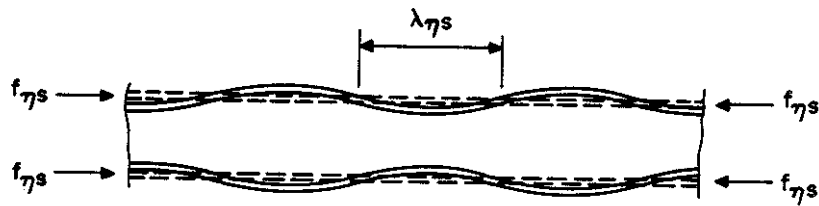
2.2 Sign Convention and Co-ordinate Systems

$0(x, y, z)$	plate reference axes, x-direction defined by direction of principal axes of orthotropy of core (see Sketch 2.1)
$0(\eta, \zeta, z)$	reference axes, η -direction defined by direction of f_{η} (see Sketch 2.1)

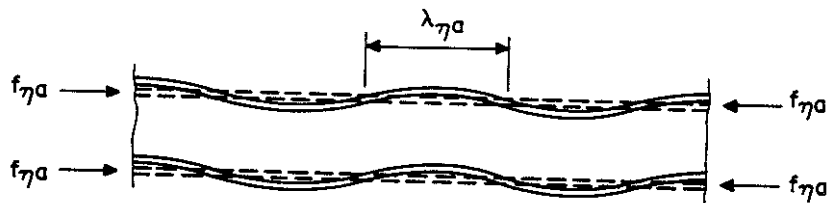
Compressive stresses positive, tensile stresses negative



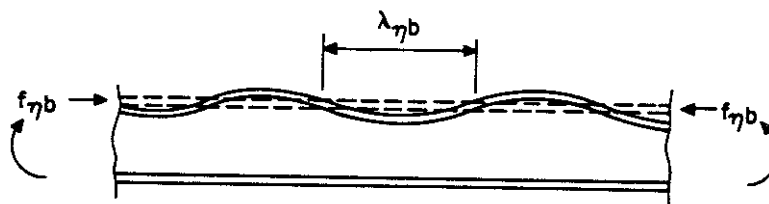
Sketch 2.1



Sketch 2.2 Panel face plate symmetric wrinkling in ηz -plane



Sketch 2.3 Panel face plate antisymmetric wrinkling in ηz -plane



Sketch 2.4 Panel compression face plate wrinkling under bending in ηz -plane

3. INITIALLY FLAT PANELS UNDER IN-PLANE LOADING

Under the action of compressive in-plane loads sandwich panel face plates can wrinkle in either symmetric or antisymmetric modes as illustrated in Sketches 2.2 and 2.3. The two modes are independent of one another and therefore both should be checked to ensure that wrinkling does not occur. The following three sub-sections describe the data provided and their application. Section 3.4 details the assumptions made in the theoretical analysis.

All the curves provided in this Item for in-plane loadings may be calculated using the computer program given in Data Item No. 88015* if the appropriate isotropic face-plate and orthotropic core properties are employed.

3.1 Symmetric Wrinkling

The face-plate stress at symmetrical wrinkling is given by the equation

$$\frac{f_{\eta s}}{E_t} = \left\{ \frac{2}{3} \frac{E_c}{E_t(1 - \nu^2)} \frac{t}{h} \right\}^{1/2}. \quad (3.1)$$

Figure 1 presents curves of $f_{\eta s}/E_t$ against h/t for various values of E_c/E_t when $\nu = 0.3$. Also provided on Figure 1 is a scale of $\lambda_{\eta s}/t$. The value of $f_{\eta s}$ is the largest compressive principal stress, the value of the corresponding principal stress $f_{\zeta s}$ does not influence $f_{\eta s}$.

As Equation (3.1) shows, symmetric wrinkling is dependent upon E_t , E_c , h and t and is independent of both $G_{c\eta}$ and $G_{c\zeta}$. It is assumed to be independent of the panel boundary conditions and has the values of $\lambda_{\eta s}$ and $\lambda_{\zeta s}$ which accord with the lowest wrinkling load. For any value of f_{ζ}/f_{η} other than unity the minimum value of $f_{\eta s}/E_t$ occurs when $\lambda_{\zeta s} \rightarrow \infty$. Thus, in practice, $\lambda_{\zeta s}$ is the panel width. However, provided that the panel width is greater than about $10h$, $f_{\eta s}/E_t$ is not significantly reduced by increasing $\lambda_{\zeta s}$ and Figure 1 may be used. In the special case when $f_{\eta} = f_{\zeta}$ all possible combinations of $\lambda_{\eta s}$ and $\lambda_{\zeta s}$ have the same value of $f_{\eta s}$, which is the value given by Figure 1. In practice the initial irregularities present in the face plate will govern the values of $\lambda_{\eta s}$ and $\lambda_{\zeta s}$ that occur (see Section 5.4).

3.2 Antisymmetric Wrinkling

Figures 2 to 5 present curves of $f_{\eta a}/E_t$ against h/t for a range of values of E_c/E_t for $G_{c\eta}/E_c = 0.1, 0.2, 0.3$ and 0.4 , all for $\nu = 0.3$. The curves are terminated at the value of h/t below which antisymmetric wrinkling does not occur.

Antisymmetric wrinkling is assumed to be independent of panel boundary conditions and to take up values of $\lambda_{\eta a}$ and $\lambda_{\zeta a}$ such that $f_{\eta a}$ is a minimum. Under uniaxial loading the lack of a buckle wave in the ζ -direction ($\lambda_{\zeta a}$) requires less energy than does its imposition. In consequence the minimum value of $f_{\eta a}$ is associated with $\lambda_{\zeta a} \rightarrow \infty$ and a finite value of $\lambda_{\eta a}$, usually in the region of $20t$ to $100t$. The curves of Figures 2 to 5 are therefore plotted for $\lambda_{\zeta a} \rightarrow \infty$. Because $f_{\eta a}$ is always associated with a very large value of $\lambda_{\zeta a}$ the core transverse shear stiffness $G_{c\zeta}$ has negligible influence on the value of $f_{\eta a}$.

For biaxial loading wrinkling is governed by the value of f_{η} irrespective of f_{ζ} . As with symmetric wrinkling when $f_{\eta} = f_{\zeta}$ and $G_{c\eta} = G_{c\zeta}$ all possible combinations of $\lambda_{\eta a}$ and $\lambda_{\zeta a}$ have the same value of $f_{\eta a}$ which is given by Figures 2 to 5.

* Data Item No. 88015 "Elastic wrinkling of sandwich panels with laminated fibre reinforced face plates (Face plates of AsBoDs, AsBIDs, and AsBsDs types) (computer program)".

Since $f_{\eta a}$ is dependent upon the value of $G_{c\eta}$, in cases where $G_{c\zeta} < G_{c\eta}$ and f_{ζ} approaches f_{η} it is necessary to check whether or not $G_{c\zeta}$ is such as to allow wrinkling to occur in the ζ -direction prior to the η -direction. This is done by interchanging the η - and ζ -axes, so interchanging the stresses f_{ζ} and f_{η} and also interchanging $G_{c\zeta}$ and $G_{c\eta}$, and repeating the calculation. Example 7.2 shows how this may be done.

In the case of uniaxial loading aligned with the x -axis or biaxial loading where the loads are aligned with the x - and y -axes, $G_{c\eta}$ and $G_{c\zeta}$ accord with G_{cx} and G_{cy} respectively. When shear loading, q_{xy} , is present resolution of the stresses to determine f_{η} and f_{ζ} will generally lead to f_{η} not being aligned with the axes of orthotropy of the core. In this condition an effective value of the core shear modulus in the direction of f_{η} should be used. In the absence of experimental values or rigorous analysis the values of $G_{c\eta}$ and $G_{c\zeta}$ may be estimated from known orthogonal values of G_{cx} and G_{cy} using the relationships

$$G_{c\eta} = G_{cx} \cos^2 \gamma + G_{cy} \sin^2 \gamma$$

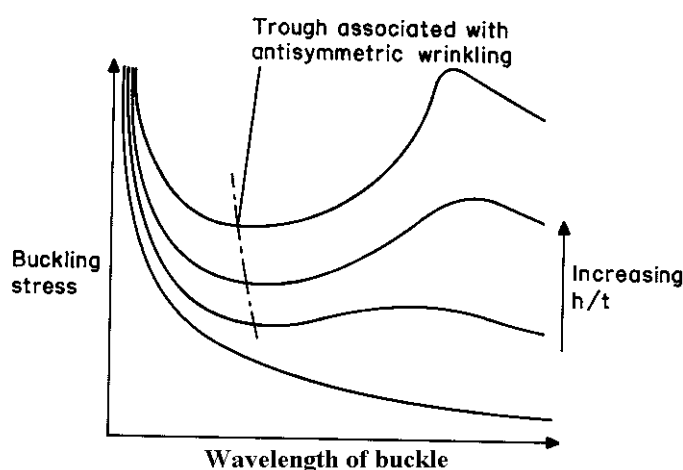
(3.2)

and

$$G_{c\zeta} = G_{cx} \sin^2 \gamma + G_{cy} \cos^2 \gamma.$$

Values of the shear moduli of honeycomb cores along and across the ribbon direction may be estimated using Data Item No. 67022*.

Sketch 3.1 shows the form of curves of $f_{\eta a}$ plotted against $\lambda_{\eta a}$ for various values of h/t and $\lambda_{\zeta a} \rightarrow \infty$. These illustrate the trough in the curve where antisymmetric wrinkling occurs and show that it becomes less pronounced with decreasing h/t and that below a certain value of h/t it ceases to exist. If there is no trough the curve falls continuously from $\lambda_{\eta a} = 0$. In this region the lowest value of $f_{\eta a}$ is associated with the longest allowable $\lambda_{\eta a}$ which is the panel length. This is overall buckling. It follows that if the trough associated with wrinkling is shallow, as it is near the termination of the curves on Figures 2 to 5, then overall buckling may be close to $f_{\eta a}$.



Sketch 3.1 Buckling stress versus wavelength for fixed values of h/t (antisymmetric mode)

3.3 Comparison of Symmetric and Antisymmetric Wrinkling Modes

Figure 6 compares the symmetric and antisymmetric mode wrinkling curves for values of E_c/E_t of 0.0002 and 0.0010 for a range of values of $G_{c\eta}/E_c$. These show that the symmetric mode usually has the lowest wrinkling stress and that the antisymmetric mode has the lowest wrinkling stress only over a relatively

* Data Item No. 67022 "Modulus of rigidity of sandwich panels with hexagonal cell cores".

narrow band near to the termination point of the curves for antisymmetric wrinkling. This band may be wider than that obtained by comparing Figure 1 with Figures 2 to 5 if the face plates are worked above their limit of proportionality.

3.4 Theoretical Basis for Figures

The figures for both symmetric and antisymmetric modes of wrinkling are based upon elastic, thin plate, small deflection theory. The face plates are taken to be isotropic, homogeneous and identical but their through-the-thickness shear stiffnesses are neglected. The core direct stiffness in the η - and ζ -directions and the shear moduli in the $\eta\zeta$ -plane are all assumed to be zero. The remaining core stiffnesses are E_c , which is assumed to have the same value in tension and in compression, and the two shear moduli, $G_{c\eta}$ and $G_{c\zeta}$, which need not be equal. With these three stiffnesses the core fulfils its structural functions but is not homogeneous since $G_{c\eta}$, $G_{c\zeta}$ and E_c are assumed to be independent and uncoupled. All the applied in-plane loads are assumed to be taken by the face plates which are assumed to be initially flat and to be held flat by the core until they wrinkle.

The above assumptions are appropriate to cellular cores, such as honeycomb cores, provided that λ_η is significantly greater than the cell width. For foam type cores, in which the core is homogeneous, the above assumptions are conservative so that values taken from the figures are likely to underestimate the stress at wrinkling.

Values obtained from the Figures 1 to 5 are valid for all panel widths greater than about $3h$ and they are therefore valid for all practical panels.

4. WRINKLING OF PANELS SUBJECT TO OUT-OF-PLANE BENDING

A complete analysis of the wrinkling of the compressive faces of sandwich panels subject to bending out-of-plane is not available. A solution is available for narrow sandwich beams subject to bending in the xz -plane only. This two-dimensional beam solution has been factored by $(1 - \nu^2)^{-1}$ to introduce a correction for the influence of the wide face plates of the panel and is presented in Figures 7, 8, 9 and 10. The figures show $f_{\eta b}/E_t$ plotted against h/t for a range of values of E_c/E_t and $G_{c\eta}/E_c$ for $\nu = 0.3$ and $f_\zeta = 0$. The value of $G_{c\eta}$ is that in the direction of the applied moment. If the moment is not aligned with the x -axis but is at angle γ to it, Equation (3.2) may be used to estimate $G_{c\eta}$ for use with the figures. The assumptions regarding sandwich construction used in predicting symmetric and antisymmetric wrinkling stated in Section 3.4 also apply.

Wrinkling under bending is influenced by the core shear modulus $G_{c\eta}$. This is because the tension face of the sandwich is assumed to provide a rigid boundary for the core against wrinkling of the compression face plate and to remain undeflected in the z -direction during wrinkling. This assumption is not usually fully met in practice since, to some extent, the tension face plate is influenced by wrinkling of the compression face plate. Because of this, values of $f_{\eta b}$ obtained from Figures 7 to 10 are likely to overestimate the wrinkling stress. This is confirmed by a limited number of experiments and by simple finite element studies. These show that when h/t is large (greater than about 50) the actual wrinkling stress can be as low as $0.75f_{\eta b}$ (Derivation 10). Alternatively, when h/t is very low (less than about 20) reasonable agreement between experiment and theory is found (Derivation 7).

Because the solution is essentially two-dimensional the value of $G_{c\zeta}$ does not appear. The behaviour of a panel subject to symmetric wrinkling in compression indicates that the value of $G_{c\zeta}$ has negligible effect on wrinkling under bending.

The computer program in Data Item No. 87013* may be used to calculate $f_{\eta b}$ for cases not covered by

Figures 7 to 10. The program provides the load per unit width in the face plate which must be divided by t and by $(1 - \nu^2)$ to obtain $f_{\eta b}$.

5. FACTORS INFLUENCING WRINKLING

5.1 Variations in Core Properties and Dimensions

Values of the core stiffnesses E_c , G_{cx} and G_{cy} are difficult to establish and experimental values are subject to wide variations. Tests on nominally identical sandwiches indicate that E_c can vary by ± 15 per cent and G_{cx} can vary by ± 30 per cent. Also, the face-plate thickness can vary by up to ± 6 per cent. The resulting variation in $f_{\eta a}/E_t$ for the combined effect of such large variations is shown by the hatched area on Figure 6 for the arbitrarily chosen ratios of $h/t = 65$, $E_c/E_t = 0.0002$ and $G_{c\eta}/E_c = 0.3$. It can be seen that $f_{\eta a}/E_t$ for the mean values is 0.00173 and that its maximum is 0.0020 and its minimum is 0.0013, that is, a variation of + 15 per cent and -25 per cent. Note that although the tolerances were symmetrical the variation in $f_{\eta a}/E_t$ is not.

As previously stated all the curves are based on the assumption that E_c has the same value in tension and compression. If this is not so the lower modulus should be used.

5.2 Effect of Face Plate Material Plasticity

In practical sandwiches the facings are often worked into the region where their modulus decreases as a result of the onset of material plasticity. This can be allowed for by using E_t throughout. Values of E_t , for a given stress and material, can be obtained from Data Item No. 76016. The value of E_t should correspond to the wrinkling stress, $f_{\eta a}$, $f_{\eta s}$ or $f_{\eta b}$, and must therefore be found by trial, see Example 7.2.

5.3 Effect of Adhesive Layer

For sandwiches using thin face plates the layer of core to face plate adhesive may significantly modify the wrinkling stress. Approximate allowance can be made for this by calculating an equivalent modulus for the face plate and its associated adhesive and modifying h to measure between the neutral planes of the equivalent face plates. Alternatively the sandwich can be treated as a laminated plate with the adhesive forming a layer. This can then be analysed using the computer program in Data Item No. 87013 for bending or Data Item No. 88015 for in-plane compression.

5.4 Effect of Initial Irregularities of the Face Plates on Symmetric Wrinkling

Any initial waviness of the face plates will, on the start of application of loading, begin to grow in amplitude. As the applied load increases those waves that most nearly correspond to the preferred wrinkling wavelength of the sandwich become largest. The initial irregularities of particular interest are therefore those with a half wavelength in the region of $\lambda_{\eta s}$ as determined from Figure 1. As the load is increased the amplitude of the preferred wave increases until core failure, in tension and/or compression, or failure in core to face plate adhesion occurs. This will happen before the theoretical flat plate buckling load is reached.

Estimates of how much an initial wave of given amplitude reduces the theoretical flat-plate wrinkling stress have been formulated for sandwich columns only (Derivation 4). The analysis is therefore restricted to a

* Data Item No. 87013 "Elastic wrinkling of sandwich columns and beams with unbalanced laminated fibre reinforced face plates (Face plates of AsBoDs, AsBiDs and AsBsDs types) (computer program)".

single η -direction wave and to uniaxial loading and yields the following equation

$$f_{\eta cf} = \frac{f_{\eta s}}{2 \left[\frac{\delta}{t} \frac{t}{h} \frac{E_c}{f_{cf}} \right] + 1.0} \quad (5.1)$$

in which the value of δ is that occurring over lengths equal to λ_s .

Application of equation (5.1) to panels should be treated with caution. As described in Section 3.1 the ζ -direction wave is usually very large so that account of the η -direction wave only in these cases may be sufficient. Thus, provided f_ζ is significantly less than f_η , Equation (5.1) may provide reasonable guidance on the effect of initial waviness using a value of δ related to $\lambda_{\eta s}$.

In the special case where $f_\eta = f_\zeta$ many values of $\lambda_{\eta s}$ and $\lambda_{\zeta s}$ are possible at the same value of $f_{\eta s}$ and $f_{\zeta s}$ (see Section 3.1). Because of this the value of δ related to the half wavelength of $\lambda_{\eta s}$ given on Figure 1 should not be used in Equation (5.1). The value of δ used should relate to that of the largest amplitude initial irregularity likely to be present in the plate. This will equal or exceed the δ relating to the $\lambda_{\eta s}$ of Figure 1 so giving a lower value of $f_{\eta cf}$.

Values of δ are difficult to determine since variations in sandwich outer surface flatness reflect both core and face plate thickness changes as well as waviness. Figure 11 shows Equation (5.1) in graphical form. Note that the abscissa δ/h can be represented as $(\delta/t)(t/h)$ in which, for a particular sandwich, t/h is usually fixed.

5.5 Effect of Initial Irregularities of the Face Plates on Antisymmetric Wrinkling

Owing to the form of antisymmetric wrinkling the critical wavelengths for initial irregularities are in the region of $0.25h$ to $2h$. The effects of waviness have been studied theoretically without useful result.