



# ESDU 88033

Issued December 1988

With Amendment A

June 2012

**The design of spur and helical involute gears. A procedure compatible with BS 436: Part 3: 1986 – method for calculation of contact and root bending stress limitations for metallic involute gears.**

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The Mechanisms Committee, which first met in 1975, is continuing the work that previously was the responsibility of the former Gearing and Mechanisms Committee, which first met in November 1964. Like the earlier Committee it has the cooperation of many engineers and scientists in industry, research establishments and universities from whom much assistance and information is being received.

The ESDU Mechanisms Series is recognised as a valuable source of validated data and methods by the Institution of Mechanical Engineers and its Mechatronics, Informatics and Control Group is involved in continually reassessing the scope and focus of the content of this Series.

## THE DESIGN OF SPUR AND HELICAL INVOLUTE GEARS. A procedure compatible with BS 436: Part 3: 1986 – method for calculation of contact and root bending stress limitations for metallic involute gears

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## THE DESIGN OF SPUR AND HELICAL INVOLUTE GEARS. A procedure compatible with BS 436: Part 3: 1986 – method for calculation of contact and root bending stress limitations for metallic involute gears

### 1. NOTATION AND UNITS

The dimensional parameters in this notation are given with SI units. The symbols are based on BS 2519: Part 2: 1976.

|               |                                                                            | Units                     | Computer Program Equivalent |
|---------------|----------------------------------------------------------------------------|---------------------------|-----------------------------|
| $a$           | centre distance (negative for internal gears)                              | mm                        | a                           |
| $b$           | facewidth                                                                  | mm                        | b                           |
| $b_{eff}$     | effective facewidth                                                        | mm                        | —                           |
| $C_I, C_{II}$ | tip and root relief, respectively                                          | $\mu\text{m}$             | —                           |
| $c$           | facewidth ratio                                                            | —                         | —                           |
| $c_{eff}$     | effective case depth                                                       | mm                        | ceff                        |
| $c'$          | maximum tooth stiffness of one tooth pair in normal section                | N $\mu\text{m}/\text{mm}$ | c'                          |
| $c_\gamma$    | mean value of total tooth stiffness (or mesh stiffness) per unit facewidth | N $\mu\text{m}/\text{mm}$ | cgamma                      |
| $d$           | reference diameter                                                         | mm                        | d                           |
| $d_1$         | reference diameter of pinion                                               | mm                        | d1                          |
| $d_2$         | reference diameter of wheel                                                | mm                        | d2                          |
| $d_\alpha$    | tip diameter                                                               | mm                        | da                          |
| $d_b$         | base diameter                                                              | mm                        | db                          |
| $d_f$         | root diameter                                                              | mm                        | df                          |
| $d_w$         | pitch diameter                                                             | mm                        | dw                          |
| $d_{w1}$      | pitch diameter of pinion                                                   | mm                        | dw1                         |
| $d_{w2}$      | pitch diameter of wheel                                                    | mm                        | dw2                         |

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|---------------|---------------------------------------------------------|-------------------|---------|
| $E$           | Young's modulus of elasticity                           | MN/m <sup>2</sup> | —       |
| $F_f$         | involute tolerance without tip and root relief          | μm                | —       |
| $F_n$         | normal contact load                                     | N                 | —       |
| $F_t$         | nominal tangential force at reference circle            | N                 | Ft      |
| $F_{bt}$      | tangential tooth load at mean operating torque          | MN/m <sup>2</sup> | —       |
| $F_{\beta x}$ | mesh misalignment prior to running-in                   | μm                | Fbetax  |
| $f_p$         | adjacent pitch error                                    | mm                | —       |
| $f_{ma}$      | mesh misalignment due to manufacturing tolerance        | μ m               | fma     |
| $f_{pe}$      | tolerance on pitch                                      | μm                | fpe     |
| $f_{sh}$      | mesh misalignment due to shaft deflections              | μ m               | fsh     |
| $G$           | modulus of rigidity                                     | MN/m <sup>2</sup> | —       |
| $HB$          | Brinell hardness value                                  | —                 | —       |
| $HV$          | Vickers hardness value                                  | —                 | —       |
| $h$           | tooth depth                                             | mm                | h       |
| $h_c$         | height of crowning                                      | μm                | —       |
| $h_f$         | bending moment arm                                      | mm                | hF      |
| $I$           | polar moment of inertia                                 | kg m <sup>2</sup> | I       |
| $K$           | bearing arrangement factor                              | —                 | —       |
| $K_A$         | application factor                                      | —                 | KA      |
| $K_v$         | dynamic factor                                          | —                 | Kv      |
| $K_{F\alpha}$ | transverse load factor for bending stress               | —                 | KFalpha |
| $K_{F\beta}$  | face load factor for bending stress                     | —                 | KFbeta  |
| $K_{H\alpha}$ | transverse load factor for contact stress               | —                 | KHalpha |
| $K_{H\beta}$  | face load factor for contact stress (Hertzian pressure) | —                 | KHbeta  |
| $K_{v\alpha}$ | dynamic factor for spur gears                           | —                 | —       |

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|              |                                                                      |                |       |
|--------------|----------------------------------------------------------------------|----------------|-------|
| $K_{v\beta}$ | dynamic factor for helical gears ( $\varepsilon_\beta \geq 1$ )      | —              | —     |
| $K_{v350}$   | dynamic factor for $F_t K_A / b = 350$                               | —              | —     |
| $l$          | bearing span                                                         | mm             | L     |
| $m$          | module in mm                                                         | —              | —     |
| $m_n$        | normal module in mm                                                  | —              | mn    |
| $N_L$        | number of pinion load cycles                                         | —              | —     |
| $n$          | rotational speed                                                     | rev/min        | n     |
| $P$          | transmitted power                                                    | kW             | P     |
| $P_e$        | transverse base pitch                                                | mm             | —     |
| $p$          | contact load (see Appendix A)                                        | N              | —     |
| $R$          | roughness                                                            | $\mu\text{ m}$ | R     |
| $R_a$        | arithmetic average roughness (CLA value)                             | $\mu\text{ m}$ | Ra    |
| $S$          | safety factor                                                        | —              | S     |
| $S_F$        | actual safety factor for bending stress (against breakage)           | —              | SF    |
| $S_{Fmin}$   | minimum demanded safety factor for bending stress (against breakage) | —              | SFmin |
| $S_H$        | actual safety factor for contact stress                              | —              | SH    |
| $S_{Hmin}$   | minimum demanded safety factor for contact stress                    | —              | SHmin |
| $s_{Fn}$     | thickness of virtual tooth at critical section                       | mm             | sFn   |
| $T$          | torque                                                               | N m            | T     |
| $T_1$        | pinion torque                                                        | N m            | T1    |
| $T_2$        | wheel torque                                                         | N m            | T2    |
| $u$          | gear ratio (negative for internal gears)                             | —              | u     |
| $v$          | pitch line velocity                                                  | m/s            | v     |
| $w$          | specific loading                                                     | N/mm           | —     |
| $w_m$        | mean specific load intensity                                         | N/mm           | —     |

|              |                                                                     |                         |          |
|--------------|---------------------------------------------------------------------|-------------------------|----------|
| $w_{max}$    | max specific load intensity                                         | N/mm                    | —        |
| $x$          | addendum modification coefficient                                   | —                       | x        |
| $Y$          | factor for bending stress                                           | —                       | —        |
| $Y_F$        | tooth form factor for bending stress                                | —                       | YF       |
| $Y_M$        | material quality factor for bending stress                          | —                       | YM       |
| $Y_N$        | life factor for bending stress                                      | —                       | YN       |
| $Y_R$        | surface condition factor for bending stress                         | —                       | YR       |
| $Y_S$        | stress correction factor for bending stress                         | —                       | YS       |
| $Y_X$        | size factor for bending stress                                      | —                       | YX       |
| $Y_\beta$    | helix angle factor for bending stress                               | —                       | Ybeta    |
| $Y_\delta$   | sensitivity factor for bending stress                               | —                       | Ydelta   |
| $y_\alpha$   | running-in allowance for $K_{H\alpha}$                              | $\mu\text{m}$           | yalpha   |
| $Z$          | factor for contact stress                                           | —                       | Z        |
| $Z_C$        | case depth factor for contact stress                                | —                       | —        |
| $Z_E$        | elasticity factor for contact stress                                | $(\text{N/mm}^2)^{1/2}$ | ZE       |
| $Z_G$        | disc/gear correlation factor for contact stress                     | —                       | —        |
| $Z_H$        | zone factor for Hertzian pressure at pitch point for contact stress | —                       | ZH       |
| $Z_L$        | lubricant factor for contact stress                                 | —                       | ZL       |
| $Z_M$        | material quality factor for contact stress                          | —                       | ZM       |
| $Z_N$        | life factor for contact stress                                      | —                       | ZN       |
| $Z_R$        | roughness factor for contact stress                                 | —                       | ZR       |
| $Z_W$        | work hardening factor for contact stress                            | —                       | ZW       |
| $Z_X$        | size factor for contact stress                                      | —                       | ZX       |
| $Z_v$        | speed factor for contact stress                                     | —                       | ZV       |
| $Z_\epsilon$ | contact ratio factor for contact stress                             | —                       | Zepsilon |

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|                      |                                                                             |                   |          |
|----------------------|-----------------------------------------------------------------------------|-------------------|----------|
| $z$                  | number of teeth                                                             | —                 | $z$      |
| $\alpha_{en}$        | pressure angle at highest point of single tooth pair contact                | radian            | —        |
| $\alpha_n$           | normal pressure angle at reference cylinder                                 | radian            | alphan   |
| $\alpha_t$           | transverse pressure angle at reference cylinder                             | radian            | alphat   |
| $\alpha_{tw}$        | transverse pressure angle at pitch cylinder                                 | radian            | alphatw  |
| $\beta$              | helix angle (without subscript: at reference cylinder)                      | radian            | beta     |
| $\beta_b$            | base helix angle                                                            | radian            | betab    |
| $\varepsilon$        | contact ratio                                                               | —                 | —        |
| $\varepsilon_\alpha$ | transverse contact ratio                                                    | —                 | —        |
| $\varepsilon_\beta$  | overlap ratio                                                               | —                 | epsbeta  |
| $\varepsilon_\gamma$ | total contact ratio                                                         | —                 | epsgamma |
| $\Delta f_b$         | effective bending deflection                                                | mm                | —        |
| $\Delta f_t$         | effective torsional wind-up                                                 | mm                | —        |
| $\theta$             | radius of curvature of involute at reference diameter                       | mm                | —        |
| $\nu$                | Poisson's ratio                                                             | —                 | —        |
| $\sigma_B$           | ultimate tensile strength                                                   | MN/m <sup>2</sup> | sigB     |
| $\sigma_F$           | actual tooth root bending stress                                            | MN/m <sup>2</sup> | sigF     |
| $\sigma_H$           | actual contact stress (Hertzian pressure)                                   | MN/m <sup>2</sup> | sigH     |
| $\sigma_R$           | residual stress                                                             | MN/m <sup>2</sup> | sigR     |
| $\sigma_{FE}$        | fatigue endurance limit                                                     | MN/m <sup>2</sup> | —        |
| $\sigma_{FO}$        | basic endurance limit of a polished specimen under a reversing bending load | MN/m <sup>2</sup> | sigFO    |
| $\sigma_{FP}$        | permissible tooth root bending stress                                       | MN/m <sup>2</sup> | sigFP    |
| $\sigma_{FY}$        | yield strength for bending stress                                           | MN/m <sup>2</sup> | sigFY    |
| $\sigma_{Fn}$        | nominal tooth root bending stress                                           | MN/m <sup>2</sup> | —        |
| $\sigma_{HD}$        | endurance limit for contact stress for discs                                | MN/m <sup>2</sup> | —        |

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|                 |                                                            |                   |         |
|-----------------|------------------------------------------------------------|-------------------|---------|
| $\sigma_{HO}$   | nominal contact stress                                     | MN/m <sup>2</sup> | –       |
| $\sigma_{HP}$   | permissible contact stress (permissible Hertzian pressure) | MN/m <sup>2</sup> | sigHP   |
| $\sigma_{HY}$   | yield strength for contact stress                          | MN/m <sup>2</sup> | –       |
| $\sigma_{Hlim}$ | endurance limit for contact stress for gears               | MN/m <sup>2</sup> | sigHlim |

### Subscripts

|             |                               |
|-------------|-------------------------------|
| <i>a</i>    | tip                           |
| <i>b</i>    | base                          |
| <i>core</i> | core of case hardened gear    |
| <i>eff</i>  | effective values, real stress |
| <i>est</i>  | estimated value               |
| <i>i</i>    | internal                      |
| <i>lim</i>  | value of endurance limit      |
| <i>min</i>  | minimum                       |
| <i>max</i>  | maximum                       |

## 2. INTRODUCTION

A new British Standard for the calculation of contact and root bending stress for spur and helical gears requires a re-appraisal of gear design procedures. The new Standard, BS 436: Part 3: 1986 (Derivation 2), departs radically from the traditional approach to gear design embodied in the previous Standard, BS 436: 1940. The new procedure is based on a more rigorous analysis of root bending and surface stress, and is thus similar to other recent gear Standards for spur and helical gears, in other words:

- (i) ISO DS 6336 (DRAFT),
  - (ii) DIN 3990, 1987 (Reference 7)
- or
- (iii) AGMA 218 (Reference 6).

This Item largely supersedes ESDU 68040, "Design of parallel axis straight spur and helical gears – choice of materials and preliminary estimates of major dimensions". It differs from ESDU 77002, "Design of parallel axis straight spur and helical gears – geometric design", in using terminology to BS 2519: Part 2: 1976 (ISO) throughout, and tolerances to BS 436: Part 2: 1970 (see Derivation 1).

Transmission systems can fail in service due to a number of causes not associated with the gears themselves, *e.g.* failure of

- (i) bearings,
  - (ii) seals,
  - (iii) lubrication,
  - (iv) splines and keyways,
  - (v) shafts
- and
- (vi) gearcases, *etc.*

For a successful transmission system, due attention needs to be given to these components to ensure appropriate overall reliability. Engineers are referred to the relevant ESDU Data Items (References 8 to 22) for other aspects of the gearbox design.

This Item is concerned only with the design of the gears themselves. Gears can fail in three ways:

- (1) by **tooth breakage**, caused by the bending stress exceeding the fracture strength or fatigue strength of the gear material,
- (2) by **surface failure**, that is, pitting or sub-surface cracking, caused by the contact stress exceeding the fatigue strength or the crushing strength of the gear material,
- (3) by **scuffing, scoring or abrasive wear**, caused by oil film failure, inadequate lubrication or dirt.

BS 436: Part 3: 1986 deals only with tooth breakage and surface failure. Consequently only these two modes of failure will be considered in this Data Item. The new Standard takes account, quantitatively, of all factors known to affect gear performance. It is thus possible to investigate the effect on gear stressing of changes in individual parameters such as gear accuracy, bearing spacing, crowning or lead correction. When combined with adequate data on gear manufacturing costs, the new Standard is an effective procedure for designing more cost effective gearing.

Since BS 436: Part 3: 1986 takes account of all influence factors, it is also a powerful development tool when designing new gears based on operating experience.

As a result of the more accurate analysis, the new Standard is more complex and the calculation procedure relatively time-consuming when carried out long-hand. For approximate sizing this Item proposes a simplified procedure which is described in Section 3 and derived in Appendix A. To implement fully the calculations, BS 436: Part 3: 1986 is available as a computer program from the Design Unit of the University of Newcastle-upon-Tyne.

The new features of the Standard are described in Section 4, and its application in Section 6.

## 3. FIRST ESTIMATIONS OF GEAR DIMENSIONS

The new Standard for spur and helical gears, BS 436: Part 3: 1986 (Derivation 2), is primarily a procedure for stress analysis of a known gear. It is not well suited to the long-hand synthesis of a gear design. This approximate technique provides a quick method for preliminary sizing of gears based on the new Standard.

In most cases, gears will be designed against specific requirements of:

- (i) power,  $P$  (kW),
- (ii) input speed,  $n$  (rev/min),
- (iii) gear ratio,  $u$
- and (iv) gear life (in hours or numbers of revolutions).

In addition, one or more of the following may be specified:

- (v) gear centre distance,  $a$  (mm),
- (vi) pinion and wheel material and heat treatment,
- (vii) manufacturing route and associated accuracy,
- (viii) safety factors against surface failure, *e.g.* pitting,  $S_H$ , and bending failure,  $S_F$ ,
- and (ix) application factor.

The safety factors must be considered in relation to the required reliability. If the permissible probability of failure is small the safety factor should be relatively large. The permissible probability of failure should take account of the application, the cost of down-time and replacement, the possible danger to life and the consequential damage in the event of gear failure. However, it should also be remembered that rolling element bearings are commonly designed to a  $B_{10}$  life, that is, a 10% probability of failure during the design life of the bearing.

The approximate calculation procedure is summarised in the flow chart of Figure 3.7 and detailed below. If a computer program for carrying out the calculations to the new Standard is available, this should be used in preference. The derivation of Equations (3.1) to (3.10) is given in Appendix A.

### 3.1 Summary

When designing a spur or helical gearbox a number of decisions need to be made before the gear dimensions can be determined. These are:

- (i) the split of ratios between the stages (in a multi-stage gearbox) (Section 3.2),
- (ii) the gear materials and heat treatment to be used (Section 3.3)
- and (iii) the manufacturing route to be taken, and the associated accuracy (Sections 3.4 and 5.2).

If not specified, appropriate values for

- (iv) application factor
- and (v) safety factor,

must be selected.

When a fixed centre distance,  $a$ , is specified, the only primary variables are the materials and the facewidth of the gear and the size (module) of the gear teeth. Normally the material would be selected and the facewidth and module determined from the calculation procedure outlined below.

When no fixed centres are specified, the material, accuracy and the facewidth can be selected (depending on the application and the gear mounting) and the required centre distance and module calculated.

## 3.2 Ratio Split

The overall ratios that are economic in spur and helical gearboxes are given in Table 3.1. The ratio per stage can be exceeded in special cases, for example in slew-rings or drum drives.

**TABLE 3.1 Ratio Limits**

| <i>Number of Stages</i> | <i>Normal Ratio</i> | <i>Extended Ratio</i> | <i>Extreme Ratio</i> |
|-------------------------|---------------------|-----------------------|----------------------|
| Single Stage            | 6                   | 8                     | 20                   |
| Two Stage               | 30                  | 45                    | 100                  |
| Three Stage             | 150                 | 200                   | 300                  |

The split of ratios between stages should be selected to give the higher ratio(s) in the low torque, high speed stage(s) of the gearbox. There is no generally applicable formula for best ratio splits for minimum cost. Niemann and Winter (Derivation 5) give the following formulae for the best ratio split for gears of similar material to achieve the minimum total gear volume.

For two-stage gearboxes of total ratio,  $u$ , the first stage ratio,  $u_1$ , is given by

$$u_1 = 0.8 (u)^{2/3}.$$

For three-stage gearboxes of total ratio,  $u$ , the ratios for the first and second stages are given by

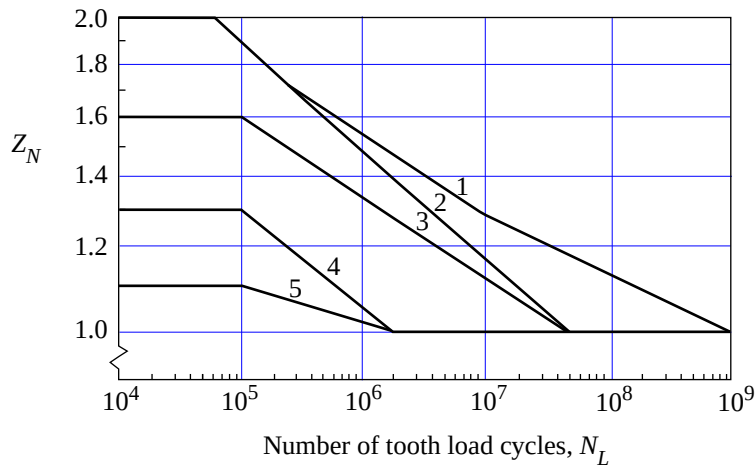
$$u_1 = 0.6 (u)^{4/7}$$

and 
$$u_2 = 1.1 (u)^{2/7}.$$

**Note.** Although achieving minimum total gear volume, these ratio splits result in high ratios in the first stages, which may not represent the most cost-effective ratio split. Consequently the ratios should be used for guidance only.

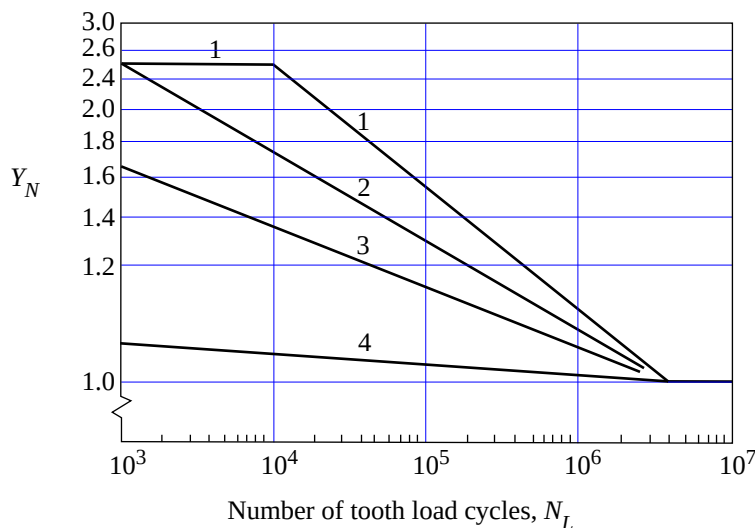
## 3.3 Gear Materials, Heat Treatment and Fatigue Strength

The most appropriate gear material and heat treatment should be selected using the guidance given in Section 5. For the selected material the surface and bending fatigue limits,  $\sigma_{Hlim}$  and  $\sigma_{FE}$ , respectively, can be taken from Table 5.1. These fatigue limits are the maximum cyclic stress levels at which the life of the gear can be assumed to be infinite. The fatigue strength can be assumed to be greater than the limits above if the required life is less than  $10^6$  to  $10^9$  load cycles (dependent on material and failure mode). When some initial, non-progressive pitting can be tolerated, as in many industrial drives, a higher surface stress can be applied in the finite life region. The fatigue strengths for finite life can be adjusted using the life factors,  $Z_N$  and  $Y_N$  in Figures 3.1 and 3.2, respectively.



- (1) Through induction and case hardened steels – some pitting permitted.
- (2) Through hardened steel – no pitting permitted.
- (3) Case and induction hardened steels – no pitting permitted.
- (4) Gas nitrided steels.
- (5) Bath nitrided steels.

**FIGURE 3.1 Life factor for contact stress,  $Z_N$**



- (1) Through and direct surface hardened steels.
- (2) Case carburised steels.
- (3) Gas nitrided steels.
- (4) Bath nitrided steels.

**FIGURE 3.2 Life factor for bending stress,  $Y_N$**

The permissible figure strengths in the finite life region are then given by

$$\sigma_{HP} = Z_N \sigma_{Hlim} / S_{Hmin} \quad (3.1)$$

and 
$$\sigma_{FP} = Y_N \sigma_{FE} / S_{Fmin} \quad (3.2)$$

### 3.4 Gear Accuracy

The design gear accuracy will depend on the proposed manufacturing route (see Section 5.2) and the accuracy required for adequate gear performance. As a general rule, the higher the peripheral speed and the smaller the specific tooth load (*i.e.* the tangential load per unit facewidth) the more accurate must be the gears. Pitch and profile errors should be small enough to avoid inducing high dynamic inertia loads caused by transmission error (kinematic gear error). Dynamic loads are a function of pitch and profile error, peripheral speed and the inertia of the gears. For geometrically similar gears the required pitch and profile accuracy is shown as a function of peripheral speed in Figure 3.3.

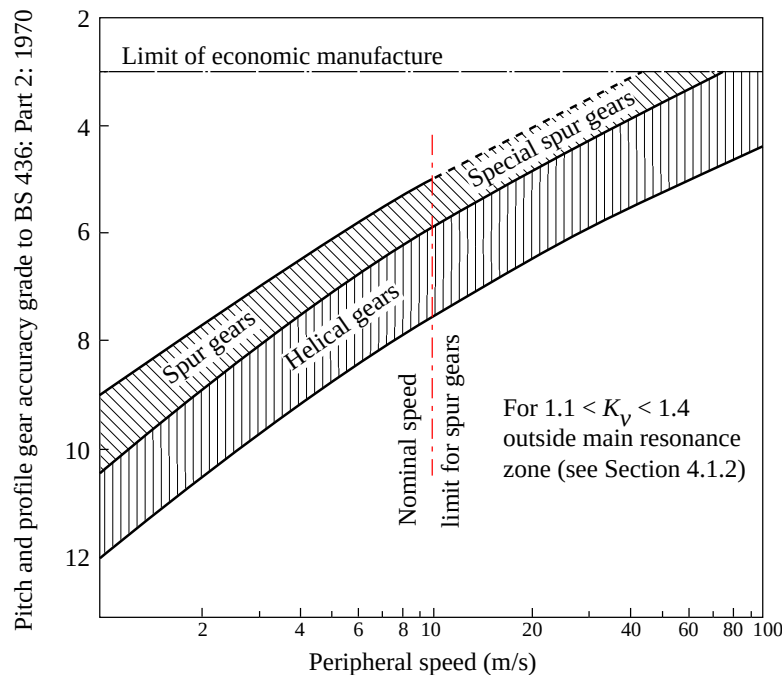


**FIGURE 3.3 Required pitch and profile accuracy**

The lead accuracy should be selected to give an even load distribution across the facewidth. This means that lead errors should be small compared to elastic tooth deflection. The tooth deflection depends on the specific load which, for materials of similar strength, is a function of gear module. The required lead accuracy thus depends on module (and facewidth and material) as shown in Figure 3.4.

**Note.** In helical gears adjacent pitch errors as well as lead errors affect the face load distribution. The pitch accuracy should be selected so that adjacent pitch errors are less than the lead error.

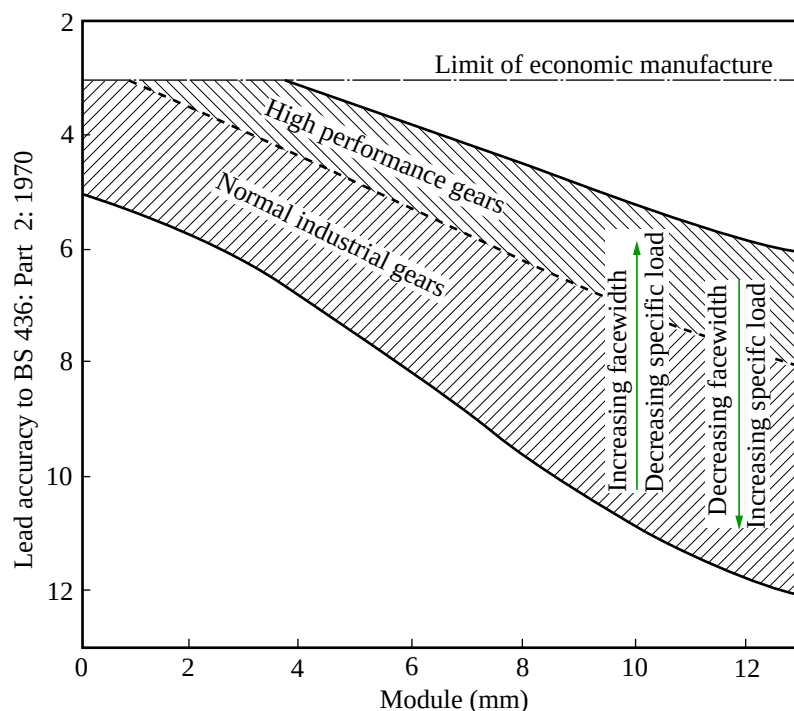


**FIGURE 3.4 Required lead accuracy**

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## 3.5 Application Factor and Safety Factors

When the application factor,  $K_A$ , and the safety factors,  $S_H$  and  $S_F$ , are not specified in the design brief then  $K_A$  should be taken from Table 4.1 and the safety factors for surface failure,  $S_H$ , and bending failure,  $S_F$ , from Figure 4.3.

## 3.6 Facewidth

The facewidth,  $b$ , of the gear, together with the accuracy and the degree of lead correction (Section A.4 of Appendix A), determines the uniformity of load distribution across the gear face. For many transmissions, gears of narrow facewidth can be very economic, but wide facewidths are necessary in some very high power drives. Maximum facewidths generally used in industrial gearboxes are shown in Table 3.2 as a function of the pinion diameter,  $d_1$ ; where possible, considerably narrower gears are desirable, with  $b$  in the range  $0.4 d_1$  to  $0.8 d_1$  (see Figure 6.3). For gears mounted on compliant shafts, for example automotive change gearing, narrow facewidth gears down to  $b = 0.2 d$  are used.

**TABLE 3.2 Maximum Facewidth Ratio**

| Type of Gear                                         | Face width ratio, $c = b/d_1$      |            |          |
|------------------------------------------------------|------------------------------------|------------|----------|
|                                                      | Gear mounting relative to bearings |            |          |
|                                                      | Symmetric                          | Asymmetric | Overhung |
| <u>Spur and Single Helical</u>                       | < 1.6                              | < 1.2      | < 0.8    |
| 1. Hardness < 180 HB                                 | < 1.4                              | < 1.1      | < 0.7    |
| 2. Hardness > 180 HB                                 | < 1.1                              | < 0.9      | < 0.6    |
| 3. Induction hardened, case carburised               | < 0.8                              | < 0.6      | < 0.5    |
| 4. Nitrided                                          |                                    |            |          |
| <u>Spur and Single Helical, with lead correction</u> |                                    |            |          |
| 5. Case carburised                                   | < 1.4                              | < 1.2      |          |
| 6. Nitrided                                          | < 1.2                              | < 1.0      |          |
| <u>Double Helical Gears</u>                          |                                    |            |          |
| 7. Induction hardened, case carburised               | < 2.0                              | < 1.6      |          |
| 8. Nitrided                                          | < 1.4                              | < 1.1      |          |

## 3.7 Face Load Factor for Contact Stress

The load factor for contact stress,  $K_{H\beta}$ , depends on the gear accuracy, the specific tooth load and the facewidth of the gears. For preliminary gear sizing,  $K_{H\beta}$  can be taken from Figure 3.5, which shows this factor as a function of pinion reference diameter,  $d_1$ , gear accuracy grade and facewidth ratio  $b/d_1$ , for surface hardened gears and for through hardened gears. In a well designed gear pair  $K_{H\beta}$  should be less than 2.0. If  $K_{H\beta}$  from Figure 3.5 is greater than 2.0 for the selected facewidth and the achievable gear accuracy then lead correction or crowning should be used (see Section A.4 of Appendix A). If lead correction cannot be utilised a smaller facewidth to give  $K_{H\beta} < 2$  should be used in all cases.

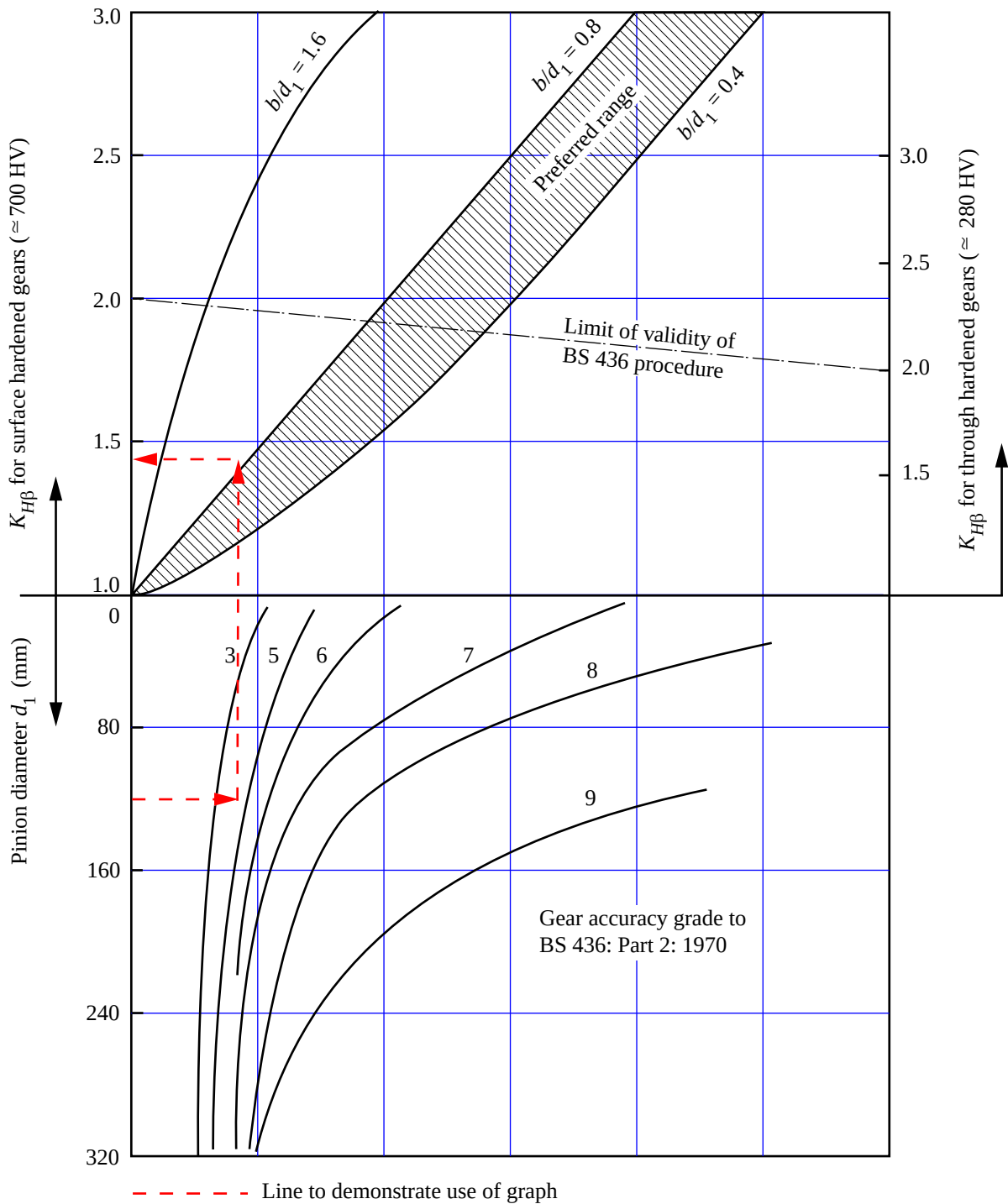


FIGURE 3.5 Face load factor selection curves

## 3.8 Approximate Sizing for Surface Strength

The derivation of the approximate Equations (3.3) to (3.10) is shown in Section A.2 of Appendix A. The equations apply only to steel gears and assume the following:

$$\begin{aligned} Z_H &= 2.5 \\ Z_E &= 189 \text{ (N/mm}^2\text{)}^{1/2} \\ Z_\epsilon &= 0.85 \\ K_v = K_{H\alpha} = K_{F\alpha} &= 1.05 \\ K_{F\beta} &= 0.91 K_{H\beta} \\ \text{and } Y_F Y_S Y_\beta = Y &= 3.4 \text{ for spur gears (or 2.9 for helical gears).} \end{aligned}$$

### 3.8.1 Centre distance not prescribed

With a selected facewidth ratio,  $c (= b/d_1)$ , found from Section 3.6, and the other factors selected or calculated from previous sections, the reference diameter of the pinion can be found from the pinion diameter required to transmit a given pinion torque,  $T_1$ . The pinion diameter is given by

$$d_1 = 700 \left( \left( \frac{1}{\sigma_{HP}} \right)^2 \frac{T_1}{c} \left( \frac{u+1}{u} \right) K_A K_{H\beta} \right)^{1/3}, \quad (3.3)$$

where the pinion torque is given by

$$T_1 = \left( \frac{P}{2 \pi n} \right)^3 60 \quad (3.4)$$

and the power,  $P$ , is in kilowatts and the pinion speed,  $n$ , is in revolutions per minute.

### 3.8.2 Centre distance fixed

When the centre distance,  $a$ , is specified, the pitch circle diameter of the pinion is given by

$$d_1 = 2a/(1+u). \quad (3.5)$$

The required facewidth is given by  $b = c d_1$  where

$$c = 700^3 \left( \frac{1}{\sigma_{HP}} \right)^2 \frac{T_1}{d_1^3} \left( \frac{u+1}{u} \right) K_A K_{H\beta}. \quad (3.6)$$

The following parameters contained in Equations (3.3) and (3.6) are obtained as follows:

$$\begin{aligned} \sigma_{HP} &\text{ from Equation (3.1),} \\ K_A &\text{ from Table 4.1} \\ \text{and } K_{H\beta} &\text{ from Figure 3.5.} \end{aligned}$$

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**Note.** Equations (3.3) and (3.6) apply to steel gears only.

Should  $c$  calculated from Equation (3.6) be greater than the maximum practical facewidth ratio given in Table 3.2, a stronger gear material should be used or the centre distance increased.

## 3.9 Approximate Sizing for Bending Stress

For a gear where the diameter,  $d_1$ , and facewidth,  $b$ , are sized to resist pitting, the minimum module,  $m_n$ , required to avoid bending fatigue failure is calculated from

$$m_n = \frac{2T_1 10^3}{c d_1^2} \left( \frac{1}{\sigma_{FP}} \right) Y K_A K_{H\beta}, \quad (3.7)$$

where  $Y \approx 3.4$  for spur gears  
and  $Y \approx 2.9$  for helical gears.

The following parameters contained in Equation (3.7) are obtained as follows:

$\sigma_{FP}$  from Equation (3.2),  
 $K_A$  from Table 4.1,  
 $K_{H\beta}$  from Figure 3.5  
and  $d_1$  from Equation (3.3) or (3.5).

With this value of  $m_n$ , the nearest number of teeth in the pinion is determined from

$$z_1 \equiv \frac{d_1}{m_n} \cos \beta. \quad (3.8)$$

While, kinematically, spur gears with  $z_1 = 7$  (at  $x = 0.41$ ) are possible, and helical gears with  $z_1 = 4 \dots 6$  are feasible with adequate face contact ratio, larger numbers of pinion teeth are usually required to achieve an adequate balance of strength, surface durability, noise and vibration. Figure 3.6 below shows typical numbers of pinion teeth,  $z_1$ , as a function of gear ratio.

Should  $z_1$ , calculated from Equation (3.8), fall outside the range shown in Figure 3.6 it will be necessary to re-calculate the minimum pinion diameter  $d_1$  (for the assumed facewidth ratio  $c = b/d_1$ ) or the minimum facewidth,  $b$  (when the centre distance,  $a$ , is fixed), to achieve adequate gear bending strength. For strength, the required pinion diameter for the selected values of  $z_{1min}$  and  $c$  are given by

$$d_1 = \left( \frac{2z_{1min}}{c \cos \beta} T_1 10^3 \left( \frac{1}{\sigma_{FP}} \right) Y K_A K_{H\beta} \right)^{1/3} \quad (3.9)$$

or, when the centre-distance is fixed,

$$c = \frac{2z_{1min}}{d_1^3 \cos \beta} T_1 10^3 \left( \frac{1}{\sigma_{FP}} \right) Y K_A K_{H\beta}. \quad (3.10)$$

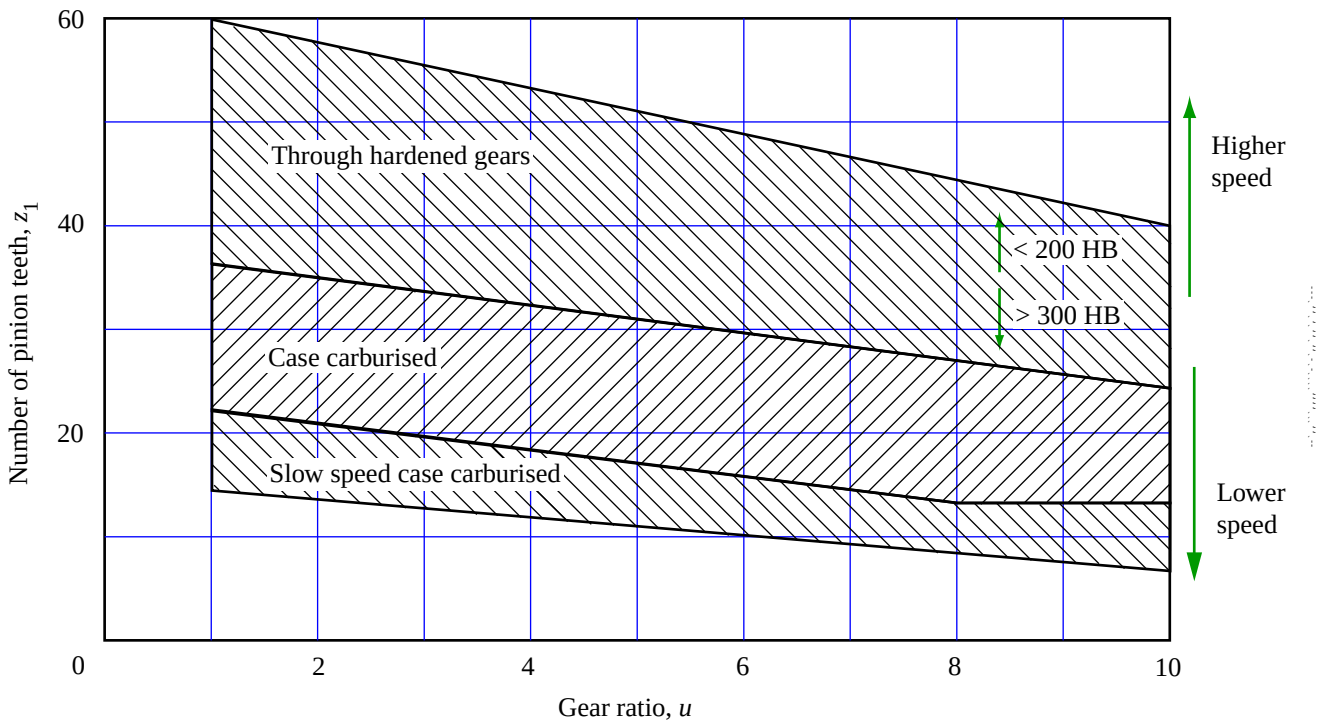


**FIGURE 3.6 The minimum number of gear teeth in the pinion**

The minimum number of gear teeth in the pinion is taken from Figure 3.6, for the appropriate ratio, the selected material and in accordance with the running speed.

For the first layout, the larger value of  $d_1$  resulting from Equation (3.3) or (3.9) should be taken. The value of  $c$  calculated from Equation (3.10) should be less than the maximum facewidth ratio shown in Table 3.2. If the calculated value of  $c$  is too large, then a stronger material must be selected or the centre distance increased.

For an accurate check of the proposed gear pair, a full analysis to BS 436: Part 3: 1986 should be carried out, using software from the Design Unit of the University of Newcastle-upon-Tyne.

The approximate calculation procedure is summarised in the flowchart of Figure 3.7.

**Note.** The above equations for centre distance,  $a$ , facewidth ratio,  $c$ , and module,  $m_n$ , are based on a dynamic factor  $K_v = 1.05$ . When leading dimensions have been established the actual dynamic factor should be found from Figures 4.1 and 4.2 and the gear sizing should be checked.

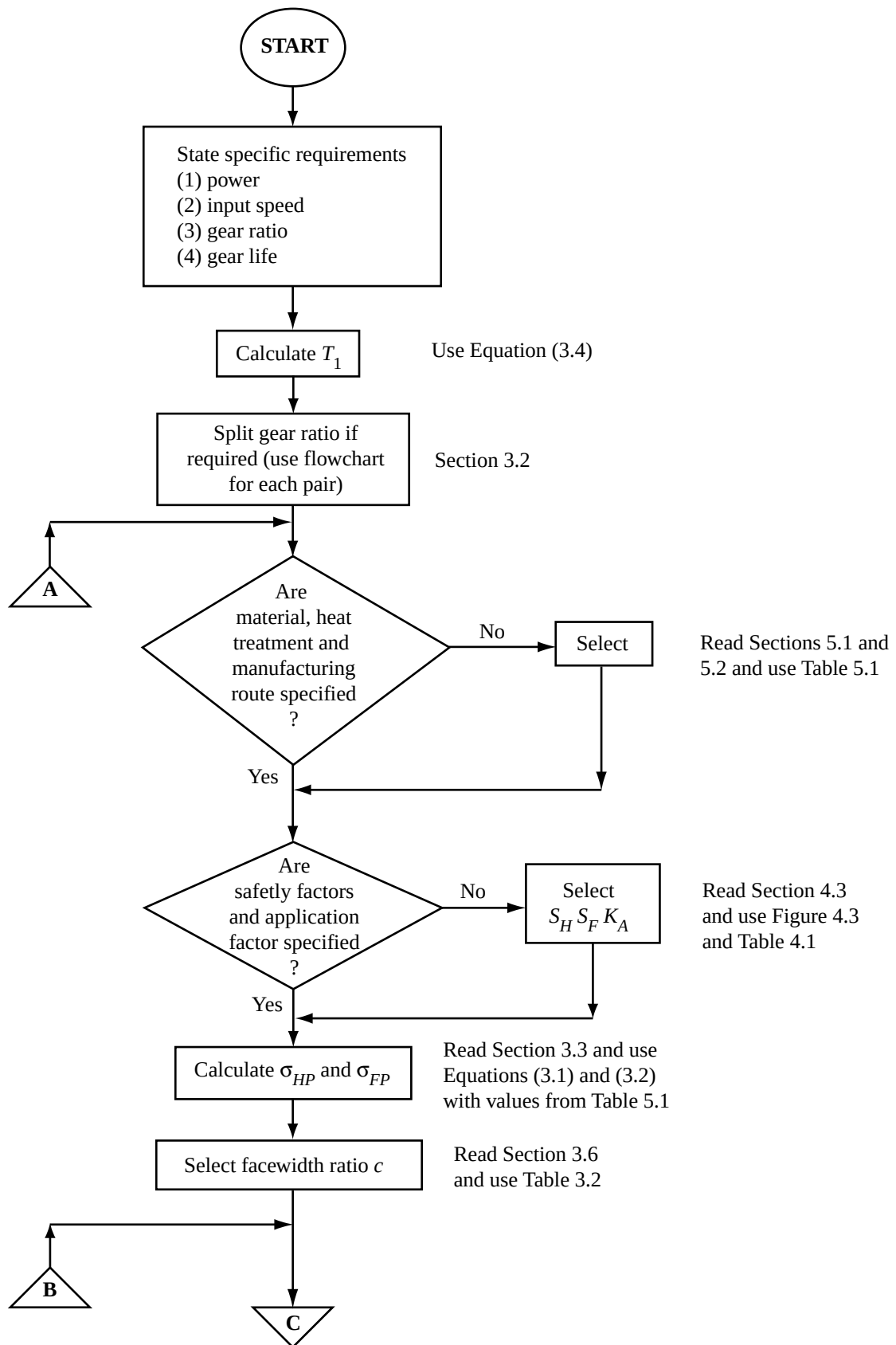


**FIGURE 3.7** Flowchart of approximate calculation procedure

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