

AVOIDANCE OF BUCKLING OF SOME ENGINEERING ELEMENTS (STRUTS, PLATES AND GUSSETS)

1. INTRODUCTION

The term buckling describes non-linear elastic behaviour which can occur when a component is subject to compressive load or to other load actions that cause compressive stress. It is accompanied by transverse movement of part of the component out of the line of the applied load which greatly reduces its stiffness in resisting the applied load, and which generally leads to collapse.

It should be noted that there are many forms of buckling, and that this Item is designed to cover only those cases likely to be encountered in general mechanical engineering. For structures and for some special plant equipment, such as pressure vessels, codes and standards provide all the necessary data on buckling and these should be used where appropriate. The purpose of this Item is to give a general description of buckling and to give details of simple calculation procedures that may be used to estimate the buckling loads of mechanical engineering components so as to ensure that buckling does not occur in service.

2. NOTATION

Both SI and British units are quoted but any coherent system of units may be used.

A	area enclosed by tube cross-section (see Section 5)	m^2	in^2
a, b	plate length and width in x- and y-directions respectively	m	in
d	dimension of section (see Table 4.2 and Sketch 4.3)	m	in
E	modulus of elasticity of material	N/m^2	lbf/in^2
e	effective distance along triangular plate edge at which applied load is taken to act (see Sketch 7.2)	m	in
f_b	elastic buckling stress	N/m^2	lbf/in^2
f_{bq}	compression stress at which plate first buckles when shear stress in plate is q_{bf}	N/m^2	lbf/in^2
f_{bt}	theoretical elastic buckling stress for a geometrically perfect cylinder	N/m^2	lbf/in^2
f_{btf}	average stress at which torsional-flexural buckling occurs in a strut	N/m^2	lbf/in^2
f_{bx}, f_{by}	combination of stresses in x- and y-directions respectively at which a plate first buckles	N/m^2	lbf/in^2
f_c	collapse stress for a rectangular tube	N/m^2	lbf/in^2

f_{Ex}, f_{Ey}	Euler buckling stresses about x- and y-axes respectively where $f_{Ex} = \pi^2 E (k_x / l')^2$ $f_{Ey} = \pi^2 E (k_y / l')^2$	N/m ²	lbf/in ²
f_p	0.2 per cent proof stress or yield stress of material	N/m ²	lbf/in ²
f_T	average compressive stress at torsional instability	N/m ²	lbf/in ²
f_0	elastic buckling stress of a long plate with simply-supported edges loaded on short ends under uniaxial compression, given by $f_0 = \frac{\pi^2 E}{3(1 - \nu^2)} \left(\frac{t}{b} \right)^2$	N/m ²	lbf/in ²
f_1, f_2	longitudinal stresses at plate edges AB and CD (see Figure 8)	N/m ²	lbf/in ²
f_{1b}, f_{2b}	critical values of f_1 and f_2 at which a plate first buckles	N/m ²	lbf/in ²
G	modulus of rigidity or shear modulus of material	N/m ²	lbf/in ²
h	dimension of section (see Table 4.2 and Sketch 4.3)	m	in
I_p	polar second moment of area of cross-section about shear centre	m ⁴	in ⁴
I_y	second moment of area of cross-section about neutral axis parallel to y-axis (see Section 8)	m ⁴	in ⁴
J	torsion constant of section	m ⁴	in ⁴
K, K_{min}	buckling stress coefficient and minimum buckling stress coefficient respectively		
k	radius of gyration of section	m	in
k_x, k_y	radii of gyration of section about x- and y-axes respectively	m	in
l, l'	actual and equivalent length of strut (see Section 4.1) or cylinder acting as strut (see Sections 4.2 and 5)	m	in
M_b	bending moment	N m	lbf in
P_b	critical point load	N	lbf
Q	coefficient modifying f_{bt} for end support conditions		
q_b	stress at which plate buckles in shear alone	N/m ²	lbf/in ²
q_{bf}	shear stress at which plate first buckles when compressive stress in plate is f_{bq}	N/m ²	lbf/in ²

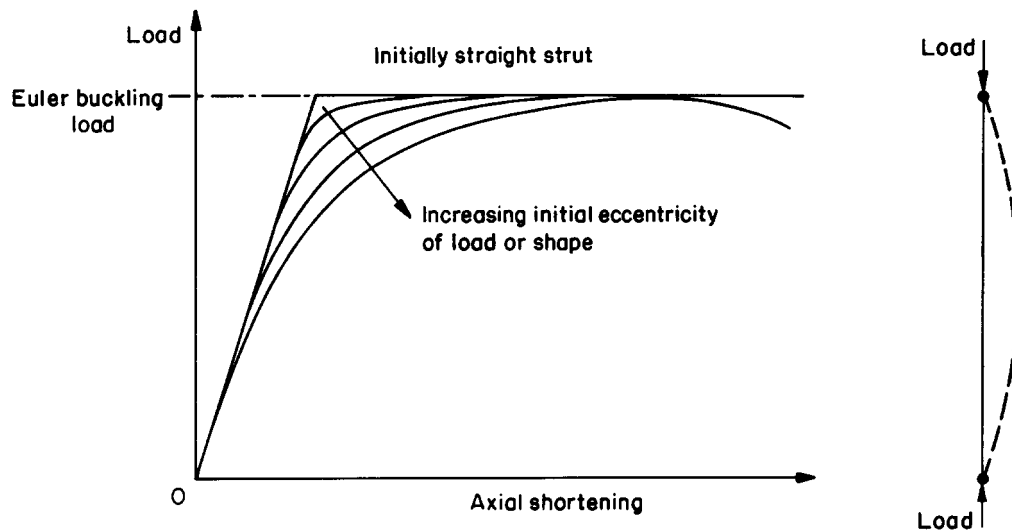
R	radius of cylinder	m	in
T_b	torque at which tube wall buckles in shear (tubes composed of flat plates)	N m	lbf in
t	thickness	m	in
t_d, t_h	thickness of section of dimensions d and h respectively	m	in
w	uniformly distributed load per unit length	N/m	lbf/in
w, v	lengths of sides of a triangular plate (see Section 7)	m	in
x_0, y_0	co-ordinates of shear centre with reference to principal axes of section	m	in
Γ	warping constant of section	m ⁶	in ⁶
δ	maximum inward initial deviation of mid-line of wall of an unloaded cylinder	m	in
ν	Poisson's ratio for material		
ρ	polar radius of gyration of section about shear centre	m	in

Sign Convention

Compressive stresses positive, tensile stresses negative

3. DESCRIPTION OF BUCKLING AND ASSOCIATED BEHAVIOUR

A buckle develops in a non-linear fashion, the load not being proportional to the deflection, even though the elastic limit of the material has not been reached. This is illustrated by Sketch 3.1 which shows the load versus deflection (shortening) behaviour of a slender elastic strut. As buckling is developed a small increase in load will cause a large increase in deflection. Continuing to apply the load will cause the deflection to grow rapidly. This can occur elastically but in practice this growth is usually such that yielding occurs followed by collapse.

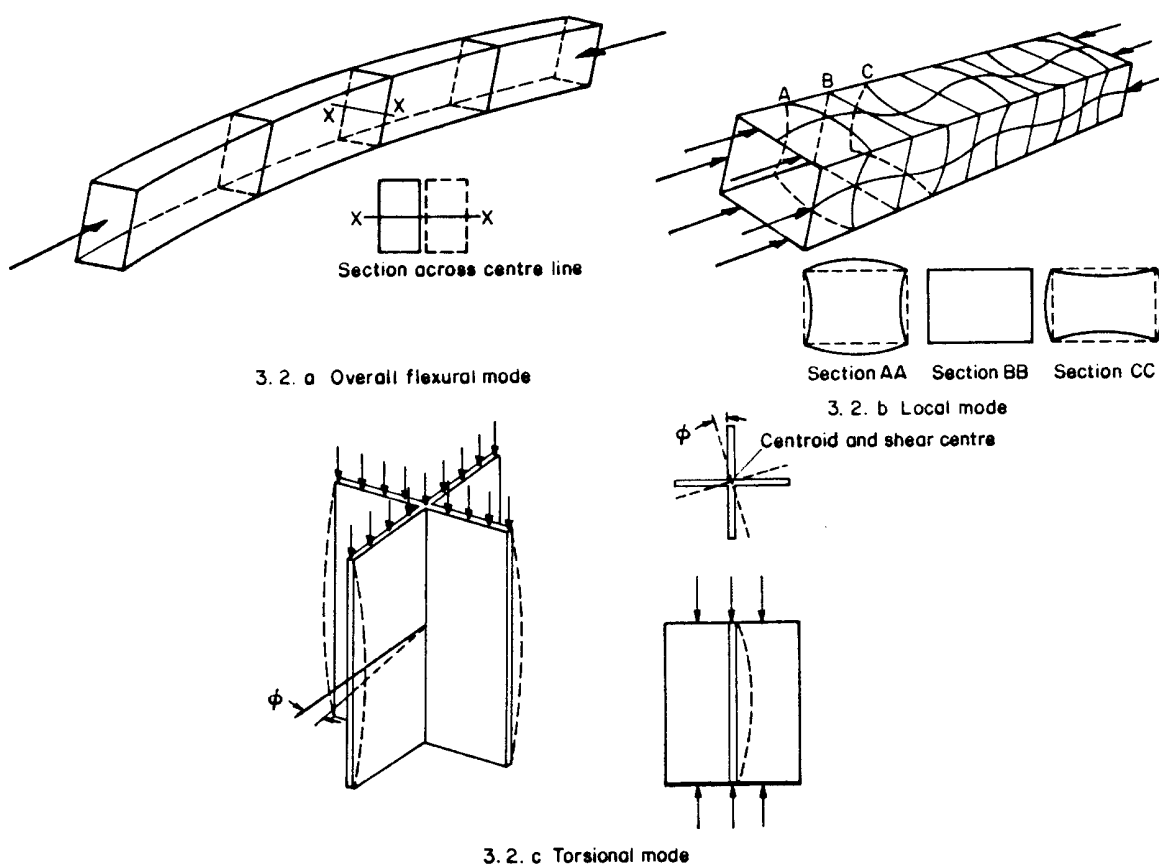


Sketch 3.1 Behaviour of an elastic strut

The buckling load is a function of cross-section geometry as well as material properties. The relationship between buckling load and the dimensions of the cross-section is not linear, being dependent upon the square or cube of the minimum dimensions. Thus, although reducing the cross-sectional dimensions of a critical section by a small amount leads to only a small increase in the compressive stress, it can lead to a large reduction in the buckling load.

For the majority of components in which it is necessary to avoid buckling, simple checks using the assumption that buckling occurs elastically will suffice. Throughout this Item small deflection theory applies and, for plates, thin plate theory is used. Poisson's ratio is 0.3 where not stated, and the materials are assumed to be homogeneous and isotropic. (The majority of engineering materials are approximately isotropic and the data may be used for these without significant error.)

Buckling can occur in a number of basic modes, each of which is associated with a particular wavelength and form of distortion. The major modes for components subject to compression are overall flexural buckling, overall torsional buckling and local buckling. These are illustrated in Sketch 3.2, using, as examples, struts of various types of cross sections.



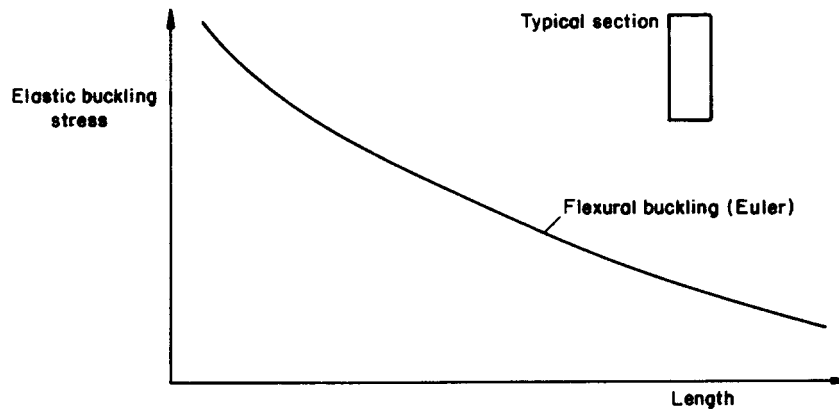
Sketch 3.2 Buckling modes of rectangular section, tubular and X-section struts subject to compressive loading.

Sketch 3.2a illustrates the overall buckling mode. This is the mode that occurs most frequently and is sometimes referred to as flexural buckling. For pinned ends the half-wavelength of buckle is equal to the distance between the ends of the strut. The section across the centre-line indicates that the section translates but does not rotate. Sketch 3.2b shows the local buckling mode, in which the buckle half-wavelength is comparable to the principal dimensions of the strut cross-section. Here the cross-section distorts without translation or rotation.

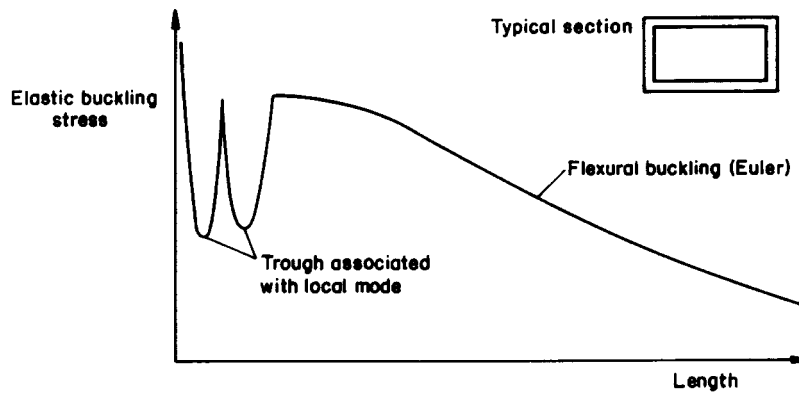
The torsional buckling mode is shown in Sketch 3.2c. This introduces buckling in the walls of the section having a half-wavelength of at least half the length of the strut. Torsional buckling does not occur in all sections but is a characteristic of open sections, especially those that are asymmetric. It occurs by rotation of the cross-section. It is unusual for torsional buckling to occur alone; it usually occurs in conjunction with another buckling mode, most commonly flexural, or, very rarely, local buckling.

Sketch 3.3 compares, for three typical strut cross-sections, the relationship between the various modes of buckling and the length of a strut. The three curves represent different sections, ranging from Sketch 3.3a which is that for a simple solid section strut, to Sketch 3.3c which is for a beam with a complex section, such as an I-section. For a simply-supported strut the half-wavelength of buckle must equal the strut length or be a sub-multiple of it. Thus, as Sketch 3.3c shows, if the length is sufficiently great the flexural or torsional-flexural mode of buckling will occur at the lowest load. However, for shorter struts the lowest stress at which buckling will occur will be near to one of the troughs towards the left of Sketch 3.3. In practice it is conservative to assume that for short wavelength modes the buckle wavelength adjusts itself to the value corresponding to the trough whether or not this is an exact sub-multiple of the strut length. Basic buckling calculations for plate and/or strut elements, and their behaviour will be examined in the next two sections.

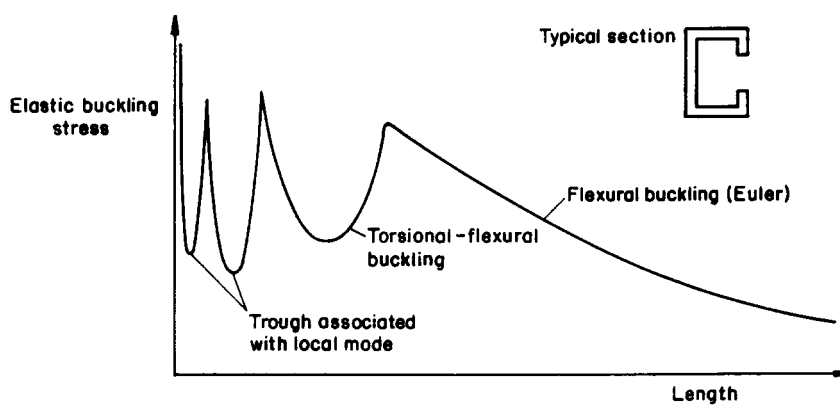
Derivations 25 and 28 give more information on buckling, as do the Data Items on buckling in the Structures Sub-series, which cover a wider range of elements and conditions than the data presented in this Item.



3.3. a



3.3. b



3.3. c

Sketch 3.3 Effect of strut length on the elastic buckling stress of some typical sections

4. BUCKLING OF STRUTS SUBJECT TO COMPRESSION

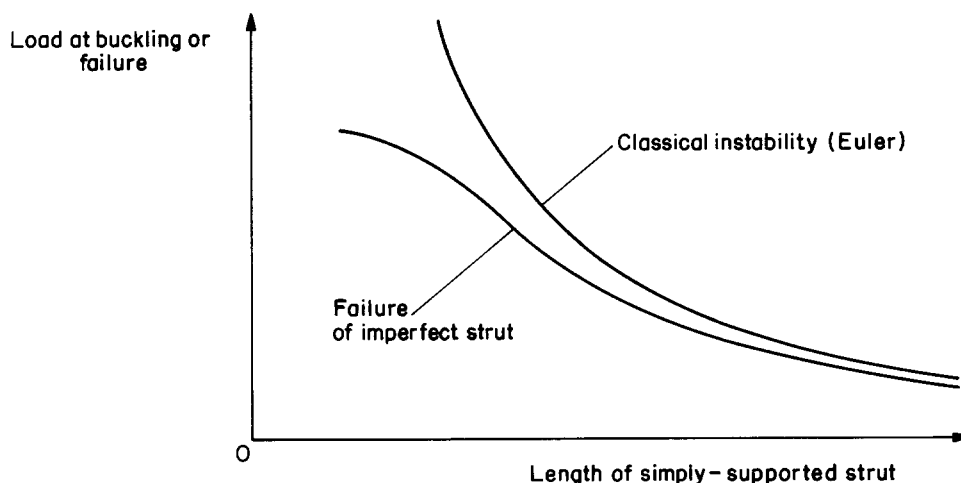
A strut is a structural component having cross-sectional dimensions small in comparison with its length that is loaded in compression parallel to that length. The cross-section may be solid but, for reasons of structural efficiency, the strut is more likely to be constructed from material that is itself thin in relation to the overall dimensions of the cross-section. The cross-sectional shape is usually uniform along the length of the strut but the shape may change locally near the ends in order to introduce the load uniformly to the cross-section.

Most calculations relating to buckling deal with theoretically perfect geometry and precise axial alignment of the load. The most elementary of these is the classical Euler calculation for flexural buckling. Similar analyses can be performed for torsional-flexural and torsional buckling. Calculation of the local buckling load is usually carried out by idealising the section into an assembly of flat plates and analysing these using flat plate theory.

Struts can fail in all three buckling modes, flexural, torsional-flexural and local, and these are dealt with in the following sections. These modes are independent of one another so that all must be checked to avoid buckling.

4.1 Overall Flexural Buckling

For normally manufactured struts subject to flexural buckling the effect of the initial deviation from straightness due to manufacturing inaccuracy is usually significant and should be allowed for. This results in an analysis that predicts the load at which an extreme fibre reaches a prescribed stress, usually a yield stress, prior to buckling. Although such calculations are dependent on the form of the initial deviation from straightness it is usual, and for most practical cases adequate, to assume a sinusoidal form. The resulting loads may conveniently be plotted against the length of the simply-supported strut to give the familiar strut curves of the type shown in Sketch 4.1. The abscissa of Sketch 4.1 could equally be the buckle half-wavelength since the overall mode of buckling always occurs at the highest wavelength possible.



Sketch 4.1 Typical Euler buckling curves for struts

The curves presented in Figure 1 refer to flexural failure, and are based on Equation (4.1).

$$\frac{f_b}{f_E} = \frac{1}{2} \left(\frac{f_p}{f_E} + (1 + \eta) \right) - \left[\frac{1}{4} \left(\frac{f_p}{f_E} + (1 + \eta) \right)^2 - \frac{f_p}{f_E} \right]^{1/2}, \quad (4.1)$$

where η is an empirically determined factor accounting for imperfections, taken as $0.003l'/k$ for general engineering structures. It must be noted that this is a general correction factor – more specific factors for differing section shapes may be obtained from the necessary standards, for example, References 31, 32, 35 and 36.

Equation (4.1) is represented as a series of curves plotted as f_b/E against l'/k for a range of values of f_p/E in Figure 1. The curves are presented for zero rotational restraint and infinite lateral restraint, that is pinned, at both ends of the strut. The curves can be used to take account of different values of rotational and/or lateral restraint by replacing the actual strut length by an equivalent value. A table of equivalent lengths relating to various end conditions is provided by Table 4.1 below. (In some codes of practice the fixed-fixed equivalent length is assumed to be greater than $0.5l$ because perfect fixing of the ends is impossible in practice.)

TABLE 4.1 Table of Equivalent Lengths

End Conditions	<i>Pinned</i>	<i>Pinned</i>	<i>Fixed</i>	<i>Fixed</i>	<i>Fixed</i>
	<i>Pinned</i>	<i>Fixed</i>	<i>Fixed</i>	<i>Free</i>	<i>Fixed in rotation, with no lateral restraint</i>
Equivalent length, l'	$1.0 \times l$	$0.7 \times l$	$0.5 \times l$	$2.0 \times l$	$1.0 \times l$

It should be noted that l' may have different values in different planes, and the minimum value of k/l' for the section should be used. When the strut is attached continuously to a plate along its length the appropriate value of k is that about the neutral axis parallel to the plane of the plate and k should include an allowance for that part of the plate that acts with the strut.

For most isotropic engineering materials and commonly employed strut sections containing initial irregularities the shear deformation has a negligibly small effect on the buckling load and, unless very precise values are required, can be neglected. For struts in which the shear stiffness is unusually low the shear deformation may seriously reduce the buckling load, for example, in the case of a weakly braced strut.

4.2 Torsional Buckling

Torsional buckling can occur as a result of compressive loads applied axially to the ends of a strut. Torsional buckling can also occur in conjunction with flexural buckling, that is torsional-flexural buckling, as a result of compression. Details of these modes of buckling are given in the next sections. Section 5 deals with buckling due to applied torsional couples.

4.2.1 Torsional buckling due to axial compression

Some thin-walled sections, with relatively low torsional stiffness may, under axial compression alone, buckle torsionally while the longitudinal axis remains straight. The subject of torsional buckling in this mode is treated extensively in Derivation 18 and Reference 34.

The elastic buckling stress at which torsional buckling due to axial loading occurs is given by Equation (4.2):

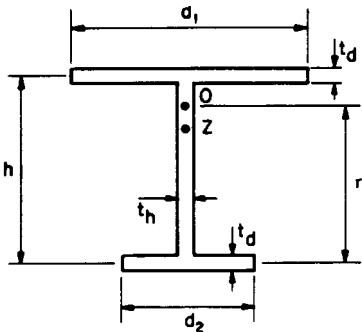
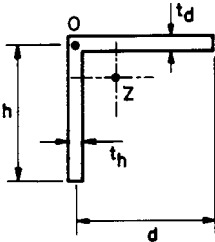
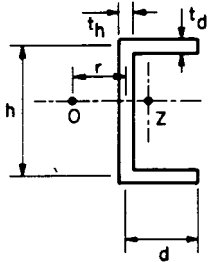
$$f_T = \frac{GJ}{I_p} + \frac{\pi^2 E \Gamma}{I_p l'^2}, \quad (4.2)$$

where Γ is the warping constant of the strut. It should be noted that Γ is zero for shapes consisting of thin rectangular elements that intersect at a common point.

For a strut with ends fixed against rotation but free to warp, the effective length, l' , is equal to the actual strut length, l . If, however, the ends are prevented from warping the effective length will be only half the actual length, that is $l/2$. An end of a strut is free to warp when the cross-section of the end is free to distort out of its original plane.

Equation (4.2) is applicable to doubly symmetric sections, such as I-sections, whose shear centre and centroid coincide. For the common single-angle strut, Equation (4.2) will provide a reasonable approximation. Expressions for the warping constant, Γ , and torsion constant, J , for various shapes are given in Table 4.2. More examples of Γ can be found in Derivation 18 and References 32 and 35.

TABLE 4.2 Torsional Properties for some Sections

SECTION	TORSIONAL PROPERTIES
	$J = (d_1 t_d^3 + d_2 t_d^3 + h t_h^3)/3$ $\Gamma = \frac{t_d h^2}{12} \left(\frac{d_1^3}{d_1^3 + d_2^3} \right)$ $r = h \frac{d_1^3}{d_1^3 + d_2^3}$ Note that two special cases exist; an I-section where $d_1 = d_2$ and a T-section where $d_2 = 0$. The above equations hold for these cases.
	$J = (d t_d^3 + h t_h^3)/3$ $\Gamma = (d^3 t_d^3 + h^3 t_h^3)/36 \approx 0 \text{ for small } t_d \text{ and } t_h$
	$J = (2d t_d^3 + h t_h^3)/3$ $\Gamma = \frac{t_d d^3 h^2}{12} \left(\frac{3d t_d + 2h t_h}{6d t_d + h t_h} \right)$ $r = \frac{3d^2 t_d}{h t_h + 6d t_d}$
O = shear centre Z = centroid	

In the general case where the shear centre does not coincide with the centroid, buckling is a combination of flexure and torsion, and is described in the next section.

4.2.2 Torsional-flexural buckling

In the general case of a column of thin-walled open cross-section, buckling failure usually occurs by a combination of torsion and flexure. For struts of arbitrary cross-section the buckling stress may be found using Equation (4.3). More detailed information can be obtained from Reference 39.

The buckling stress, f_{btf} , is given by the smallest root of the following equation.

$$\left[1 - \frac{f_{Ex}}{f_{btf}}\right] \left[1 - \frac{f_{Ey}}{f_{btf}}\right] \left[1 - \frac{f_T}{f_{btf}}\right] - \left[1 - \frac{f_{Ex}}{f_{btf}}\right] \left[\frac{y_0}{\rho}\right]^2 - \left[1 - \frac{f_{Ey}}{f_{btf}}\right] \left[\frac{x_0}{\rho}\right]^2 = 0, \quad (4.3)$$

where f_{Ex} and f_{Ey} are the Euler buckling stresses about the x - and y -axes respectively. The torsional buckling stress, f_T , can be obtained from Equation (4.2).

If f_{Ex} , f_{Ey} and f_T differ very widely, the buckling stress will be, in general, only a little smaller than the smallest of these stresses and the instability will be predominantly in the corresponding mode.

For struts having a cross-section with one axis of symmetry, for example the x -axis, the instability occurs either in pure flexure about the y -axis or in combined flexure about the x -axis and torsion about the shear centre, whichever mode results in the lower buckling stress. In this case the buckling stress can be obtained from Equation (4.4), which is a simplified form of Equation (4.3) having only two roots.

$$f_{btf} = 2 \left[\frac{1}{f_{Ex}} + \frac{1}{f_T} \pm \left[\left(\frac{1}{f_{Ex}} - \frac{1}{f_T} \right)^2 + \frac{4}{f_{Ex} f_T} \left(\frac{x_0}{\rho} \right)^2 \right]^{1/2} \right]^{-1}. \quad (4.4)$$

4.3 Local Buckling

Calculation of the local buckling loads of sections composed of flat plate elements normally uses plate theory modified to take account of the connections between the elements. The plates are usually assumed not to translate in their own planes and this implies that the plate junctions remain straight along the length of the strut.

Calculation of local buckling loads also usually ignores the effects of initial irregularities, through-the-thickness shear deformations and the effects of the local radius at the junctions of the plate elements. The error incurred by ignoring shear deformations and initial irregularities is usually negligible and ignoring the local radius results in a slight under-estimate of the buckling load, thus leading to a conservative estimate of the buckling stress.

Local buckling is not significantly affected by any form of end restraint. In the extreme case of a very short strut, with a length less than about twice the major dimension of the cross-section, end restraints may increase the buckling stress but not greatly.

For the simple case of elastic buckling, Equation (4.5) may be used to calculate the buckling stress:

$$f_b = KE(t_h/h)^2 \text{ or } KE(t_d/d)^2. \quad (4.5)$$