

MAXIMUM INELASTIC BENDING MOMENTS AND FORM FACTORS FOR SYMMETRIC BENDING OF BEAMS

1. APPLICATION AND DERIVATION OF PROGRAM

1.1 Introduction

This Data Item, together with ESDUpac A8835, presents a Fortran computer program for calculating the bending moment associated with the maximum allowable extreme fibre stress in a beam under symmetric bending. The program allows for the non-linear inelastic behaviour of the beam beyond the limit of proportionality in the generalised form given in Data Item No. 76016*. In addition to the maximum bending moment a form factor based on an elastic stress equivalent to the 0.2 per cent proof stress is given.

This Data Item is arranged in three sections. Section 1 gives information on the application of the program, examples of its application and a summary of the theory from which the program is derived. Section 2 gives the instructions necessary for operating the program, guidance in interpreting the output and an example of input to and output from the program. Section 3 gives a description of the program structure and details of the tests that have been performed on the program.

The analysis assumes that the beam material is homogeneous. The strain is considered to vary linearly across the section in the y -direction, plane sections remain plane, but the beam may be stressed beyond the limit of proportionality to an extent that may be defined by the user, as either a maximum strain or a maximum stress. The stress-strain curve of the material is represented by the generalised form given in Data Item No. 76016 and is assumed to be the same in tension and compression. The curve is defined by either two proof stresses, the material coefficients m and f_n or two values of strain and their complementary stresses. Details of the theoretical basis of the program are given in Section 1.3.

The beam cross section may take the form of any multi-segment polygon provided that its first and second moments of area are balanced about the neutral x -axis. The cross section does not have to be symmetric about the y -axis. Sections that are not symmetric about the x -axis but satisfy the above requirements, *eg.* Z-sections, can be analysed provided bending about the y -axis is constrained. Examples on the application of the program are given in Section 1.4.

1.2 Notation

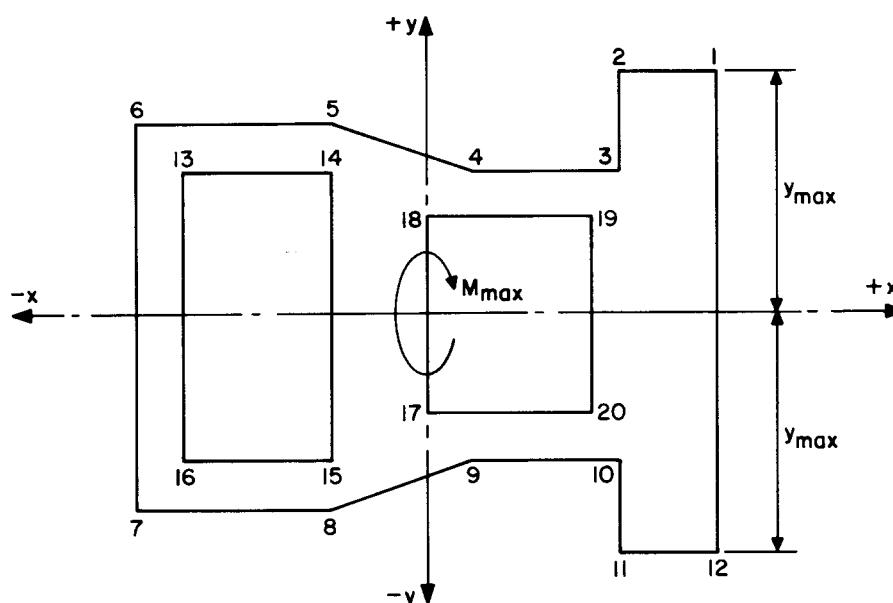
Both S.I. and British Units are quoted but any coherent system of units may be used.

a, b	identification symbols of vertices of a segment (see Sketch 1.2)		
C	identifier for maximum stress or strain		
E	modulus of elasticity of material	N/m ²	lbf/in ²
E_t	tangent modulus of material	N/m ²	lbf/in ²
f	stress	N/m ²	lbf/in ²
f_{max}	maximum stress in beam material	N/m ²	lbf/in ²

* Data Item No. 76016 "Generalisation of smooth continuous stress-strain curves for metallic materials".

f_n	stress at which $E_t = E/2$	N/m ²	lbf/in ²
f_t	ultimate tensile stress in beam material	N/m ²	lbf/in ²
f_1, f_2	stresses provided at any two points on stress-strain curve	N/m ²	lbf/in ²
H	number of holes in cross section		
I	second moment of area of beam cross section	m ⁴	in ⁴
K	form factor, defined by $K = M_{max}/M_e$		
m	material characteristic (see Data Item No. 76016)		
M_e	moment about x-axis derived from linear analysis with extreme fibre stress equal to t_2 ($M_e = Zt_2$)	N m	lbf in
M_{max}	moment about x-axis derived from non-linear analysis with inelastic stress or strain at extreme fibre	N m	lbf in
M_{xs}	moment about x-axis on segment of beam cross section during integration	N m	lbf in
N	total number of vertices in polygonal beam cross section		
NE	number of vertices of external shape of beam cross section		
NH	number of vertices of holes in beam cross section		
S	indicator for stress-strain curve input selection		
t_A, t_B	proof stresses $t_A < t_B$	N/m ²	lbf/in ²
t_1	0.1 per cent tensile proof stress	N/m ²	lbf/in ²
t_2	0.2 per cent tensile proof stress	N/m ²	lbf/in ²
t_5	0.5 per cent tensile proof stress	N/m ²	lbf/in ²
w_1, w_2	width of parts of segment for integration	m	in
x, y	coordinate system (see Sketch 1.1)		
y_{max}	distance from neutral axis of furthest vertex of cross section	m	in
Z	section modulus of beam cross section, $Z = I/y_{max}$	m ³	in ³
ϵ	strain		
ϵ_{max}	maximum total strain in beam material		
ϵ_1, ϵ_2	total strains appropriate to stresses f_1 and f_2 (not necessarily t_1 and t_2)		

- ϵ_5 total strain appropriate to 0.5 per cent proof stress
- $\epsilon_{pA}, \epsilon_{pB}$ permanent strains corresponding to t_A and t_B , respectively (proof strains)
- $\epsilon_{p1}, \epsilon_{p2}$ permanent strains corresponding to t_1 and t_2 , respectively (proof stress)



Sketch 1.1 Coordinate system, moment configuration and typical identification of vertices

1.3 Derivation

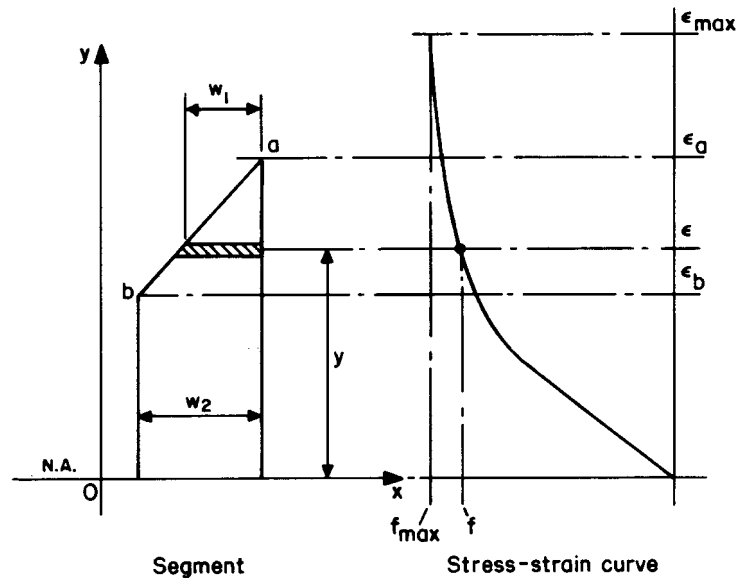
1.3.1 Sources

The non-linear inelastic analysis on which the program of ESDUpac A8835 is based was provided by Derivation 1. It was developed using the analysis and program of Derivation 2 as a starting point and guide.

1.3.2 Theory

The analysis is based upon small deflection, non-linear, inelastic theory. A bending moment is determined that gives the maximum allowable extreme fibre stress or strain specified by the user. It is assumed that the strain distribution under the action of the bending moment is linear across the section, plane sections remain plane, but that the stress distribution may be non-linear inelastic.

The moment taken by each segment of the cross section between adjacent vertices and the neutral axis is considered in turn by integrating the stress between the neutral axis and the furthest vertex. The sum of the segment moments provides the moment on the entire cross section.



Sketch 1.2 General segment for integration and its stress-strain curve

Sketch 1.2 shows a typical segment between vertices a and b and the neutral axis (x). It also shows the idealised inelastic stress distribution applied to the segment; the strain is assumed to vary linearly from vertex a to the neutral axis. The expression obtained for the moment on the segment by integrating the stresses between the neutral axis and y_b and between y_b and y_a is of the form

$$M_{xs} = \int_0^{y=b} f w_2 y dy + \int_{y=b}^{y=a} f w_1 y dy. \quad (1.1)$$

The stresses within these expressions are based on the linear strain proportional to the maximum allowable strain applied at the furthest vertex of the segment furthest from the neutral axis. They are obtained from the expression:

$$\epsilon = \frac{f}{E} \left[1 + \frac{1}{m} \left(\left[\frac{f}{f_n} \right]^2 \right)^{\frac{m-1}{2}} \right]. \quad (1.2)$$

This expression is derived from Equation (3.1) of Data Item No. 76016 and is reformulated to avoid negative indices and to give an identical stress-strain curve in compression and tension.

The moment appropriate to the allowable inelastic stress at the extreme fibres of the cross section is obtained by summing the moments on all the segments thus

$$M_{max} = \sum_{1, 2}^{N, 1} M_{xs}. \quad (1.3)$$

The form factor is the ratio of the maximum inelastic moment to the maximum elastic moment for the

cross-section, thus

$$K = \frac{M_{max}}{M_e}, \quad (1.4)$$

where M_{max} is obtained from Equation (1.3) and

$$M_e = Z t_2. \quad (1.5)$$

The section modulus Z is computed by the program. The 0.2 per cent proof stress, t_2 , used here is that defined in entry 4 of the input (see Table 2.1). If t_2 has not been used to define that curve in entry 2 of the program input then it may not exactly correspond with the generalised stress-strain curve of Equation (1.2).

Where a cross section can be defined by continuously connected vertices, as shown for an I-section in the example in Section 1.4, changes of direction of vertices and superposition of spaces on solid material are automatically accounted for by the sign of the integration. Where holes are present and therefore the sequence of vertices defining the shape is broken, as shown in Sketch 1.1, those defining the hole are allocated in the opposite sense (clockwise) to those for the parent body (anti-clockwise) to ensure that the hole is subtracted.

1.3.3 Derivation and references

Derivation

This section lists selected sources that have assisted in the presentation of this Item.

1. SNELL, L.B. Unpublished Reports, British Aerospace plc, Civil Aircraft Division, Hatfield, January 1977.
- ROBINSON, J.R.
2. MITCHELMORE, P.J. Unpublished Communications, British Aerospace plc, Civil Aircraft Division, Hatfield, September 1988.

References

The references given are recommended sources of information supplementary to that in this Item.

3. COZZONE, F.P. Bending strength in the plastic range. *Jl aero. Sci.*, Vol. 10, No. 5, pp. 137-151, May 1943.
4. BARRETT, A.J. The bending of some common beam sections into the plastic range. *Jl R. aeronaut. Soc.*, Vol. 57, pp. 110-115, February 1953.
5. ESDU Information on the use of Data sheets 01.06, Data Item No. Struct 01.06.00, ESDU International plc, November 1956.

1.4 Application of the Program of ESDUpac A8835

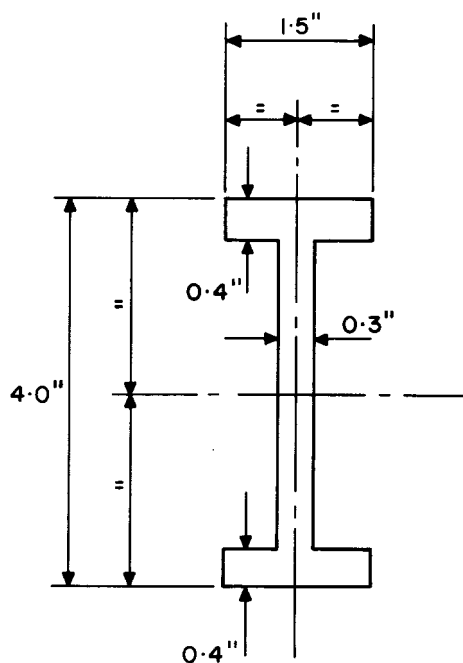
The program can be used to obtain the maximum moment that may be applied to a beam of any polygonal cross section that is symmetric about the neutral, x -axis. The maximum moment, M_{max} , is that which is appropriate to the maximum inelastic stress or strain, as defined by the user, applied to the extreme fibres of the cross section. In default the program will provide the moment related to the 0.5 per cent proof strain at the extreme fibre. Values higher than this should be used with caution since there may be problems in satisfying proof (yield) requirements and also high shear stresses can occur. The program also provides a form factor, K , and the section modulus, Z , used in the calculation of K .

The program is applicable to any beam cross section, for which the first and second moments of area are symmetric about the x -axis, that can be defined by line-linked points and for which the material modulus of elasticity and any two points or characteristics of the material stress-strain curve and the 0.2 per cent proof stress are known.

Typical beam cross sections to which the program is applicable are symmetric I-sections, H-sections, solid and hollow square, rectangular and diamond sections. It is applicable to shapes such as channels or Z-sections that are not symmetric about the x -axis provided the width of material at any distance y from the x -axis is the same as that at $-y$ on the opposite side of the x -axis and that the section is constrained from bending about the y -axis.

The program has been used to analyse an I-section beam for comparison with Data Item No. Struct 01.06.02. Data Item No. Struct 01.06.02 assumes that the material stress-strain curve can be idealised as an elastic-perfectly plastic curve with a maximum stress equal to the 0.2 per cent proof stress. The program has been used with a shallow non-linear curve based on the same 0.2 per cent proof stress, $t_2 = 40\,000 \text{ lbf/in}^2$, with $m = 100$, $f_n = 38\,450 \text{ lbf/in}^2$, $E = 10 \times 10^6 \text{ lbf/in}^2$, $f_t = 41\,000 \text{ lbf/in}^2$ and the default value of 0.5 per cent proof strain.

The I-section beam used in the comparison is shown in Sketch 1.3



Sketch 1.3

The parameters required for use with Data Item No. Struct 01.06.02, derived from the above information, are:

$$\frac{t}{h} = 0.10, \frac{b}{d} = 0.20 \text{ and } \frac{Ph}{m} = 0.$$

The form factors obtained from the two sources are tabulated below.

Data Item No. Struct 01.06.02	Program of ESDUpac A8835
$K = 1.210$	$K = 1.215$

The section modulus obtained by the program was

$$Z = 2.362 \text{ in}^3$$

and the maximum inelastic moment

$$M_{max} = 114\,200 \text{ lbf in.}$$