

FLEXURAL AND TORSIONAL-FLEXURAL BUCKLING OF THIN-WALLED OPEN SECTION STRUTS

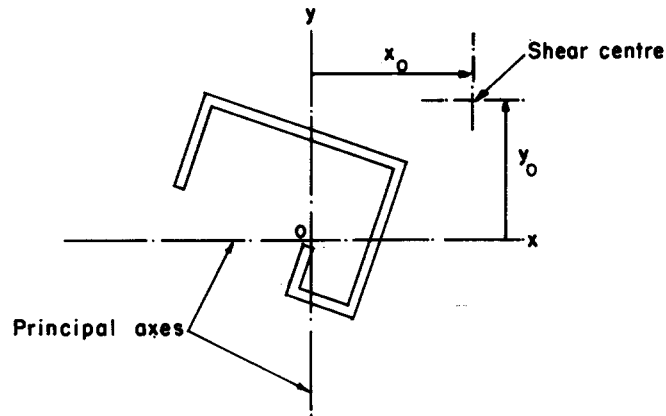
1. NOTATION

A	cross-sectional area of strut section	m^2	in^2
E_t	tangent modulus of elasticity of strut material at stress f	N/m^2	lbf/in^2
f	minimum compressive stress at onset of buckling	N/m^2	lbf/in^2
f_T	torsional buckling stress	N/m^2	lbf/in^2
f_x	Euler buckling stress for flexure about x -axis	N/m^2	lbf/in^2
f_y	Euler buckling stress for flexure about y -axis	N/m^2	lbf/in^2
G	shear modulus of strut material	N/m^2	lbf/in^2
I_p	polar moment of inertia of section about shear centre	m^4	in^4
I_x	second moment of area of section about x -axis	m^4	in^4
I_y	second moment of area of section about y -axis	m^4	in^4
J	torsion constant of section as defined in Data Item No. Struct 00.07.00	m^4	in^4
l	length of pin-ended strut	m	in
x_0	distance of shear centre from y -axis (see Sketches 2.1 and 2.2)	m	in
y_0	distance of shear centre from x -axis (see Sketch 2.1)	m	in
Γ	warping constant of section as defined in Data Item No. Struct 00.07.00	m^6	in^6
ρ	polar radius of gyration of section about shear centre	m	in
ν	Poisson's ratio of strut material		
$o(x, y)$	principal axes of cross section		

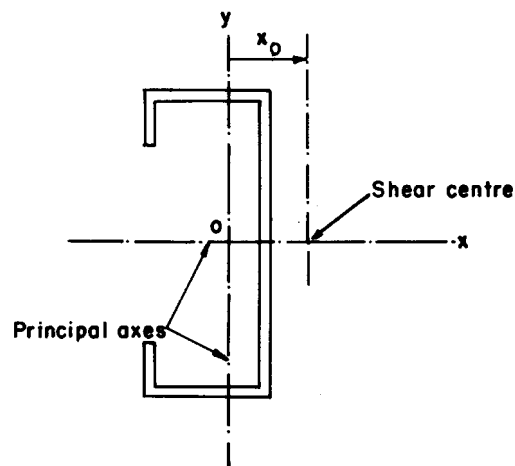
Both SI and British units are quoted but any coherent system of units may be used.

2. NOTES

This Item gives the method of calculating the critical compressive stress, f , at which buckling of thin-walled open section struts occurs. The ordinary Euler buckling of a compressed bar does not always occur when the section is torsionally weak. The strut may buckle by twisting, or by a combination of flexure and twist. This Item shows how to determine which mode of instability will be critical and the value of this critical buckling stress.



Sketch 2.1 General, unsymmetric, section



Sketch 2.2 Section with symmetry about x-axis

For struts of arbitrary cross section the buckling stress may be found from the largest root of the following cubic in E_t/f .

$$\left[1 - \frac{E_t f_x}{f E_t}\right] \left[1 - \frac{E_t f_y}{f E_t}\right] \left[1 - \frac{E_t f_T}{f E_t}\right] - \left[1 - \frac{E_t f_x}{f E_t}\right] \left[\frac{y_0}{\rho}\right]^2 - \left[1 - \frac{E_t f_y}{f E_t}\right] \left[\frac{x_0}{\rho}\right]^2 = 0, \quad (2.1)$$

where f_x is the Euler buckling stress defined by

$$\frac{f_x}{E_t} = \frac{\pi^2 I_x}{l^2 A}, \quad (2.2)$$

f_y is the Euler buckling stress defined by

$$\frac{f_y}{E_t} = \frac{\pi^2 I_y}{l^2 A} \quad (2.3)$$

and f_T is the torsional buckling stress defined by*

$$\frac{f_T}{E_t} = \frac{G J}{E_t I_p} + \frac{\pi^2 \Gamma}{I_p l^2}. \quad (2.4)$$

If f_x , f_y and f_T differ very widely, the buckling stress will be, in general, only a little smaller than the smallest of these stresses and the instability will be predominantly in the corresponding mode.

For struts having a cross section with one axis of symmetry, say the x -axis, the instability occurs either in pure flexure about the y -axis or in combined flexure about the x -axis and torsion about the shear centre, whichever is the lower. For such a strut the flexural buckling stress is given by

$$\frac{f_y}{E_t} = \frac{\pi^2 I_y}{l^2 A} \quad (2.5)$$

and the torsional-flexural buckling stress is given by the larger root, E_t/f , of the equation

$$\left[1 - \frac{E_t f_x}{f E_t}\right] \left[1 - \frac{E_t f_T}{f E_t}\right] = \left[\frac{x_0}{\rho}\right]^2. \quad (2.6)$$

Equation (2.6) is a quadratic in E_t/f so that

$$\frac{E_t}{f} = \frac{1}{2} \left[\frac{E_t}{f_x} + \frac{E_t}{f_T} \pm \left\{ \left(\frac{E_t}{f_x} - \frac{E_t}{f_T} \right)^2 + \frac{4 E_t^2}{f_x f_T} \left[\frac{x_0}{\rho} \right]^2 \right\}^{1/2} \right]. \quad (2.7)$$

One of the values of f/E_t given by the above equation is always less than the smaller f_x/E_t and f_T/E_t and the other value exceeds the greater of f_x/E_t and f_T/E_t .

Data Item No. 76023[†] gives curves for the flexural and torsional-flexural buckling of channel section and lipped channel section struts.

* As a first approximation G may be assumed to be proportional to E_t . This assumption will produce a conservative answer. If necessary a more accurate allowance can be made for the variation of G in a second approximation.

[†] Data Item No. 76023 "Buckling of struts. Lipped and unlipped channel sections".