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Natural frequencies of singly-curved laminated plates with simply-supported edges

Associated software: ESDUpac A8911

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THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item was monitored and guided by the Acoustic Fatigue Panel on behalf of the Vibration and Structural Dynamics Steering Group:

Chairman

Prof. R.G. White	—	Institute of Sound and Vibration Research, University of Southampton

Members

Prof. R.D. Adams	—	University of Bristol
Mr B.P. Broadley	—	Rolls-Royce Ltd, Derby
Dr D.J. Cottney	—	Short Brothers plc, Belfast
Mr R.J. Cummins	—	British Aerospace (Commercial Aircraft) Ltd, Bristol
Mr D.C.G. Eaton*	—	The European Space Research and Technology Centre, Domeinweg, The Netherlands
Mr D. Sims	—	British Aerospace (Dynamics) Ltd, Hatfield.

* Corresponding Member

The Vibration and Structural Dynamics Steering Group first met in 1976 and now has the following membership:

Chairman

Mr D.C.G. Eaton	—	The European Space Research and Technology Centre, Domeinweg, The Netherlands

Members

Mr T.V. Lawson	—	University of Bristol
Dr M.R. Smith	—	University of Newcastle-upon-Tyne
Dr H.H.B.M. Thomas	—	Independent.

The technical work involved in the assessment of the available information and the construction and subsequent development of the Data Item was undertaken by

Mr R.F. Lambert	—	Head of Noise and Structural Dynamics Group.

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NATURAL FREQUENCIES OF SINGLY-CURVED LAMINATED PLATES WITH SIMPLY-SUPPORTED EDGES

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NATURAL FREQUENCIES OF SINGLY-CURVED LAMINATED PLATES WITH SIMPLY-SUPPORTED EDGES

1. NOTATION

A	in-plane stiffness matrix for plate	N/m	lbf/in
a	length of plate in x-direction (see Sketch 1.1)	m	in
B	coupled in-plane flexural stiffness matrix for plate	N	lbf
b	length of plate in y-direction, or arc length of curved plate (see Sketch 1.1)	m	in
D	flexural stiffness matrix for plate	N m	lbf in
E_α, E_β	lamina modulus of elasticity in α-, β-direction respectively	N/m^2	lbf/in^2
$G_{\alpha\beta}$	shear stiffness of lamina in (α, β) plane	N/m^2	lbf/in^2
m	mode number of layers in x-direction		
N	number of layers in plate		
N_s	static force per unit width in y-direction, or hoop force per unit width in s-direction of curved plate (tensile forces positive)	N/m	lbf/in
N_x	static force per unit width in x-direction (tensile forces positive)	N/m	lbf/in
n	mode number in y-direction, or circumferential mode number of curved plate		
p	pressure loading on concave surface of curved plate	N/m^2	lbf/in^2
s	circumferential distance from $y = 0$ axis (see Sketch 1.1)	m	in
R	plate mean radius (see Sketch 1.1)	m	in
t	laminated plate thickness	m	in
x, y, z	plate coordinate system (see Sketch 1.1)		
U_{mn}, V_{mn}, W_{mn}	modal amplitude parameters [see Equations (2.1), (2.2) and (2.3)]	m	in
u, v, w	axial, circumferential and radial displacements of plate mid-plane, respectively	m	in

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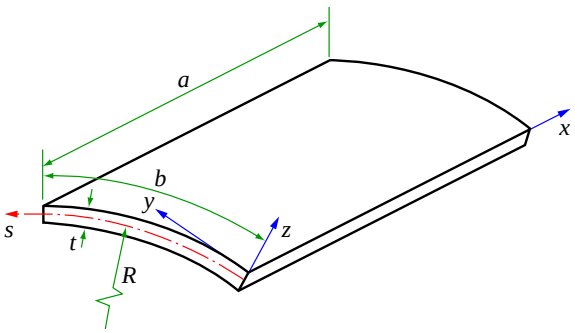
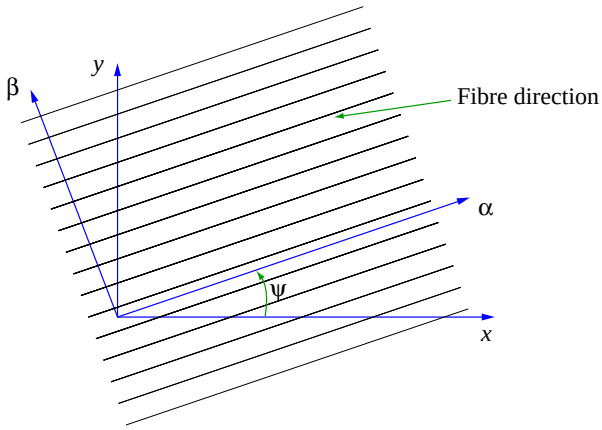
α, β	lamina coordinates aligned with principal axes of orthotropy of lamina (see Sketch 1.2)		
$\nu_{\alpha\beta}$	Poisson's ratio* for lamina resulting from load applied in α -direction		
ρ	density of plate material	kg/m^3	lb s^2/in^4
τ	time	s	s
ψ	angle of principal axis of orthotropy, α , of lamina measured from plate principal axis, x (see Sketch 1.2)	degree	degree
ω	circular natural frequency	rad/s	rad/s

* In some previous Items in this series, Poisson's ratio has been designated σ



Sketch 1.1 Plate dimensions



Sketch 1.2 Relationship between plate and layer axes

2. NOTES

2.1 General

This Data Item provides a means of estimating the lower natural frequencies of singly-curved laminated plates. The considered plates are made up from an arbitrary number of layers bonded together. The orthotropic axes of material symmetry of the individual layers need not coincide with the x, y-axes of the cylindrical plate. The data are applicable to thin, rectangular plates of shallow curvature having all edges simply-supported. Static in-plane forces, uniformly distributed along the plate edges, may be considered but in-plane shear forces are assumed to be zero. The static in-plane forces are assumed to act on the middle surface of the plate in the x-direction and tangentially to the plate straight edges. The data presented in this Item are particularly applicable to fibre-reinforced composite laminated plates.

Because of the large number of practical laminate constructions, and consequent large range of elastic properties, it is not possible to present natural frequency data in graphical form. Therefore, data are provided in the form of a computer program contained in ESDUpac A8911 with example input and output files. ESDU 83036* provides a computer program for the estimation of natural frequencies of initially unstressed

* Natural frequencies of rectangular, specially orthotropic laminated plates.

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flat plates having combinations of simply-supported and clamped edges.

Data presented in this Item are applicable to flat plates as well as singly-curved plates. Therefore, one plate dimension is defined in terms of s for curved plates or y for flat plates.

2.2 Method of Analysis

The method of analysis is based on that given in Derivation 3 and is applicable to thin laminated plates of shallow cylindrical curvature. The laminated plate constitutive relations are substituted into the equations of motion and, using the strain-displacement and curvature-displacement relationships obtained from the classical theory of elasticity, a set of three displacement equations is derived. In forming the strain-displacement relationships, it is assumed that tangential displacements are linear functions of the z-coordinate and that transverse displacements do not vary through the plate thickness. Substitution of appropriate displacement relationships into the displacement equations leads to a cubic equation in frequency. The lowest of the three natural frequencies is evaluated. For that natural frequency the predominant displacement is normal to the surface of the plate.

The displacement relationships used, which satisfy the boundary conditions of a plate simply-supported on all edges, are as follows.

$$u = U_{mn} e^{i\omega\tau} \cos\frac{m\pi x}{a} \sin\frac{n\pi s}{b}, \tag{2.1}$$

$$v = V_{mn} e^{i\omega\tau} \sin\frac{m\pi x}{a} \cos\frac{n\pi s}{b}, \tag{2.2}$$

and

$$w = W_{mn} e^{i\omega\tau} \sin\frac{m\pi x}{a} \sin\frac{n\pi s}{b}. \tag{2.3}$$

For the case of simply-supported plates, using the above displacement functions restricts the method of analysis to a class of orthotropic laminates in which the shear coupling terms in the constitutive relationships vanish (see Section 2.6).

2.3 Mode Numbers

For simply-supported plates, the mode numbers m and n are the number of half waves across the plate in the x-direction and y-direction (or circumferential direction), respectively. That definition is consistent with the definition, in terms of nodal lines, used in other Items in the Series.

2.4 Effect of Plate Curvature

The natural frequency of a mode of vibration increases with increased plate curvature. This is illustrated in Figure 1 where, for a typical fibre-reinforced laminated plate, the lower natural frequencies are plotted against a curvature parameter, b^2/Rt. The most rapid increase in natural frequencies with b^2/Rt occurs in modes having only one half wave in the circumferential direction (*i.e.* $n = 1$).

To illustrate both the effects of plate curvature and plate aspect ratio, a/b, the ratio of curved plate frequency to flat plate frequency is plotted against a plate parameter, a/mb, for a range of values of b^2/Rt for modes having $n = 1$ and $n = 2$ in Figures 2a and 2b. The plate construction used to generate the curves of Figures 2a and 2b is that of the symmetric laminated plate given in Figure 1.

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2.5 Effect of In-plane Loading

The general information on the effects of static in-plane loading on the natural frequency of flat plates, given in ESDU 90016*, is applicable to laminated plates. The natural frequency of a mode of vibration increases with increasing static tensile loading (*i.e.* N_x and N_s positive) but with the application of, and initial increase in, compressive loading the natural frequency decreases.

Under high static compressive loading, as the buckling load is approached, natural frequencies may increase with increasing loading. However, this Item is applicable only to static compressive loading much lower than the plate buckling load. For the analysis used in this Item it is recommended, based on flat plate test data, that the maximum compressive loading should not exceed 30 per cent of the buckling load.

As the static in-plane loading is varied, the rate of change of natural frequency is a function of the mode of vibration, the plate aspect ratio and the plate curvature. In the case of increasing plate curvature, the rate of change of frequency decreases most significantly for modes having only one half wave in the circumferential direction. This effect is illustrated in Figures 3a and 3b where natural frequencies are shown, for values of b^2/Rt of 10 and 30, respectively, for a plate subjected to a static loading representative of pressure loading (N_s/N_x = 2.0). Consequently, with a change in in-plane loading, the order in which modes occur as frequency increases may vary.

2.6 Accuracy and Limitations of Data

In this Item a means is provided for the estimation of the lower natural frequencies of singly-curved laminated plates having edge conditions that can be modelled as simply-supported. The data are applicable to rectangular, laminated plates. The method of analysis is outlined in Section 2.2 and is constrained by the assumptions detailed in that section.

The method of analysis uses small deflection, linear plate theory. Plates are assumed to be thin, a criterion that is satisfied if both t/a and t/b are less than 0.05. The data are applicable to plates of shallow curvature having a value of t/R that is negligible compared with unity. The shallow shell criterion is satisfied if both a/R and b/R are very much less than 1.0.

The effects of shear deformation and rotary inertia have been neglected in the analysis. This simplification is usually acceptable if the ratio of half wavelength to plate thickness is greater than 10. However, for a unidirectional fibre-reinforced plate, the criterion for neglecting shear deformation may require a higher half wavelength to plate thickness ratio.

The data provided are not restricted to symmetric laminates but it is required that the shear coupling terms within the stiffness matrices are negligible. The data are therefore applicable to laminates having a stacking geometry such that the stiffness matrix elements A_{13}, A_{23}, B_{13}, B_{23}, D_{13}, and D_{23}† are all zero. If the stiffness matrices are evaluated within the computer program, a check is carried out on the shear coupling terms and a warning message is given if any of those terms is not zero.

It has not been possible to validate fully the method of estimating natural frequencies of simply-supported curved plates due to the lack of test data for that edge condition. However, variational trends of the ratio of curved plate natural frequency to flat plate natural frequency, for plates that are most closely modelled as clamped on all edges, are reasonably predicted using the program. Until additional test data are available for validation of the estimation procedure, it is recommended that estimated natural frequencies should be treated with caution if either of the mode numbers exceeds 3, or the plate is subjected to more than light static in-plane loading.

* Natural frequencies of isotropic and orthotropic rectangular plates under static in-plane loading (including shear loading).

† Matrix elements A_{13}, A_{23}, *etc* are often referred to as A_{16}, A_{26}, *etc*.

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3. COMPUTATION OF NATURAL FREQUENCIES

The computer program contained in ESDUpac A8911 provides a means of estimating the natural frequencies of curved laminated plates having simply-supported edge conditions. The effect of direct in-plane static loading may be considered. The method of analysis is described in Section 2.2 and a flow chart for the computer program is presented in Figure 4.

3.1 Data Input

The input data may be entered either from the keyboard or from a file with directed input. Using a file input allows data correction, or case data changes, to be made using file editing facilities. Entering case data from the keyboard provides the opportunity for the program to provide visual requests for each item of data as it is required.

As an illustration of the form of the input file, the file structure required for the example calculation in Section 5 is listed in that Section.

Data are input in free format in the following order and any coherent system of units may be used.

3.1.1 Stiffness sub-matrices and material density

Are stiffness sub-matrices to be calculated?
Answer 1 (yes) or 0 (no).

If the answer to the above question is 'yes' go to Section 3.1.2 for the next input. Otherwise continue with following input.

Elements of the in-plane stiffness sub-matrix.
A_{11} , A_{12} , A_{22} , A_{33}

Elements of the coupled in-plane flexural stiffness sub-matrix.
B_{11} , B_{12} , B_{22} , B_{33}

Elements of the flexural stiffness sub-matrix.
D_{11} , D_{12} , D_{22} , D_{33}

Plate material density*.
ρ

Input continues from Section 3.1.3.

3.1.2 Data for calculation of plate stiffness sub-matrices

Number of layers in plate, symmetry code.
N, integer number

Set symmetry code = 1 for a symmetric lay-up.
Set symmetry code = 2 for an anti-symmetric lay-up.
Set symmetry code = 3 for an asymmetric lay-up.

* A density value expressed in British units as lb/in^3 has to be divided by 386.4 before being input into the computer program. (A force of 1 lbf acting on a mass of 1 lb produces an acceleration of 386.4 in/s^2.)

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