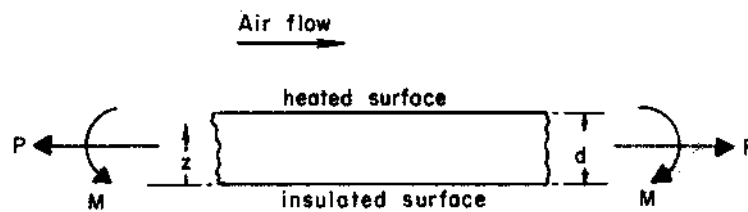


INFORMATION ON THE USE OF DATA ITEMS ON HOT STRUCTURES

1. INTRODUCTION

Data Items Nos 89023 to 89025 give temperature time variations and thermal loadings in thermally thick plates subjected to aerodynamic heating on one surface, the other surface being insulated. In these Data Items a thermally thick plate is defined as one in which the temperature gradients across the plate thickness are not negligible and this has been taken as being presented by Biot numbers $B > 0.1$.

The sheets consider the case of the recovery temperature (see Data Item No. 89022) of the boundary layer at the heated surface varying linearly with time, that is, $T_r = at + \text{constant}$.



Sketch 1.1

At the heated surface of the plate it is assumed that the rate at which heat is absorbed by the plate is equal to the rate of heat input by forced convection, that is, at any instant

$$k \frac{\partial T}{\partial z} = h(T_r - T) \text{ at } z = d, \quad (1.1)$$

where k and h are the thermal conductivity and the heat transfer coefficient respectively. (See Data Item No. 89022.) The opposite surface of the plate is assumed to be insulated, that is,

$$\frac{\partial T}{\partial z} = 0 \text{ at } z = 0. \quad (1.2)$$

2. THERMAL STRESSES

The stress-strain relationships in an elastic solid under a temperature distribution T are given by

$$\epsilon_x - \alpha(T - T_0) = \frac{1}{E}[f_x - \nu(f_y + f_z)]$$

with similar equations for the strains ϵ_y and ϵ_z , where T_0 is a datum temperature at which there are no thermal stresses, f , and α is the average coefficient of expansion over the range T_0 to T . Modulus of elasticity, E , and Poisson's ratio, ν , will, in general, depend on the local temperature. In a plate in the xy -plane the stress f_z is zero and the above equations can be expressed in the form

$$f_x = \frac{E}{1 - \nu^2}[\epsilon_x + \nu\epsilon_y - \alpha(T - T_0)(1 + \nu)]$$

with a similar expression for f_y .

2.1 Completely Restrained Plate

In calculating the thermal stresses in a plate with temperature variation through the thickness only, it is convenient to consider first a plate completely restrained against expansion in its own plane and against bending. Under these conditions the direct stresses, f , parallel to the surface of the plate due to an arbitrary temperature variation $T(z)$ are given by

$$f = f_x = f_y = -E^* \alpha (T - T_0), \quad (2.1)$$

where
$$E^* = \frac{E}{1 - \nu}.$$

This stress system has resultant forces P in the neutral plane of the plate and bending moments M about the neutral plane. These are obtained by integrating through the thickness of the plate, to give

$$P = \int_0^d f dz = - \int_0^d E^* \alpha (T - T_0) dz \quad (2.2)$$

and

$$M = \int_0^d f (z - \bar{z}) dz = - \int_0^d E^* \alpha (T - T_0) (z - \bar{z}) dz \quad (2.3)$$

where $\bar{z}$, which defines the position of the neutral surface, is given by

$$\bar{z} = \frac{\int_0^d E^* z dz}{\int_0^d E^* dz} \quad (2.4)$$

(if E^* is constant, $\bar{z} = d/2$).

The sign convention of P and M is shown in Sketch 1.1.

For a clamped plate in which one face is subjected to a recovery temperature varying linearly with time and the other face is insulated, the values of f may be obtained using the temperatures given in Data Item No. 89023.

2.2 Plate with Free Edges

To represent a free edge it is necessary to add to the clamped plate a system of end loads $-P$ and bending moments $-M$ uniformly distributed along the edges. In the literature these quantities are often denoted by $+P_t$ and $+M_t$.

Away from the edges of the plate these will produce stresses given by

$$f_P = \frac{-E^* P}{\int_0^d E^* dz} \quad \left(\text{if } E^* \text{ is constant, } f_P = -\frac{P}{d} \right) \quad (2.5)$$