

TEMPERATURES IN PLATES SUBJECTED TO A RECOVERY TEMPERATURE VARYING LINEARLY WITH TIME

1. NOTATION

SI units only are quoted but any coherent system of units may be used.

a	rate of change of T_r with time	K/s
B	Biot number, hd/k	
d	plate thickness	m
d_i	insulation thickness	m
h	heat transfer coefficient of plate material	W/(m ² K)
k	thermal conductivity of plate material	W/(m K)
k_i	thermal conductivity of insulation material	W/(m K)
T_r	recovery temperature	K
t	time	s
α	coefficient of linear thermal expansion of plate material	K ⁻¹
κ	thermal diffusivity	m ² /s

2. NOTES

The curves on the figures show the variation with time of the temperature change at the heated and insulated surfaces of a thermally thick plate and at three planes at equal intervals of $d/4$ across the plate, which is heated on one surface, the other surface being insulated. It is assumed that all the physical and mechanical properties are constant throughout the plate thickness over the time interval considered.

The heated surface is subjected to a recovery temperature (see Data Item No. 89022^{*}) that varies linearly with time, that is, $T_r = at + \text{constant}$. The curves are plotted for various values of the Biot number.

It is assumed that the initial temperature of the plate is constant and equal to the initial value of T_r .

The curves presented may be used for plates with a thin layer of insulation on the heated face provided that the insulation has a negligible heat capacity and that an effective Biot number is used. This effective Biot number is defined by the equation

$$B_{\text{effective}} = \frac{B}{1 + B_i}, \quad (2.1)$$

where $B_i = \frac{hd_i}{k_i}.$ (2.2)

^{*} Data Item No. 89022 "Heat transfer definitions and equations".

3. DERIVATION

This section gives the selected source that has been used in the compilation of this Data Item.

1. PRZEMIENIECKI, J.S. Design charts for transient temperature and thermal stress distributions in thermally thick plates. *Aeronaut. Q.*, Vol. XI, pp. 269-284, August 1960.

4. EXAMPLE

The outer panel of a windscreen is 0.5 inch thick low expansion glass and is separated from the remainder of the windscreen by an air space that is assumed to be a perfect insulator. The aircraft is initially cruising at a height and speed such that the recovery temperature is 115°C. The aircraft descends and decelerates so that after 8½ minutes the height and speed are such that the recovery temperature is –21°C.

It is required to determine the temperatures at the heated and insulated surfaces of the outer windscreen, assuming that during descent the recovery temperature changes linearly with time. The average heat transfer coefficient, h , is 0.006 67 CHU/ft² s K, and the low expansion glass has the following properties.

$$k = 0.17 \times 10^{-3} \text{ CHU/ft s K}, \quad \kappa = 5 \times 10^{-6} \text{ ft}^2/\text{s} \quad \text{and} \quad \alpha = 5.5 \times 10^{-6} \text{ K}^{-1}.$$

4.1 Rationalisation of Quantities into SI Units

The example is not stated in a coherent system of units. It is therefore necessary to convert all the given data into a coherent system. The “Conversion factors” Data Item in Volume 1 will be used to convert all the given data into SI units.

The temperatures are stated in Celsius. Since the calculation uses only temperature differences, calculations of these in Celsius and Kelvin give identical values. The temperature difference experienced is therefore

$$115 - (-21) = 136^\circ\text{C} = 136 \text{ K}.$$

Other data are converted as follows.

$$\begin{aligned} d &= 0.5 \times 0.0254 &= 0.0127 &\text{m} \\ h &= 0.006\,67 \times 20\,441.8 &= 136 &\text{W/(m}^2\text{ K)} \\ k &= 0.17 \times 10^{-3} \times 6230.68 &= 1.06 &\text{W/(m K)} \\ \kappa &= 5 \times 10^{-6} \times 92.90304 \times 10^{-3} &= 0.465 \times 10^{-6} &\text{m}^2/\text{s} \\ t &= 8.5 \times 60 &= 510 &\text{s} \end{aligned}$$

4.2 Calculation of Required Temperatures

The rate of change of recovery temperature is then

$$a = \frac{-136}{510} = -0.267 \text{ K/s.}$$

Also, $B = 136 \times 0.0127/1.06 = 1.63$

and $\kappa(t/d^2) = 0.465 \times 10^{-6} \times 510/0.0127^2 = 1.47.$

From Figure 1, for the heated surface, interpolating for the value of B calculated above, gives

$$\text{Temperature change} \times \kappa/(ad^2) = 0.98$$

so that at the heated surface:

$$\begin{aligned} \text{Temperature change} &= 0.98 \times (-0.267) \times 0.0127^2 / 0.465 \times 10^{-6} \\ &= 91 \text{ K (} \equiv 91^\circ\text{C)}. \end{aligned}$$

The temperature at the heated surface is therefore

$$115 - 91 = 24^\circ\text{C}.$$

From Figure 5, for the insulated surface, interpolating for the value of B calculated above, gives

$$\begin{aligned} \text{Temperature change} &= 0.60 \times (-0.267) \times 0.0127^2 / 0.465 \times 10^{-6} \\ &= 55.6 \text{ K (} \equiv 55.6^\circ\text{C)}. \end{aligned}$$

The temperature at the insulated surface is therefore

$$115 - 55.6 = 59.4^\circ\text{C}.$$