

THERMAL STRESSES IN UNRESTRAINED PLATES SUBJECTED TO A RECOVERY TEMPERATURE VARYING LINEARLY WITH TIME

1. NOTATION

SI units only are quoted but any coherent system of units may be used.

a	rate of change of T_r with time	K/s
B	Biot number, hd/k	
d	plate thickness	m
d_i	insulation thickness	m
E	modulus of elasticity of plate material	N/m ²
f	thermal stress	N/m ²
h	heat transfer coefficient of plate material	W/(m ² K)
k	thermal conductivity of plate material	W/(m K)
k_i	thermal conductivity of insulation material	W/(m K)
T_r	recovery temperature	K
t	time	s
α	coefficient of linear thermal expansion of plate material	K ⁻¹
κ	thermal diffusivity	m ² /s
ν	Poisson's ratio for plate material	

2. NOTES

The curves on the figures show the variation with time of the thermal stresses at the surface of a thermally thick unrestrained plate that is heated on one surface, the other surface being insulated. It is assumed that all the physical and mechanical properties are constant throughout the plate thickness over the time interval considered.

The heated surface is subjected to a recovery temperature (see Data Item No. 89022^{*}) that varies linearly with time, that is, $T_r = at + \text{constant}$. The curves are plotted for various values of the Biot number.

It is assumed that the initial temperature of the plate is constant and equal to the initial value of T_r .

The curves presented may be used for plates with a thin layer of insulation on the heated face provided that the insulation has a negligible heat capacity and that an effective Biot number is used. This effective Biot

^{*} Data Item No. 89022 "Heat transfer definitions and equations".

number is defined by the equation

$$B_{\text{effective}} = \frac{B}{1 + B_i}, \quad (2.1)$$

where

$$B_i = \frac{hd_i}{k_i}. \quad (2.2)$$

3. DERIVATION

This section gives the selected source that has been used in the compilation of this Data Item.

1. PRZEMIENIECKI, J.S. Design charts for transient temperature and thermal stress distributions in thermally thick plates. *Aeronaut. Q.*, Vol. XI, pp. 269-284, August 1960.

4. EXAMPLES

4.1 Example 1

For the example on the outer windscreen panel described in Data Item No. 89023, it is required to determine the stresses at the heated and insulated surfaces after the descent of 8½ minutes when $E = 10 \times 10^6$ lbf/in² and $\nu = 0.21$.

4.1.1 Rationalisation of quantities into SI units

The example is not stated in a coherent system of units. It is therefore necessary to convert all the given data into a coherent system. The "Conversion factors" Data Item in Volume 1 has been used in the example of Data Item No. 89023 to convert all but one of the quantities into SI units. The remaining quantity is E and this is converted as follows.

$$E = 10 \times 10^6 \times 6894.76 = 68\,960 \text{ MN/m}^2.$$

4.1.2 Calculation of stresses at the heated and insulated surfaces

From Data Item No. 89023,

$$B = 1.63, \quad \frac{\kappa t}{d^2} = 1.47$$

and

$$\frac{\kappa(1 - \nu)}{\alpha d^2} = \frac{0.465 \times 10^{-6} \times (1 - 0.21)}{5.5 \times 10^{-6} \times (-0.267) \times 0.0127^2} = -1550.$$

Therefore, from Figure 1 of this Data Item,

$$\text{stress at heated face} = \frac{68\,950 \times 10^6 \times (-0.066)}{-1550} = 2.94 \text{ MN/m}^2,$$