

STEADY STATE TEMPERATURES AND THERMAL STRESSES IN A SPHERICAL OR CYLINDRICAL LAMINATED SHELL

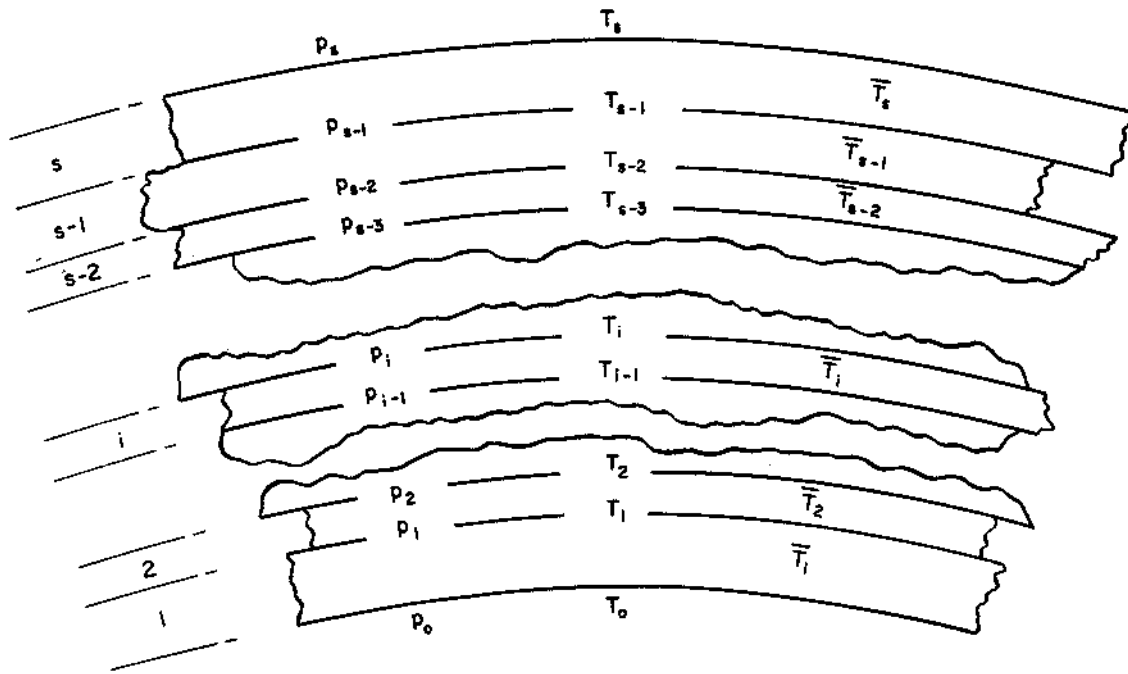
1. NOTATION

SI units only are quoted but any coherent system of units may be used.

d	thickness of layer	m
d_j	j th value of d (see Equation (2.8))	m
E	modulus of elasticity of material of layer	N/m ²
f_z	axial tensile stress in cylinder in layer	N/m ²
f_θ	circumferential tensile stress in shell in layer	N/m ²
$\bar{f}_\theta$	mean value of f_θ in layer	K
k	thermal conductivity of material of layer	W/m K
p	pressure between a layer and next outer layer	N/m ²
p_j	see Equations (2.4) and (2.8)	N/m ²
p_0	internal pressure	N/m ²
R	mean radius of sphere or cylinder	m
s	total number of layers	
T	temperature	K
T_a	temperature at which shell is free of thermal stress	K
T_i	temperature at interface between a layer and next outer layer	K
T_s	temperature at outer surface of shell	K
T_0	temperature at inner surface of shell	K
$\bar{T}$	mean temperature of layer	K
α	coefficient of thermal expansion of material of layer	K ⁻¹
ν	Poisson's ratio of material of layer	

1.1 Suffixes

i relates to i th layer of laminate from inside of shell (see Sketch 1.1)



Sketch 1.1

2. NOTES

A method is given for calculating the temperatures and stresses in the layers of a laminated shell in the form of a sphere or circular cylinder when the outer surface is at a constant temperature, T_s , and the inner surface is at a constant temperature, T_0 .

The shell is assumed to be free of thermal stress at temperature T_a and the total thickness is assumed to be less than 0.1 of the radius.

The shell is made up of a number of layers of materials that may be different and it is assumed that each layer is maintained in mechanical contact with its adjacent layer. The materials that make up the shell are assumed to be perfectly elastic and the material constants independent of temperature. The thermal resistance of a bond between adjacent layers may be allowed for by treating the bond as an additional layer.

In steady heat flow the temperature varies linearly across each layer and the change in temperature across the i th layer is given by

$$(T_i - T_{i-1}) = \frac{(d_i/k_i)(T_s - T_0)}{\sum_{i=1}^s d_i/k_i} \quad (2.1)$$

This expression enables the temperature distribution through the shell to be obtained.

2.1 Spheres

The circumferential stress at any point in the i th layer of a spherical shell is given by

$$f_{\theta} = \frac{E_i}{1 - \nu_i} \left(-\alpha_i (T - T_a) + \frac{A + p_0/2}{B} \right), \quad (2.2)$$

where

$$\left. \begin{aligned} A &= \frac{1}{R} \sum_{i=1}^s \frac{E_i d_i \alpha_i (\bar{T}_i - T_a)}{1 - \nu_i} \\ \text{and} \\ B &= \frac{1}{R} \sum_{i=1}^s \frac{E_i d_i}{1 - \nu_i} \end{aligned} \right\} \quad (2.3)$$

and in which T is the temperature at the point and $\bar{T}_i$ is the mean temperature of the i th layer obtained by means of Equation (2.1).

There are no shear stresses in the layers of material or in the bonds between layers.

The pressure exerted on the bonds between layers is obtained from the expression

$$\left. \begin{aligned} p_i &= p_0 + \sum_{j=1}^i \Delta p_j, \\ \text{where} \quad \Delta p_j &= -2\bar{f}_{\theta} d_j / R \end{aligned} \right\} \quad (2.4)$$

and $\bar{f}_{\theta}$ is the mean circumferential stress in the j th layer.

2.2 Cylinders

The circumferential and axial stresses at any point in the i th layer of a cylinder at sections remote from the ends are given by

$$f_{\theta} = \frac{E_i}{1 - \nu_i} \left[-\alpha_i \left[(T - T_a) + \frac{A}{B} + \frac{p_0}{(1 + \nu_i)(C^2 - D^2)} \right] \left[\left(1 + \frac{\nu_i}{2} \right) C - \left(\frac{1}{2} + \nu_i \right) D \right] \right] \quad (2.5)$$

$$f_z = \frac{E_i}{1 - \nu_i} \left[-\alpha_i \left[(T - T_a) + \frac{A}{B} + \frac{p_0}{(1 + \nu_i)(C^2 - D^2)} \right] \left[\left(\frac{1}{2} + \nu_i \right) C - \left(1 + \frac{\nu_i}{2} \right) D \right] \right], \quad (2.6)$$