

A PROCEDURE FOR FORCE-BALANCING PLANAR LINKAGES USING COUNTERWEIGHTS

1. NOTATION

		Units
a_i	principal length between joints of link	m
b_i	secondary length between joints of link with three or more joints	m
c	minimum number of counterweights required to achieve force-balance	—
h	number of revolute joints, on counterweighted link, to which fictitious masses are assigned	—
i	complex operator	—
j	total number of joints between pairs of links in linkage	—
ℓ	minimum number of independent loops	—
l_j	length between reference revolute joint and revolute joint to which fictitious mass is assigned	m
l_{qj}	length between reference revolute joint and revolute joint to which fictitious mass has been assigned (on link connected to counterweighted link by sliding joint)	m
m_i	mass of link	kg
m_q	mass of link connected to counterweighted link by sliding joint	kg
m_{fj}	fictitious mass	kg
m_{qfj}	fictitious mass assigned to revolute joint of link connected to counterweighted link by sliding joint	kg
n	total number of links	—
r_i	length between reference revolute joint and link centre of mass	m
r'_i	length between second revolute joint and link centre of mass	m
r_q	length between reference revolute joint and centre of mass of link connected to counterweighted link by sliding joint	m
R_I	inner radius of sectorial counterweight	m
R_O	outer radius of sectorial counterweight	m
s	number of sliding joints on counterweighted link	—
t_i	thickness of counterweight	m

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v	number of revolute joints, on link connected to counterweighted link by slider, to which fictitious masses are assigned	–
α_{ij}	angle between reference direction and line joining reference revolute joint to revolute joint on counterweighted link	degree
α_{qj}	angle between reference direction and line joining reference revolute joint to revolute joint on uncounterweighted link connected to counterweighted link by sliding joint	degree
β_i	angle between reference direction and counterweight offset length	degree
γ_i	angle between reference direction and line joining the reference revolute joint to centre of mass of link i	degree
γ'_i	angle between reference direction and line joining second revolute joint to centre of mass of link i	degree
γ_q	angle between reference direction and line joining reference revolute joint to centre of mass of link connected to counterweighted link by sliding joint	degree
θ_i	half angle of sectorial counterweight	radian
λ_i	length between reference revolute joint and counterweight centre of mass	m
μ_i	mass of counterweight	kg
ρ_i	density of counterweight	kg/m ³
ϕ	general angle	degree
ψ_{fj}	fictitious offset angle associated with fictitious mass on counterweighted link	degree
ψ_{qfj}	fictitious offset angle associated with fictitious mass on uncounterweighted link connected to counterweighted link by sliding joint	degree

Subscripts

f	refers to fictitious property
i	refers to number of link
j	refers to letter of joint
q	refers to properties of link connected to counterweighted link by sliding joint

Superscripts

$'$	refers to secondary length or angle
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2. INTRODUCTORY NOTES

This Data Item presents a procedure for the elimination of the frame shaking force in single degree-of-freedom, planar, multibar linkage mechanisms. The method is applicable to linkages which contain revolute and/or sliding joints and in which the link lengths are known. The sizes, shapes and locations of a set of counterweights are determined such that the total linkage centre of mass remains stationary throughout the motion and no net force is transmitted to the frame. The procedure is suitable for use in the design of new mechanisms or in the redesign of existing machinery in which an improved dynamic performance is to be investigated. Balancing is necessary when the dynamic forces due to the motion of link masses have an adverse effect on the operation of the machine.

It should be noted that the implementation of frame shaking force-balance by this method will not necessarily provide the best solution to a dynamic problem in a linkage. Despite this, however, the method has the major advantages of generality, simplicity and flexibility, and is especially suitable in the design of complex multibar linkages. This is in contrast to other methods which require the derivation and manipulation of a large number of kinematic equations.

2.1 Assumptions and Prerequisite Information

The following assumptions are implicit in the method. If an actual mechanism has properties which deviate widely from those assumed then a more complete analysis must be undertaken.

The assumptions are that:

- (a) the linkage is planar – this means that all of the link centres of mass are in the same plane, and if there is more than one set of identical links then there must be a central plane of symmetry for the global mass centres of each set of identical links,
 - (b) the links are rigid,
 - (c) the joints are revolute and/or sliding
- and (d) the joints have negligible clearance.

This Item is intended for use at the detail design phase after kinematic synthesis and analysis are complete.

Some information is required to determine the counterweight design. This must include:

- (i) the lengths of each link between the joints,
 - (ii) the proposed (or actual) size and shape of each link,
 - (iii) the material density of each link,
 - (iv) the spatial constraints imposed by linkage geometry, other machine members and casings,
- and (v) other constraints which may necessitate one or more of the links to remain uncounterweighted.

Links which, for reasons identified by (iv) and (v), cannot be counterweighted are termed "proscribed links".

2.2 Limitations and Important Considerations

The use of this method will not guarantee an acceptable solution to all balancing problems in linkages.

The following points should be considered prior to the use of the procedure in this Item.

- (a) Full shaking force-balance almost inevitably leads to increases in the magnitudes of shaking moment, pin forces and input torque fluctuations because of the use of counterweights which add mass to the links. In many linkages the increase in these force and moment quantities will be of the order of 50%, (see Reference 11) but, in some cases, they may rise by a factor of 10 or more. The procedure in this Data Item, by itself, does not provide any indication of the expected rise in these quantities. For a fully verified linkage design, these quantities must be calculated throughout the cycle using methods outlined in Reference 7.
- (b) In this method it is assumed that the link dimensions, the link geometry, the link materials, the bearing types and the bearing sizes have already been chosen. Link geometry and materials determine the link masses, moments of inertia and the allowable stresses in each link. Bearing types and sizes influence the link geometry and determine the allowable pin force at each revolute joint. The actual stresses and pin forces are also dependent on the initially chosen values and the addition of any balancing counterweights alters the masses and moments of inertia of some links and hence changes the stresses in the links and the forces at the joints. This makes an optimal design very difficult and highlights the importance of making the initial estimates of the linkage elements as realistic as possible.
- (c) If due attention is not paid to reducing other potential sources of mechanism vibration these may become more important than the frame shaking force. These other effects include:
 - (1) backlash due to clearance,
 - (2) torsional shaft vibrations,
 - (3) flexural vibration of links and counterweight armsand
 - (4) inadequate stiffness of the machine frame.
- (d) The addition of counterweights to provide force-balance may not be the most appropriate method in certain cases. The use of other balancing methods is outlined in References 8 and 9.

3. DESIGN PROCEDURE

The design procedure is presented as a step-by-step process as shown in the flowchart of Figure 3.1. A practical example is included in this section to illustrate each step of the procedure. Some linkages, due either to their configuration or to restrictions placed upon counterweight location, cannot have their frame shaking forces reduced to zero. These linkages are identified at Step 7 in the design procedure and the user is referred to Section 4 which provides guidance on partial balancing.

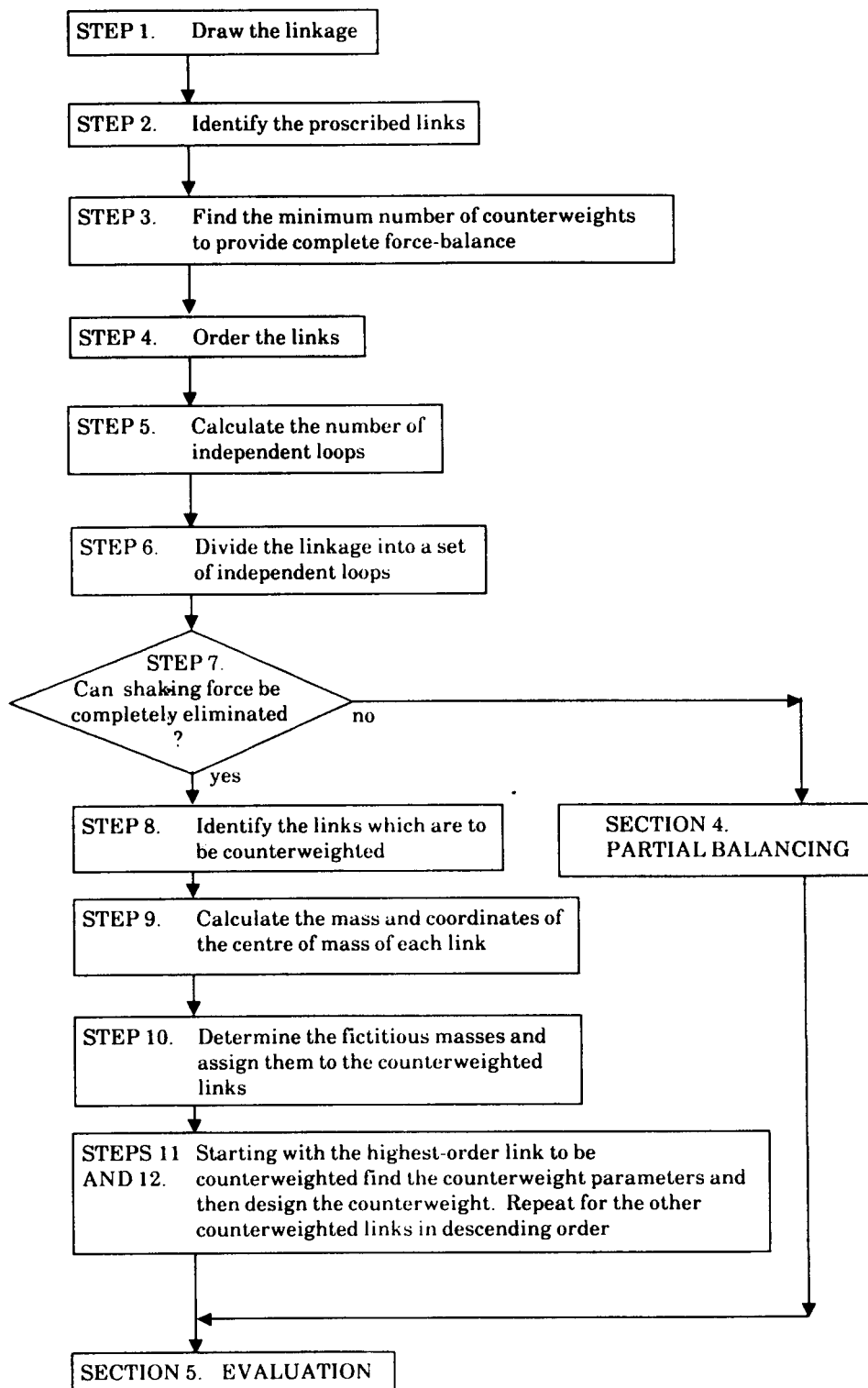
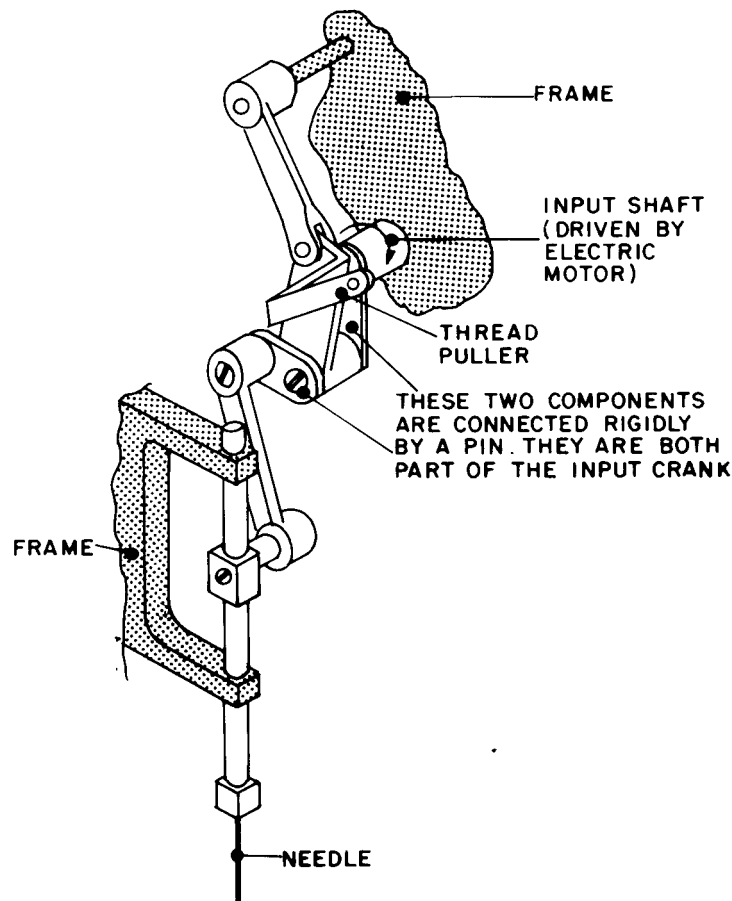


FIGURE 3.1 Flowchart of design procedure

Illustrative Example

Sketch 3.1 is a pictorial representation of a six-bar sewing machine mechanism which consists of a single input crank driving a thread puller and a needle. The machine has been in production for several years and is currently being redesigned with the aim of reducing the vibration transmitted to the frame whilst maintaining the same input motion, output motion and frame dimensions.



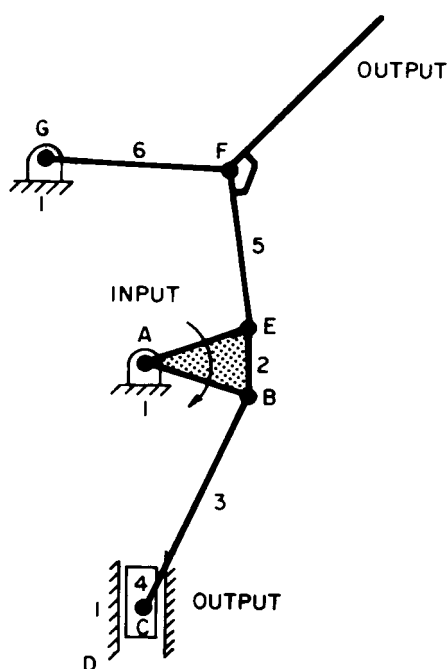
Sketch 3.1 Sewing machine mechanism

The links are manufactured from high strength aluminium alloy. Estimates of frame shaking force have been obtained using kinetostatic force analysis (see Section 5). It is now necessary to reduce the frame shaking force by the introduction of counterweights.

3.1 Draw the Linkage (STEP 1)

Draw the linkage (roughly to scale) in conventional skeleton diagram form. Number each link and letter each joint for the purposes of identification and indicate the input and output links. If the linkage incorporates a gear pair this can be represented, for the purposes of balancing, by an imaginary binary link (see the example in Section 7).

Example Step 1



Sketch 3.2 Skeleton diagram of sewing machine

3.2 Identify the Proscribed Links (STEP 2)

Identify those links, if any, to which counterweights cannot be added. These are termed “proscribed links”. Links may be unsuitable for counterweighting for many reasons and usually the reasons will be immediately apparent to the designer. Examples include:

- (a) the presence of spatial restrictions around certain joints due to the positions of other links or machine members,
 - (b) the expense involved in modifying certain links
- and
- (c) the unsuitable geometry of certain links.

Example Step 2

Link 4 (the needle) is proscribed because it is too expensive to modify. It is a standard part used on many machines and thousands are held in stock.

Counterweights of sensible proportions can be incorporated in any of the other links.

3.3 Find the Minimum Number of Counterweights Required to Provide Complete Force-Balance (STEP 3)

The minimum number of counterweights required to provide complete force balance, c , is found from the relationship (see Derivation 3)

$$c = 2(n - 1) - j, \quad (3.1)$$

where n = the total number of links (including reference frame link)

and j = the total number of joints[†] (both revolute and sliding).

Example Step 3

In the example $n = 6$ and $j = 7$,

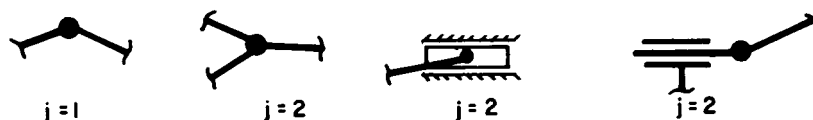
so $c = 2(6 - 1) - 7 = 3$.

3.4 Order the Links (STEP 4)

For the purposes of this procedure the moving links are ordered in the following way. Those pivoted to the frame by revolute joints are termed first-order. Those (unordered) links connected to first-order links by revolute joints are termed second-order and so on. This process is continued until either:

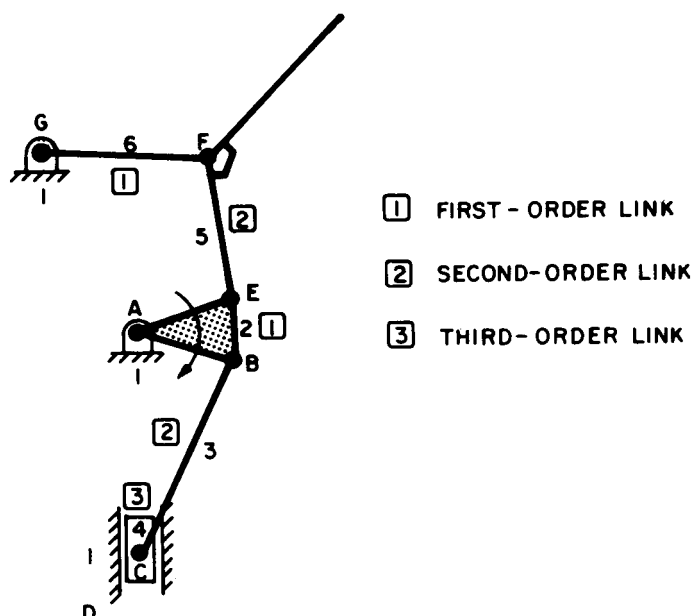
- (a) a link is connected by a sliding joint to another link,
- or
- (b) two ordered links meet at a revolute joint.

[†] If more than two links intersect at the same pivot the number of joints must be carefully ascertained. For example



Example Step 4

Links 2 and 6 are clearly first-order, Links 5 and 3 are second-order and Link 4 is third-order. The links are labelled accordingly on Sketch 3.3.



Sketch 3.3 Link ordering

3.5 Calculate the Number of Independent Loops (STEP 5)

The minimum number of independent loops (see Appendix A), ℓ , is found using the relationship (see Derivation 3)

$$\ell = j - n + 1. \quad (3.2)$$

Example Step 5

In the example $j = 7$ and $n = 6$,

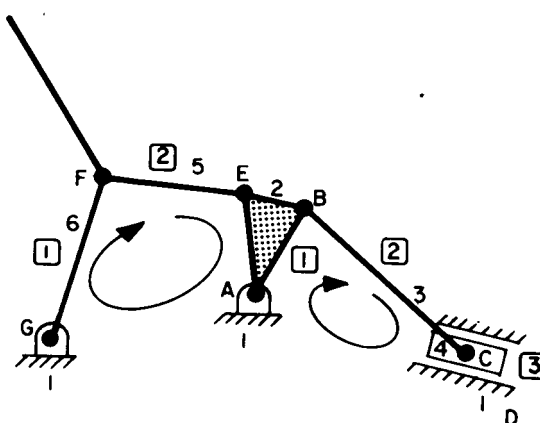
so $\ell = 7 - 6 + 1 = 2$.

3.6 Divide the Linkage into a Set of Independent Loops (STEP 6)

This step enables the user to identify what is arguably the best[†] set, or one of the best sets, of independent loops from the total number of possibilities. This procedure is necessary in order to determine the minimum number of counterweights.

- (a) Identify the links (or link) of highest-order.
- (b) Find the total number of paths by which each link identified in (a) is connected to the frame. For each path count the number of joints crossed. Paths can cross both revolute and sliding joints.
- (c) A path-pair is defined as two paths, one from each of two separate joints on a link. For each combination of path-pairs add the number of joints which have been crossed. Identify the path-pair with the lowest joint total. At this stage there may be more than one path-pair having the same number of joint totals in which case choose the one which crosses the lightest links.
- (d) The path from frame to frame including that link of highest-order identified in (c) is the first independent loop. Identify it by the letters of the joints which it includes.
- (e) Repeat steps (b) to (d) for the link(s) of highest-order which are not contained within the first independent loop.
- (f) Continue this process for links of descending order until each link is contained in at least one loop.
- (g) Ensure that the number of loops is equal to the minimum number identified in Step 5.

Example Step 6



Sketch 3.4 Independent loops

[†] The term 'best' is used here to denote that solution which, in most cases, will minimise the magnitude of counterweight mass required to balance the linkage.

Example Step 6 (continued)

- (a) Link 4 is the only third-order link (all the other links are of lower-order).
- (b) Three paths can be traced from link 4 to the frame:
 - (i) across Joint D (1 joint),
 - (ii) across Joints CBA (3 joints),
 - (iii) across Joints CBEFG (5 joints).
- (c) Path-pair (i) and (ii) cross the least number of joints (4 joints).
- (d) The first independent loop is therefore denoted ABCD.
- (e) Links 3 and 5 are second-order links but, since Link 3 is contained in Loop ABCD, Link 5 is considered. It is connected to the frame by 3 paths:
 - (i) across Joints FG (2 joints),
 - (ii) across Joints EA (2 joints),
 - (iii) across Joints EBCD (4 joints),
 Paths (i) and (ii) start at separate joints and cross the least number of joints so the second independent loop is denoted by AEFG.
- (f) Each link is contained in Loops ABCD and AEFG as shown in Sketch 3.4.
- (g) There are two loops as predicted by Equation (3.2) in Step 5.

3.7 Test to Determine whether Shaking Force can be Completely Eliminated (STEP 7)

If the number of links available for counterweight mounting is less than the required minimum number of counterweights, c , or if any loop contains:

- (a) two or more proscribed binary links which share a common joint,
 - (b) more than one sliding joint,
- or
- (c) a proscribed binary link and a proscribed link with a sliding joint,

then the linkage probably cannot be fully force-balanced. For many complex linkages it may be possible to select an alternative set of independent loops in which the prohibitive condition disappears. If this is not possible then carefully reconsider the reasons for proscribing links before referring to Section 4 for information regarding partially balancing the frame shaking forces.

Example Step 7

There are two independent loops.

Loop ABCD contains one proscribed link with a sliding joint. Loop AEFG contains no proscribed links or sliding joints. Therefore the linkage can be fully force-balanced.

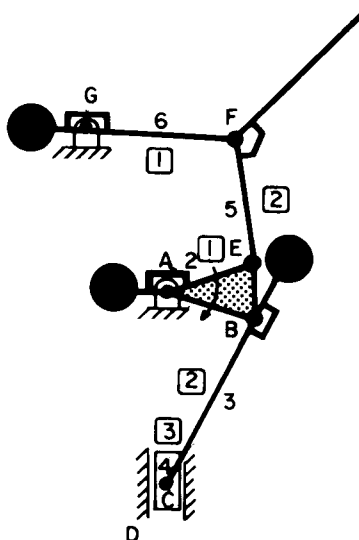
3.8 Identify the Links which are to be Counterweighted (STEP 8)

One link in each independent loop can remain uncounterweighted. Ideally the link to be left

uncounterweighted is the link of highest-order in each loop. Those links which are connected to the frame by a sliding joint should not have counterweights attached to them. If counterweights are added to high-order links the mass of those links and their counterweights must be transferred to lower order links. This results in a cascade effect so counterweights should, preferably, be placed on first-order links to ensure that the mass added to the linkage is minimised. In some linkages alternative sets of counterweights may appear equally valid. In practice it may be desirable to proceed with each alternative and to compare them in terms of the increases they cause in shaking moment, pin forces and input torque fluctuations (see Section 5). Mark the chosen positions of the counterweights on the skeleton diagram. Refer to the machine drawings and check that space is available for each counterweight.

Example Step 8

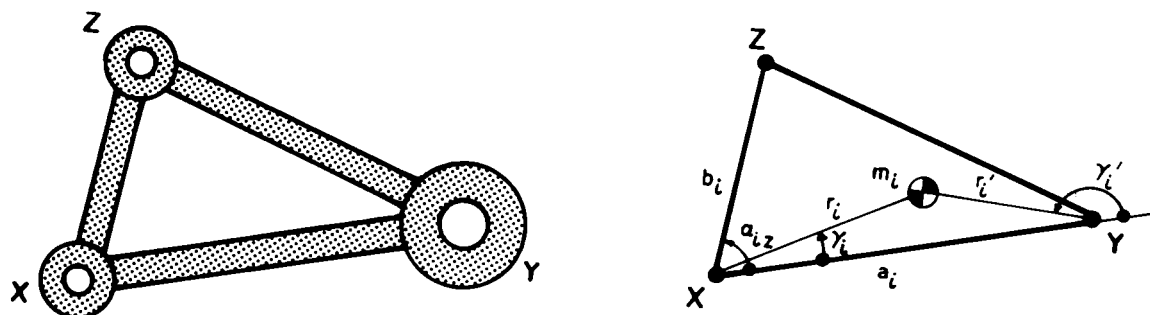
The link of highest-order in Loop ABCD is Link 4 and since this is a proscribed link it is not counterweighted. The link of highest-order in Loop AEFG is Link 5 so this is not counterweighted. The remaining links are counterweighted as shown in Sketch 3.5.



Sketch 3.5 Counterweight locations

3.9 Calculate the Mass and the Coordinates of the Centre of Mass of each Link (STEP 9)

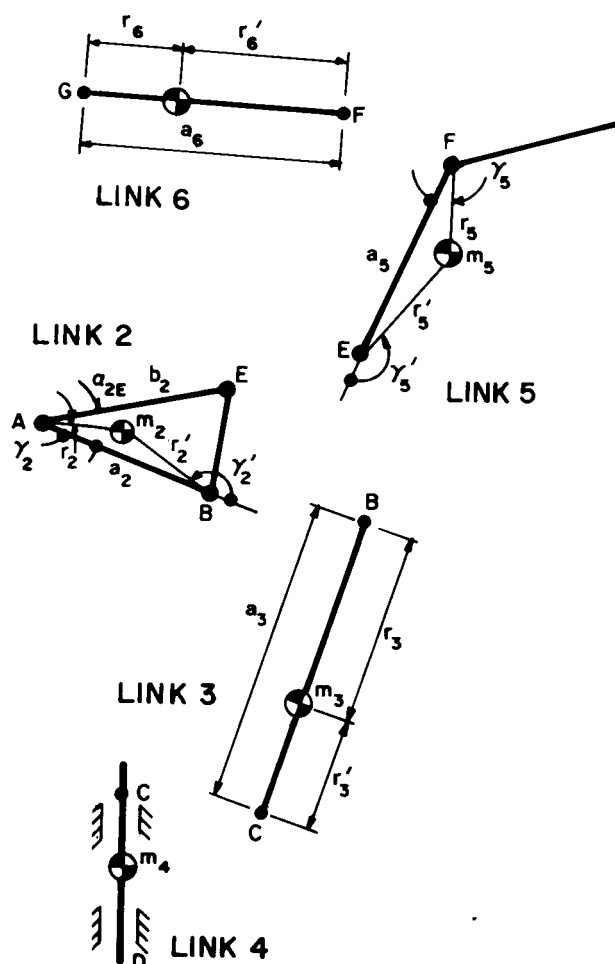
As a prerequisite to the balancing process the mass and the coordinates of the centre of mass for each moving link must be calculated. Bearing parts (pins and sleeves) are included with the link with which they move. The required properties are illustrated, for a link with three revolute joints (ternary link), in Sketch 3.6. Draw each link, define its lengths and angles (anticlockwise-positive) in the terminology of Sketch 3.6, and compile the properties into a table as shown in the example.



Sketch 3.6 Link properties (for a general ternary link)

Example Step 9

Each link is drawn separately in Sketch 3.7.



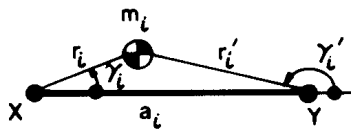
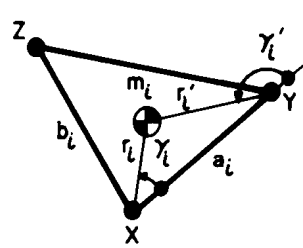
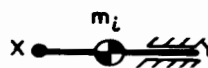
Sketch 3.7 Link properties

The following table of properties has been compiled from the known dimensions of each link

Link number	m_i [10^{-3} kg]	a_i [10^{-3} m]	b_i [10^{-3} m]	α_{ij} [degree]	r_i [10^{-3} m]	r'_i [10^{-3} m]	γ_i [degree]	γ'_i [degree]
2	42.10	16.5	16.5	35.3	1.12	15.49	24.9	178.3
3	5.66	35.0	—	—	20.27	14.73	0	180.0
4	12.56	—	—	—	—	—	—	—
5	3.45	25.0	—	—	9.59	16.46	21.5	167.6
6	2.90	26.0	—	—	8.72	17.28	0	180.0

3.10 Determine the Fictitious Masses (STEP 10)

Each uncounterweighted link, independent of the number of joints it contains, is represented by fictitious masses at two revolute joints (see Appendix B). The only exception is the slider with only one revolute joint. In this case the mass is assumed to be concentrated at the revolute joint. Each fictitious mass has a fictitious offset angle associated with it. The method for the calculation of these fictitious properties is shown in the table below for several link types. The procedure is first to determine the masses to represent the uncounterweighted link of highest-order, followed by those of any other uncounterweighted links in descending order; these masses are then assigned to revolute joints on the links to be counterweighted.

Uncounterweighted link type	Fictitious mass, m_{fi} [kg]		Offset angle ψ_{fi} [degree]
binary 	at X	$m_i r_i' / a_i$	$\gamma_i' + 180^\circ$
	at Y	$m_i r_i / a_i$	γ_i
ternary 	at X	$m_i r_i' / a_i$	$\gamma_i' + 130^\circ$
	at Y	$m_i r_i / a_i$	γ_i
	at Z	0	0
slider 	at X	m_i	0
	at Y	0	0

Example Step 10

Link 4 is the uncounterweighted link of highest-order. Since this link is a slider it can be represented by a fictitious mass of

$$m_{fC} = m_4 = 12.56 \times 10^{-3} \text{ kg}$$

at a fictitious offset angle

$$\Psi_{fC} = 0^\circ$$

assigned to Joint C on Link 3.

Link 5 is the other uncounterweighted link. It is a binary link and so it can be represented by a fictitious mass

$$m_{fF} = \frac{m_5 r_5'}{a_5} = \frac{3.45 \times 10^{-3} \times 16.46 \times 10^{-3}}{25 \times 10^{-3}} = 2.27 \times 10^{-3} \text{ kg}$$

at a fictitious offset angle

$$\Psi_{fF} = \gamma_5' + 180^\circ = 167.6^\circ + 180^\circ = 347.6^\circ$$

assigned to Joint F on Link 6 and by a fictitious mass

$$m_{fE} = \frac{m_5 r_5}{a_5} = \frac{3.45 \times 10^{-3} \times 9.59 \times 10^{-3}}{25 \times 10^{-3}} = 1.32 \times 10^{-3} \text{ kg}$$

at a fictitious offset angle

$$\Psi_{fE} = \gamma_5 = 21.5^\circ$$

assigned to Joint E on Link 2.

3.11 Find the Counterweight Parameters (STEP 11)

Determine the numerical values of the required counterweight parameters such that the shaking force transmitted to the frame of the linkage is eliminated. These parameters are:

- (a) the product of the counterweight mass, μ_i , and its offset length, λ_i ,
- and (b) the counterweight offset angle, β_i .

Beginning with the highest-order link to be counterweighted these parameters are found either:

- (a) analytically, from the balance Equations (3.3) or (3.5)
- or (b) graphically, by drawing a vector polygon as shown in Sketch 3.9 or 3.11.

For a counterweighted link containing only revolute joints the joint about which the link is balanced is termed the reference revolute joint (Joint X in Sketch 3.8) and the direction of a line drawn from this joint to another revolute joint on the same link is termed the reference direction (line XY in Sketch 3.8).

The sum of the mass-moments about the reference revolute joint, for a balanced link, is equal to zero. For each counterweighted link containing only revolute joints, substitute the known values in Equation (3.3).

$$\mu_1 \lambda_i \exp[i\beta_i] + m_i r_i \exp[i\gamma_i] + \sum_h m_{fj} l_j \exp[i(\alpha_{ij} + \Psi_{fj})] = 0. \quad (3.3)$$