



ESDU 89034

Issued October 1989

With Amendment A

August 1993

The maximum lift coefficient of plain wings at subsonic speeds

Associated software: ESDUpac A9315

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THE MAXIMUM LIFT COEFFICIENT OF PLAIN WINGS AT SUBSONIC SPEEDS

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THE MAXIMUM LIFT COEFFICIENT OF PLAIN WINGS AT SUBSONIC SPEEDS

1. NOTATION AND UNITS

		<i>SI</i>	<i>British</i>
A	aspect ratio ($= 2s/\bar{c}$)		
C_L	lift coefficient, lift/ qS		
C_{L0}	section lift coefficient at zero incidence		
C_{LL}	local lift coefficient		
C_{LLp}	peak local lift coefficient, see Sketch 6.1		
C_{Lmax}	wing maximum lift coefficient		
C_{Lm}	aerofoil section maximum lift coefficient at $M \approx 0$		
ΔC_{LM}	increment in wing maximum lift coefficient due to Mach number		
ΔC_{LR}	increment in wing maximum lift coefficient due to Reynolds number for $\Lambda_0 > 37^\circ$		
ΔC_{LT}	increment in wing maximum lift coefficient due to twist		
$\Delta C_{L\Lambda}$	increment in wing maximum lift coefficient dependent on leading-edge sweep and ζ_p		
c	local chord	m	ft
$\bar{c}$	geometric mean chord	m	ft
$\bar{\bar{c}}$	aerodynamic mean chord	m	ft
c_r	wing root chord	m	ft
c_t	wing tip chord	m	ft
M	free-stream Mach number		
q	free-stream kinetic pressure	N/m ²	lbf/ft ²
R_c	Reynolds number based on local chord and free-stream velocity		
$R_{\bar{\bar{c}}}$	Reynolds number based on aerodynamic mean chord and free-stream velocity		

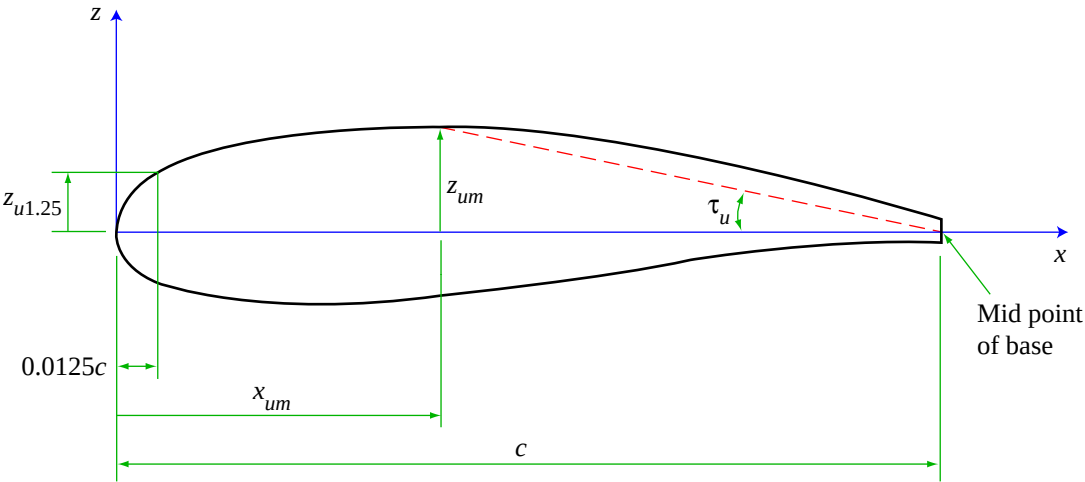
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S	wing area, $s(c_r + c_t)$	m^2	ft^2
s	wing semispan	m	ft
x	general chordwise station measured streamwise from wing leading edge, see Sketch 1.2	m	ft
x_{um}	chordwise station for z_{um} , see Sketch 1.1	m	ft
z	general section ordinate	m	ft
z_{um}	maximum upper-surface ordinate, see Sketch 1.1	m	ft
$z_{u1.25}$	upper-surface ordinate at 1.25% chord, see Sketch 1.1	m	ft
α_0	zero-lift angle of incidence for cambered two-dimensional section	degree	degree
β	compressibility parameter, $(1 - M^2)^{1/2}$		
δ	wing twist angle relative to wing root (positive leading edge up)	degree	degree
δ_e	effective twist angle	degree	degree
δ_{et}	effective tip twist angle	degree	degree
ζ_p	leading-edge shape parameter, $(z_{u1.25}/c)_p \sec \Lambda_0$		
η	spanwise distance from wing root as fraction of semispan		
η_p	value of η for μ_p		
$\bar{\eta}$	spanwise centre of pressure position, $\int_0^1 \sigma \eta \, d\eta$		
κ	wing taper parameter, $\int_0^1 (c/\bar{c}) \eta \, d\eta$		
Λ_0	wing leading-edge sweep angle	degree	degree
$\Lambda_{1/4}$	wing quarter-chord sweep angle	degree	degree
$\Lambda_{1/2}$	wing mid-chord sweep angle	degree	degree
Λ_1	wing trailing-edge sweep angle	degree	degree
λ	wing taper ratio, c_t/c_r		
μ	normalised local lift coefficient, C_{LL}/C_L		
μ_p	peak value of μ		

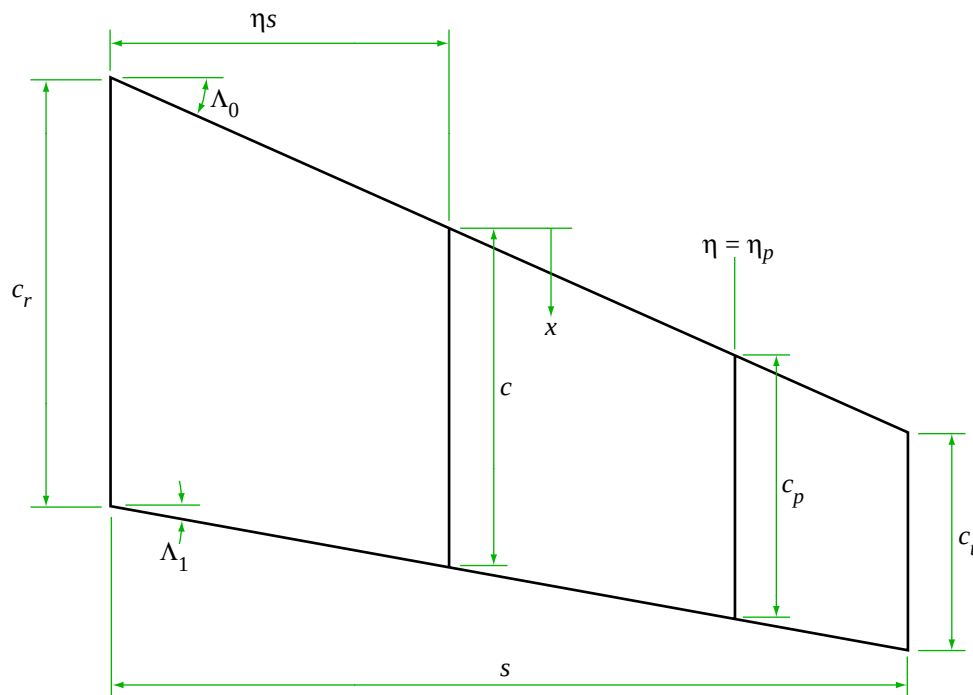
σ	spanwise loading due to incidence, $C_{LL}c/C_L\bar{c}$		
τ_u	aerofoil upper-surface angle defined by Equation (6.2), see also Sketch 1.1	degree	degree

Subscripts

$()_{expt}$	denotes experimental value
p	denotes value for section at η_p
$()_{pred}$	denotes predicted value
r	denotes value for section at wing root
t	denotes value for section at wing tip
η	denotes values for section at η



Sketch 1.1 Aerofoil geometry



Sketch 1.2 Wing notation

2. INTRODUCTION

Item No. 84026 (Derivation 5) provides a method for the prediction of maximum lift of aerofoil sections. This Item extends the scope of Item No. 84026 to apply to “plain” wings with or without camber or twist. Here, “plain” means without deflection of manoeuvre or high-lift devices such as leading-edge and trailing-edge flaps.

For subsonic speeds, the maximum lift of high aspect ratio wings is, to a first approximation, determined by the maximum lift of the aerofoil section. The main parameters which influence the maximum lift coefficient for an aerofoil section are section geometry, including camber, surface condition (*i.e.* smooth or rough), Reynolds number and Mach number. For wings, additional parameters influence the maximum lift coefficient, in particular aspect ratio, taper ratio, sweep, twist and spanwise variation of aerofoil section.

Because of three-dimensional effects, separation usually starts near the most highly loaded spanwise station and spreads rapidly with increasing incidence. For highly tapered wings separation tends to be initiated at the tip, whilst for untapered wings separation tends to be initiated at the root. For swept wings, the induced effects combine to promote tip stall. This results from the high induced incidence and negative induced camber at the tip and the spanwise pressure gradient which tends to draw the boundary layer from the root to the tip. Regardless of the spanwise location where separation first appears, the maximum lift is greatly influenced by whether separation occurs at the leading edge or trailing edge.

Trailing-edge separation, a characteristic of thick wings, normally results in a loss of maximum lift as compared with that of an aerofoil section. Leading-edge separation, where for swept wings the flow rolls up to form a spanwise vortex, results in an increase in maximum lift, the magnitude of which is related to the strength of the vortex. The strength of this leading-edge vortex is dependent on aspect ratio, sweep, incidence and the section shape near the leading edge.

As aspect ratio decreases the wing planform parameters become increasingly important, with leading-edge vortex effects becoming more dominant.

3. REQUIRED DATA ITEMS

Other ESDU Data Items that may be required in the use of this Item are:

Derivation No.	Data Item No.	For determination of
1.	Aero W.01.01.05	Slope of lift curve for two-dimensional flow, for use in Derivation 5.
2.	66034	Upper-surface ordinate at 1.25% chord.
3.	76003	Various geometrical relationships for wings, and equivalent straight-tapered planform.
4.	83040	Spanwise centre of pressure position, $\bar{\eta}$.
5.	84026	Aerofoil maximum lift coefficient for $M \approx 0$.
6.	88030	Boundaries of linear characteristics of cambered and twisted wings at subcritical Mach numbers.

4. MAXIMUM LIFT AND MAXIMUM USABLE LIFT

In making use of the data derived from this Item it is essential to distinguish between wing maximum lift and wing maximum usable lift. The lift of the wing alone, as derived here, may be significantly modified by the presence of interference effects due to fuselages, nacelles, pylons, flap track fairings or other external additions to the configuration. Moreover, maximum usable lift may be limited by other effects associated with flow separation, such as pitch-up, buffet, or wing rock. For example, prior to the attainment of maximum lift it is often the case that the pitching moment curve is characterised by a pitch break, resulting in what can be a very marked and unacceptable change in longitudinal stability. Derivations 9 and 34 indicate configuration parameters which will result in acceptable pitch characteristics. It may be noted that some improvement can be obtained by the use of leading-edge devices. Also, acceptable behaviour may result for configurations outside those guidelines, provided that the aircraft tailplane is properly positioned or adequate control effectiveness is provided.

In this Item C_{Lmax} is taken as the greatest value achieved irrespective of the pitching moment characteristics prior to the maximum value of lift. It may be noted that for some wing-body configurations at all Mach numbers, and for wings or wing-body combinations at high subsonic Mach numbers where shock separation affects the development of the stall, a first peak or plateau in C_L is often followed by a subsequent recovery to greater values. None of the data analysed for wings alone exhibited this characteristic within the range of Mach number and planform and section parameters covered by this Item.

5. EFFECTS OF MACH NUMBER AND REYNOLDS NUMBER

5.1 Mach Number Effects

In the preparation of this Item it was found that the effect of Mach number on the maximum lift coefficient for wings can be very different from that shown for two-dimensional sections in Item No. 84026. For this Item the effect has been correlated as an increment, $\Delta C_{LM}/\cos^4 \Lambda_0$, dependent on $M \cos \Lambda_0$ and on ζ_p , the value of $(z_{u1.25}/c) \sec \Lambda_0$ for the most highly loaded section, η_p .

5.2 Reynolds Number Effects

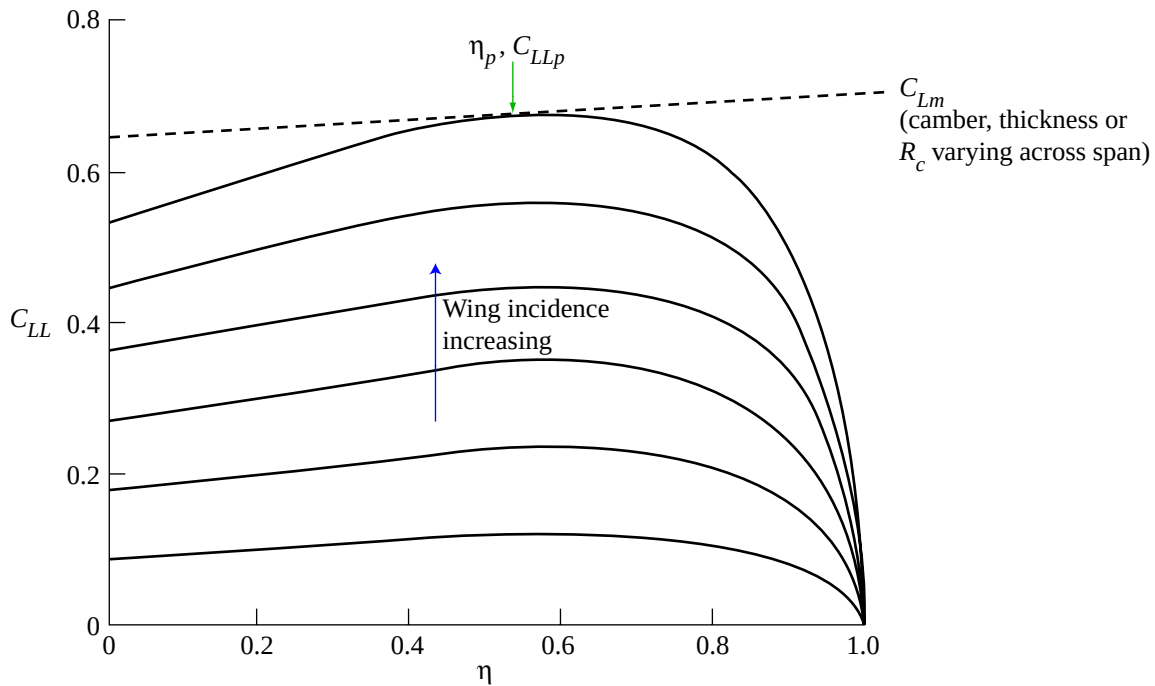
For wings with leading-edge sweep angles up to about 37 degrees the effect of Reynolds number is adequately estimated by the use of the two-dimensional section data of Item No. 84026. For wings with greater leading-edge sweep the effect of Reynolds number on maximum lift coefficient is greatly reduced. From correlations of the data used in the preparation of this Item it was found that there was a systematic trend with sweep, independent of leading-edge sectional geometry. The increment in maximum lift coefficient, ΔC_{LR} , has been correlated as a function of Λ_0 and $R_{cp} \cos^2 \Lambda_0$, which can be regarded as the local Reynolds number of the flow normal to the leading edge at η_p .

6. PREDICTION METHOD

6.1 Derivation of Method

The method is developed from that of Derivations 7, 8, 36, 37 and 39. For wings with little or no sweep, where the effects of outboard flow of the boundary layer may be neglected, it is necessary to establish the spanwise variation of C_{LL} and to compare this with the spanwise variation of C_{Lm} , the maximum lift coefficient for the aerofoil section. To obtain the spanwise lift distribution at this condition, calculations including the effects of camber and twist may be made using (for example) Item No. 83040 for a number of incidences. For the incidence at which the peak local lift coefficient, C_{LLp} , matches the local section maximum lift coefficient, C_{Lm} , the distribution of $C_{LL}c/\bar{c}$ must be obtained and integrated to obtain the value of the maximum lift coefficient for the wing, C_{Lmax} , see Sketch 6.1.

To derive the method of this Item, the same simplifying assumptions as those used in Item No. 88030 have been made to reduce the relatively complex procedure outlined above to the calculation for only the most highly loaded spanwise station. The spanwise position, η_p , of the most highly loaded section for the loading due to incidence and the corresponding maximum normalised lift coefficient, μ_p , are shown in Figures 1 and 2, which are taken from Item No. 88030. These Figures are in terms of taper ratio, λ , and the spanwise centre of pressure position, $\bar{\eta}$, corresponding to the spanwise loading due to incidence, obtainable from Item No. 83040.



Sketch 6.1

For large amounts of twist or large variation of C_{Lm} across the span due to exceptionally rapid changes of camber or a section characteristic critically dependent on Reynolds number the method of this Item may not give reliable results. However, for the usual case where these variations are more gradual the method of this Item may be used with confidence.

6.2 Unswept, Untwisted Wings

For an untwisted wing with little or no sweep ($\Lambda_0 \leq 10^\circ$, say) the maximum lift coefficient is given by:

$$C_{Lmax} = C_{Lm}/\mu_p + \Delta C_{LM} \quad (6.1)$$

where C_{Lm} is the value of maximum lift coefficient at $M \approx 0$ for the aerofoil section at η_p and may be obtained from Item No. 84026. The parameter ΔC_{LM} is the increment on wing maximum lift coefficient due to Mach number (see Section 6.3).

The spanwise position of the most highly loaded section, η_p , may be obtained from Figure 1 as a function of taper ratio, λ , and spanwise centre of pressure position, $\bar{\eta}$. The parameter $\bar{\eta}$ may be obtained from Item No. 83040 (Derivation 4) as a function of the planform parameters, reduced wing aspect ratio, βA , a wing taper parameter, κ , and mid-chord sweep parameter, $A \tan \Lambda_{1/2}$.

The ratio of the peak local lift coefficient to the wing lift coefficient, $C_{LLp}/C_L (= \mu_p)$, for the loading due to incidence may be obtained from Figure 2 as a function of λ and $\bar{\eta}$.