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Elastic stress concentration factors Geometric discontinuities in rods and tubes of isotropic materials

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THE PREPARATION OF THIS DATA ITEM

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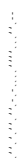
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ELASTIC STRESS CONCENTRATION FACTORS GEOMETRIC DISCONTINUITIES IN RODS AND TUBES OF ISOTROPIC MATERIALS

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ELASTIC STRESS CONCENTRATION FACTORS GEOMETRIC DISCONTINUITIES IN RODS AND TUBES OF ISOTROPIC MATERIALS

1. INTRODUCTION

This Item presents data on stress concentration effects that occur in rods and tubes when they are loaded in tension, bending or torsion. A stress concentration factor is defined as the ratio of the highest stress to a reference stress calculable from simple theory. These stresses may be either direct or shear stresses depending on the applied loading. The stress concentration factors for each set of data presented in this Item are defined in the relevant section.

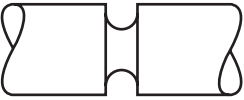
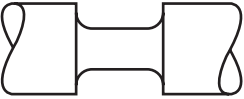
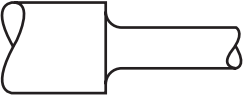
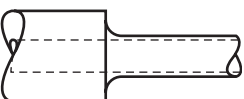
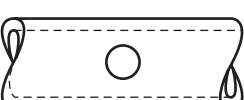
The various geometries for which data are presented are listed in Table 1.1. Section 2 deals with U-grooves, flat bottomed grooves and fillets in rods. Tubes with fillets are examined in Section 3 and diametral holes in rods and tubes are dealt with in Section 4. For each geometry, the cross section on which the reference stress is based is given in the appropriate section. The symbol K indicates that the reference stress is based on the gross cross section of the rod or tube, ignoring the discontinuity, and the symbol K' indicates that the reference stress is based on the net cross section at the discontinuity. For some geometries this reference stress will exist away from the discontinuity.

The data apply only to isotropic materials that obey Hooke's Law. However, most engineering materials conform substantially in these respects and for these the data may be used without significant error. If the stress concentration factor is such that the maximum stress is above the elastic limit of the material, the stresses will be redistributed and will give rise to residual stresses on unloading.

The notation for each section quotes both British and SI Units, but any coherent system of units may be used.

Throughout this Item the terms "rods" and "tubes" apply to bodies having solid and hollow circular sections, respectively.

TABLE 1.1

Form of discontinuity		Loading	Section number
U-grooves in rods		Tension Bending Torsion	2
Flat bottomed grooves in rods		Tension Bending Torsion	2
Fillets in rods		Tension Bending Torsion	2
Fillets in tubes		Tension Torsion	3
Diametral holes in rods and tubes		Tension Bending Torsion	4

2. GROOVES AND FILLETS IN RODS

2.1 Notation

D	maximum diameter of rod	in	m
d	minimum diameter of rod	in	m
f_{max}	maximum direct stress in rod	lbf/in ²	N/m ²
K'	stress concentration factor defined by $f_{max} = K' f_{ref}$		
	where $f_{ref} = \frac{4P}{\pi d^2}$ (for Figures 2.1 to 2.4)	lbf/in ²	N/m ²
	or $f_{ref} = \frac{32M}{\pi d^3}$ (for Figures 2.5 to 2.8)	lbf/in ²	N/m ²
	and $q_{max} = K' q_{ref}$		
	where $q_{ref} = \frac{16T}{\pi d^3}$ (for Figures 2.9 to 2.12)	lbf/in ²	N/m ²
L	length of groove	in	m
M	bending moment in rod	lbf in	N m
P	direct load in rod	lbf	N
q_{max}	maximum shear stress in rod	lbf/in ²	N/m ²
r	radius of fillet or bottom of groove	in	m
T	torque in rod	lbf in	N m

2.2 Notes

The data presented in this section are based on finite element studies and are in broad agreement with available experimental data. The finite element studies have enabled a more consistent examination of parameter variations to be made.

In recent years finite element studies have become a successful and widely used method of generating stress concentration data. Finite element methods are particularly useful for various reasons. They can provide accurate and consistent data and can be used to study the variation of parameters. Also, they have provided data for previously undocumented cases where experimental and other theoretical methods have not provided satisfactory results.

Values of K' are plotted against r/d for various values of D/d for solid circular rods with grooves and fillets in tension in Figures 2.1 to 2.4, in bending in Figures 2.5 to 2.8 and in torsion in Figures 2.9 to 2.12. Each figure gives values of K' for a particular form of groove or fillet subject to one form of loading as listed in Table 2.1. The data presented in these figures have been generated by finite element methods (Reference 11) that are in agreement with the theoretical and experimental data presented in References 5, 9 and 10. Few experimental data exist for values of stress concentration factors for geometries in the region $r/d = 0.005$ to $r/d = 0.01$. Therefore, although every care has been taken in generating the finite element data, the curves in this region are mostly unsupported by other data and so there is a greater uncertainty about them.

The maximum stress in the U-type groove occurs at the bottom of the groove. In all other cases the maximum stress occurs in the fillets close to their junction with the smaller diameter. It should be noted that Figures 2.1 to 2.8 give direct stresses and that Figures 2.9 to 2.12 give shear stresses.

In a flat bottomed groove the maximum stress concentration occurs at each fillet. As the fillet radius is increased the stress concentration is reduced. However, as the geometry of a U-groove is approached and the regions of maximum stress are close together the two stress concentrations will interact to give an increase in the overall stress concentration at each fillet. This effect can be seen in Figures 2.2, 2.6 and 2.10. In these figures some of the curves begin to rise at r/d of about 0.1. At this point the geometries corresponding to these curves are changing from fully formed* flat bottomed grooves to U-grooves. This situation does not occur for any of the curves in Figures 2.3, 2.7 or 2.11, over the range of r/d values for which data are presented.

TABLE 2.1

<i>Class of groove of fillet</i>	<i>Loading</i>	<i>Figure number</i>
U-type groove	Tension	2.1
	Bending	2.5
	Torsion	2.9
Groove with $L/d = 0.25$	Tension	2.2
	Bending	2.6
	Torsion	2.10
Groove with $L/d = 1.0$	Tension	2.3
	Bending	2.7
	Torsion	2.11
Fillet	Tension	2.4
	Bending	2.8
	Torsion	2.12

* A fully formed groove is one in which the step depth is greater than the fillet radius. The dashed line in each of Figures 2.1 to 2.12 indicates where $D - d = 2r$, that is, where the step depth equals the fillet radius.

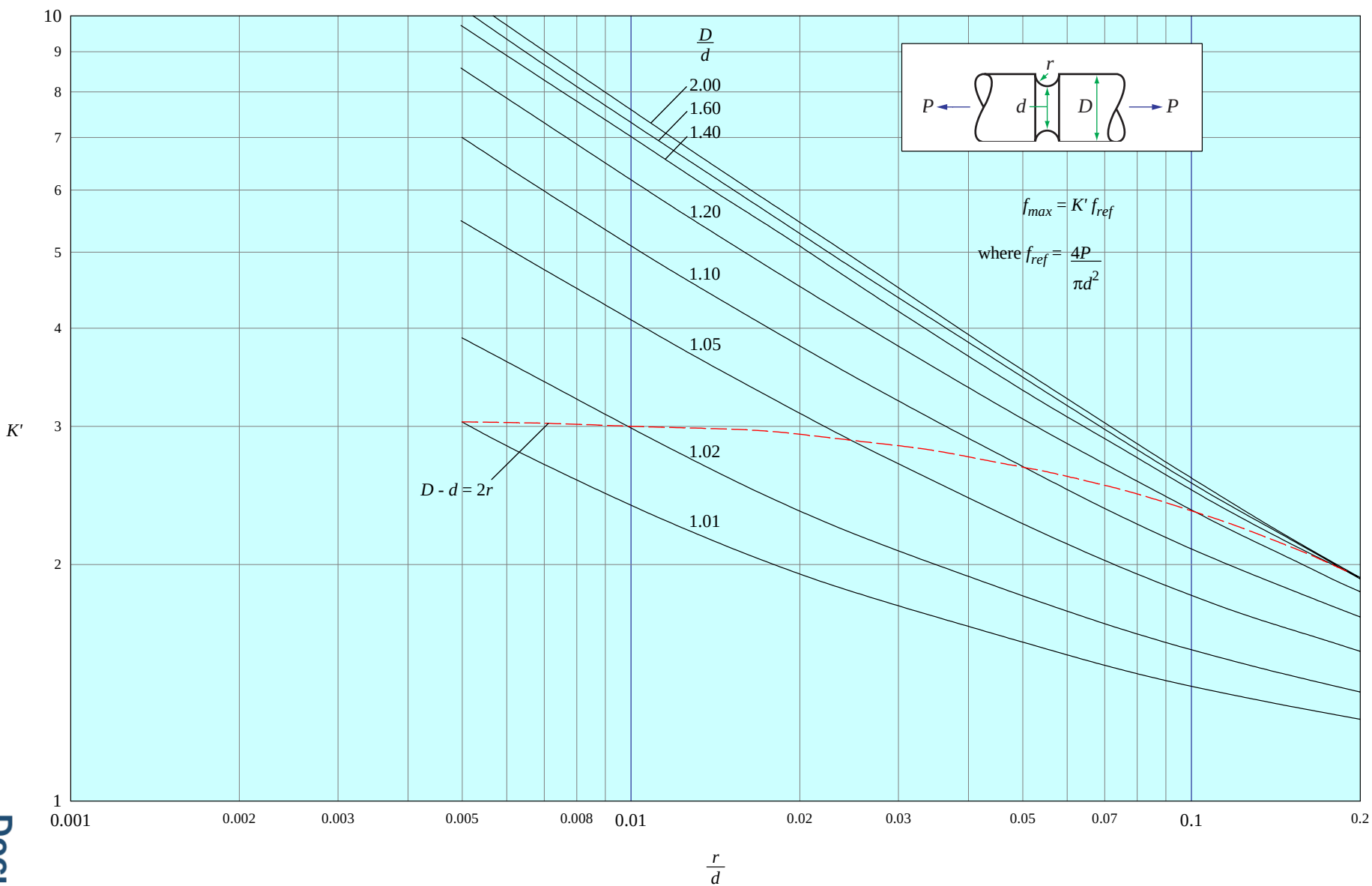


FIGURE 2.1 U-GROOVES, IN TENSION

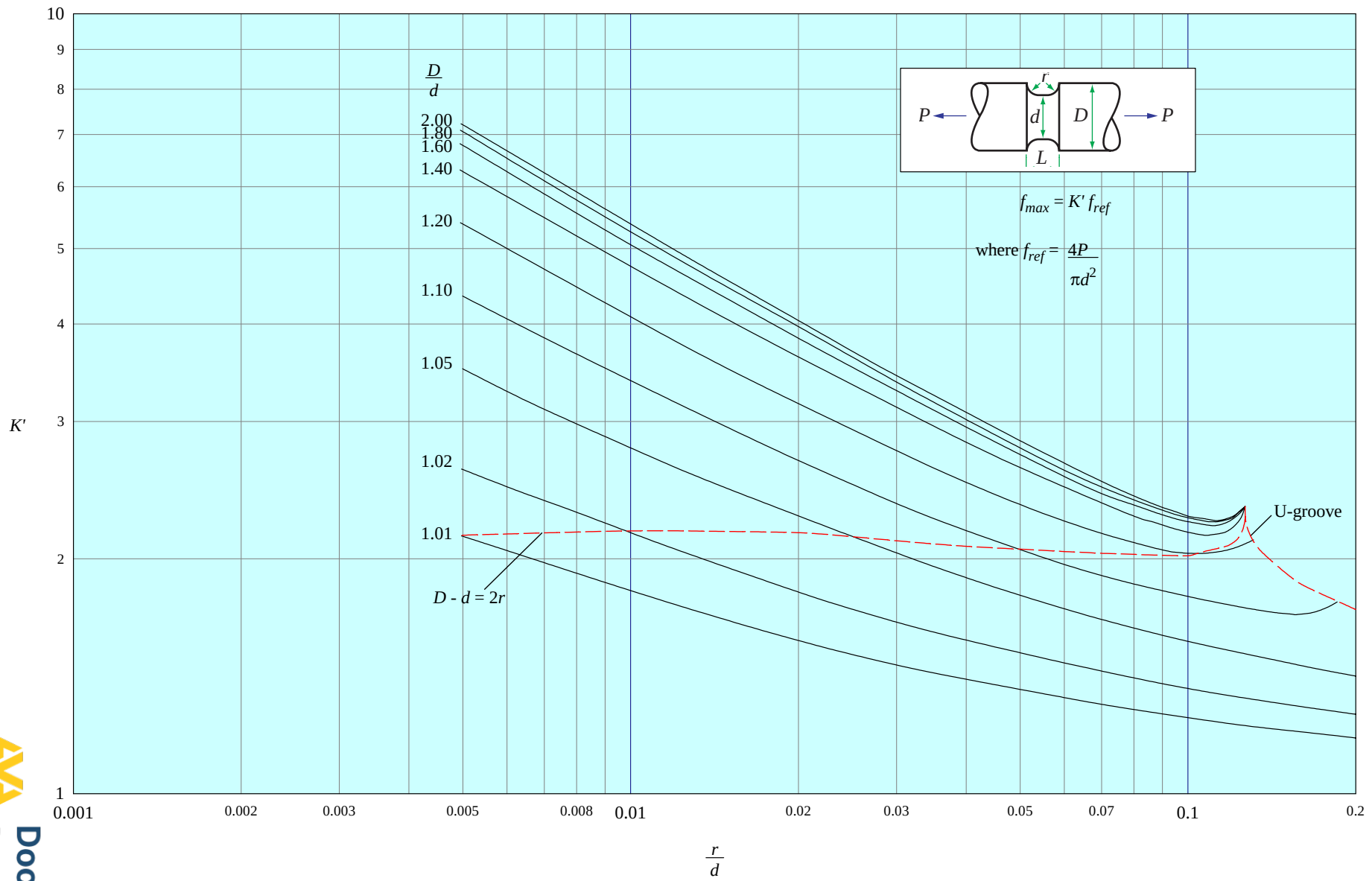
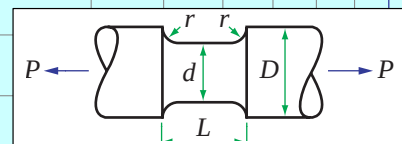


FIGURE 2.2 GROOVES WITH FLAT BOTTOMS, IN TENSION, $\frac{L}{d} = 0.25$



$$f_{max} = K' f_{ref}$$

$$\text{where } f_{ref} = \frac{4P}{\pi d^2}$$

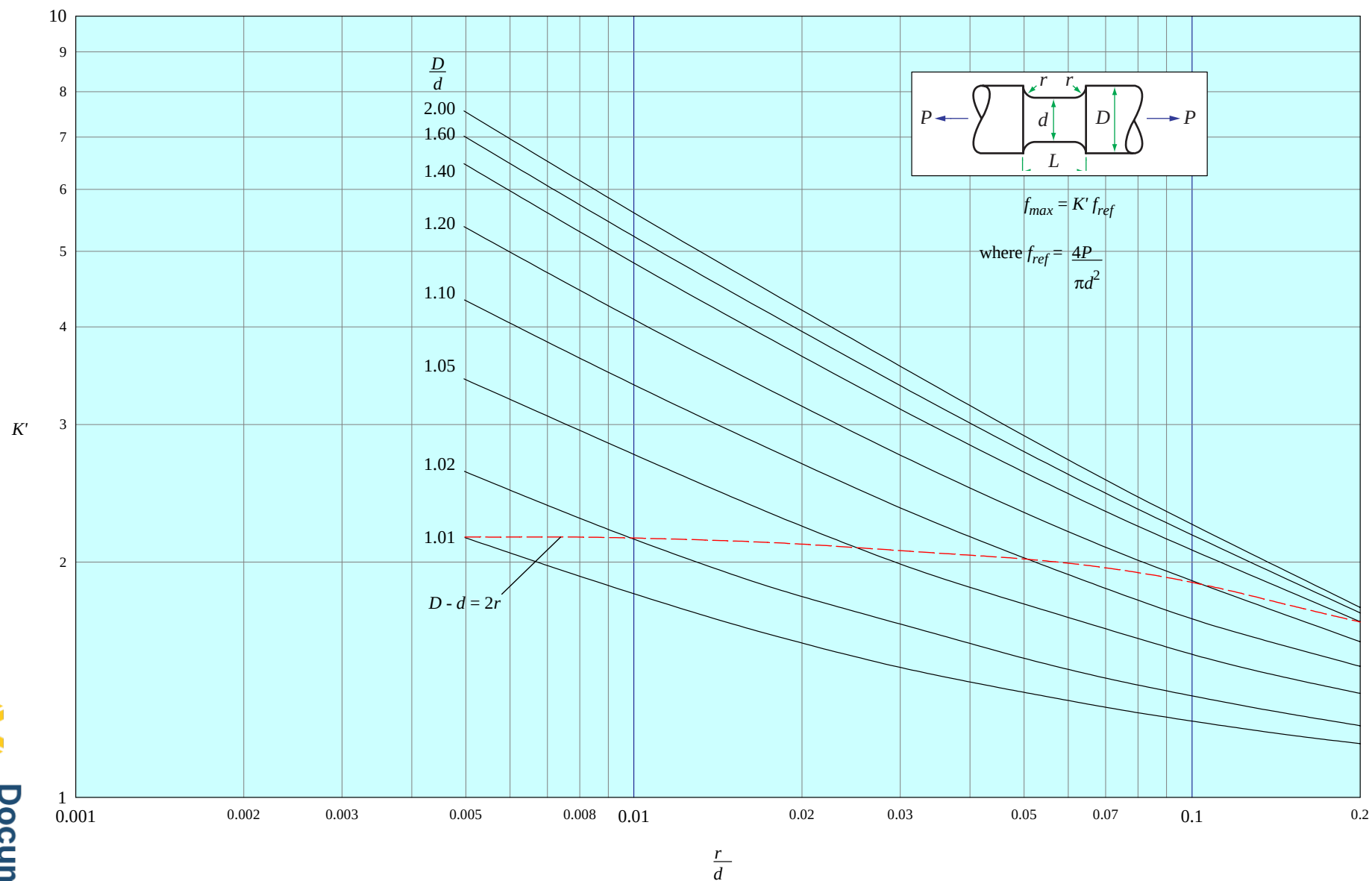


FIGURE 2.3 GROOVES WITH FLAT BOTTOMS, IN TENSION $\frac{L}{d} = 1.0$