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Response of structures to atmospheric turbulence. Response to across-wind turbulence components

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THE PREPARATION OF THIS DATA ITEM

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RESPONSE OF STRUCTURES TO ATMOSPHERIC TURBULENCE RESPONSE TO ACROSS-WIND TURBULENCE COMPONENTS

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RESPONSE OF STRUCTURES TO ATMOSPHERIC TURBULENCE

Response to Across-wind Turbulence Components

1. NOTATION AND UNITS

		Units	
		SI	British
a	structure width normal to width b	m	ft
B_B, B_R^2	load/velocity admittance factors (see Appendix B)	—	—
b	structure width; (reference area for load coefficients)/(overall height or length)	m	ft
C_D, C_L or C_C	wind-axis force coefficients	—	—
C_F	force coefficient (see Section 4.2); (force per unit length)/($\frac{1}{2}\rho V_{hc}^2 b$)	—	—
C_{FP}, C_{FQ}, C_{FR}	force coefficients in P, Q and R principal-axis directions	—	—
C_T	torsional moment coefficient (see Section 4.2); (torsional moment per unit length)/($\frac{1}{2}\rho V_{hc}^2 b(bL)^{0.5}$)	—	—
C_{TP}, C_{TQ}, C_{TR}	torsional moment coefficients about P, Q and R axes	—	—
d	displacement height of wind-speed profile zero-plane	m	ft
E_{mean}	mean load effect; $E_{mean} = \int_0^L \frac{1}{2}\rho V^2 C_F \beta b dl$	*	*
E_{ref}	reference mean load effect [†] ; $E_{ref} = \int_0^L \frac{1}{2}\rho V^2 \beta b dl$	*	*
F_j	mode-generalised mean force; $\int_0^L \frac{1}{2}\rho V^2 C_F \mu_j b dl$	N	lbf
g_B, g_R	peak response factor for background and resonant contributions to response respectively	—	—
H	overall height of vertical structure (see Sketch 1.1)	m	ft
H_i	distance up vertical cantilever structure at which response is required or distance up vertical single-support structure at which support is located (see Sketch 1.1)	m	ft
h_c	reference height (see Sketch 1.1)	m	ft

For footnotes, see end of Notation Section.

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h_0	height above ground of lowest point on structure (see Sketch 1.1)	m	ft
I_B	integral parameter; $I_B = \int_0^L m\mu_1 dl$	kg	$\frac{1}{16}$ slug
I_M, I_{Mj}	absolute and mode-generalised mass moments of inertia respectively	N m ²	lbf ft ²
I_{Rj}	integral parameter; $I_{Rj} = \int_0^L m\mu_1 \beta dl$	**	**
I_v, I_w	turbulence intensities of across-wind components of turbulence at height h_c ; $I_v = (\sigma_v/V)_{h_c}$, $I_w = (\sigma_w/V)_{h_c}$	—	—
J_B	background loading factor (see Appendix B)	rad	rad
J_{BH}, J_{BW}	background loading factors for vertical and horizontal structures respectively	rad	rad
J_R^2	resonant loading factor (see Appendix B)	rad ²	rad ²
$(J_R^2)_H, (J_R^2)_W$	resonant loading factors for vertical and horizontal structures respectively	rad ²	rad ²
K_V	weighting factor on hourly-mean wind speed; $K_V^2 = \int_0^1 (V_z/V_{hc})^2 d(l/L)$	—	—
K_ζ	aerodynamic damping factor (see Appendix B)	rad	rad
$K_{\zeta H}, K_{\zeta W}$	aerodynamic damping factors for vertical and horizontal structures respectively	rad	rad
L	structure overall length or height; H for vertical structure and W for horizontal structure	m	ft
L_i	dimension H_i or W_i as appropriate	m	ft
${}^rL_v, {}^rL_w$	compound lateral integral length scales of across-wind components of turbulence, used in evaluation of inclined structure response	m	ft
${}^xL_v, {}^xL_w$	lateral length scales of across-wind components of turbulence, in along-wind, x , direction at height h_c	m	ft
${}^yL_w, {}^zL_v$	lateral integral length scales of across-wind components of turbulence, in y and z directions respectively, at height h_c	m	ft
l	distance along structure (see Sketch 1.1)	m	ft

For footnotes, see end of Notation Section.

M	overall mass ^{††} of structure	kg	[‡] slug
M_j	overall mode-generalised mass for mode j	kg	[‡] slug
m	mass ^{††} per unit length of structure	kg/m	[‡] slug/ft
N_1, N_2	parameters in mode shape functions for cantilever first and second bending modes respectively	—	—
n	frequency	Hz	Hz
n_j	natural frequency of mode j	Hz	Hz
$0P, 0Q, 0R$	system of right-handed rectangular, principal structural axes with $0P$ axis in mean wind direction for $\phi = 0$ and $x = 0$	—	—
$P_j, \ddot{P}_j$ $Q_j, \ddot{Q}_j$	mode-generalised deflection and acceleration respectively for mode j , i.e. deflection and acceleration at location where $\mu_j = 1$	m, m/s ²	ft, ft/s ²
$p, \ddot{p}$ $q_l, \ddot{q}_l$	deflection and acceleration respectively at location l on structure, in $0P$ and $0Q$ directions	m, m/s ²	ft, ft/s ²
R_j	reference mode-generalised force ^{‡‡} ; $R_j = \int_0^L \frac{1}{2} \rho V^2 \mu_j b dl$	N	lbf
S_{FjFj}	spectral density of mode-generalised force for mode j	N ² /Hz	lbf ² /Hz
S_{vv}, S_{ww}	spectral densities of across-wind turbulence components	m ² /s ² Hz	ft ² /s ² Hz
u, v, w	fluctuating components of turbulent wind speed along x , y and z directions respectively	m/s	ft/s
V	hourly-mean wind speed	m/s	ft/s
V_z	hourly-mean wind speed at height z above wind-speed profile zero-plane	m/s	ft/s
V_m	weight hourly-mean wind speed; $V_m = K_V V_{hc}$	m/s	ft/s
W	overall length of horizontal structure (see Sketch 1.1)	m	ft
W_i	distance along horizontal cantilever structure at which response is required or distance along horizontal single-support structure at which support is located (see Sketch 1.1)	m	ft
$0X, 0Y, 0Z$	system of right-handed rectangular body axes	—	—

For footnotes, see end of Notation Section.

$0x, 0y, 0z$	system of right-handed rectangular wind axes with x -axis along mean wind direction and z -axis vertically upwards	—	—
z	height above wind-speed profile zero-plane	m	ft
z_0	terrain roughness parameter	m	ft
β	response function describing relationship between applied force or moment and resulting load effect	***	***
ζ_a, ζ_s	aerodynamic and structural damping ratios, expressed as fractions of critical damping (see Appendix B)	—	—
μ_j	mode shape function describing deflected shape of structure in mode j	—	—
ρ	air density	kg/m ³	‡slug/ft ³
σ_E	standard deviation of fluctuating load effect	*	*
$\sigma_P, \sigma_{\ddot{P}}$ $\sigma_Q, \sigma_{\ddot{Q}}$	standard deviations of mode-generalised deflection and acceleration, <i>i.e.</i> deflection or acceleration at location where $\mu_j = 1$; $\sigma_{\ddot{P}} = (2\pi n_j)^2 \sigma_P$, $\sigma_{\ddot{Q}} = (2\pi n_j)^2 \sigma_Q$	m, m/s ²	ft, m/s ²
$\sigma_{p,l}, \sigma_{\ddot{p},l}$ $\sigma_{q,l}, \sigma_{\ddot{q},l}$	standard deviations of deflection and acceleration at location l ; $\sigma_{p,l} = \sum_j \mu_{j,l} \sigma_{Pj}, \sigma_{\ddot{p},l} = \sum_j \mu_{j,l} \sigma_{\ddot{P}j},$ $\sigma_{q,l} = \sum_j \mu_{j,l} \sigma_{Qj}, \sigma_{\ddot{q},l} = \sum_j \mu_{j,l} \sigma_{\ddot{Q}j},$	m, m/s ²	ft, m/s ²
σ_v, σ_w	standard deviations of across-wind components of turbulence	m/s	ft/s
ϕ	flow incidence angle between $0P$ principal axis and mean wind direction in plane normal to $0P$ axis	rad	rad
ϕ_{vv}, ϕ_{ww}	normalised spectral density functions of across-wind components of turbulence at frequency n_j and height h_c $\phi_{vv} = n S_{vv} / \sigma_v^2$, $\phi_{ww} = n S_{ww} / \sigma_w^2$	—	—
χ	angle between $0R$ principal axis and mean wind direction in plan containing both (often referred to as yaw angle)	rad	rad
ψ	angle between $0R$ -axis of structure and vertical in plane normal to mean wind direction (sometimes referred to as pitch angle) (see Sketch 1.1)	rad	rad

Subscripts

B denotes quantity relating to background or non-resonant contribution to response

For footnotes, see end of Notation Section.

h_c	denotes value at height h_c above ground
i	general turbulence component, set to v or w as appropriate; in H_i , L_i or W_i denotes intermediate location on structure
$j, 1, 2$	denote values relating to general mode j and first and second modes
l	denotes value at distance l along structure
max	denotes expected-maximum value
$mean$	denotes quantity relating to mean contribution to response
R	denotes quantity relating to resonant contribution to response

* Units depend on load effect considered; N or lbf for forces, Nm or lbf ft for moments.

† The notation $|\beta|$ denotes the absolute value of β .

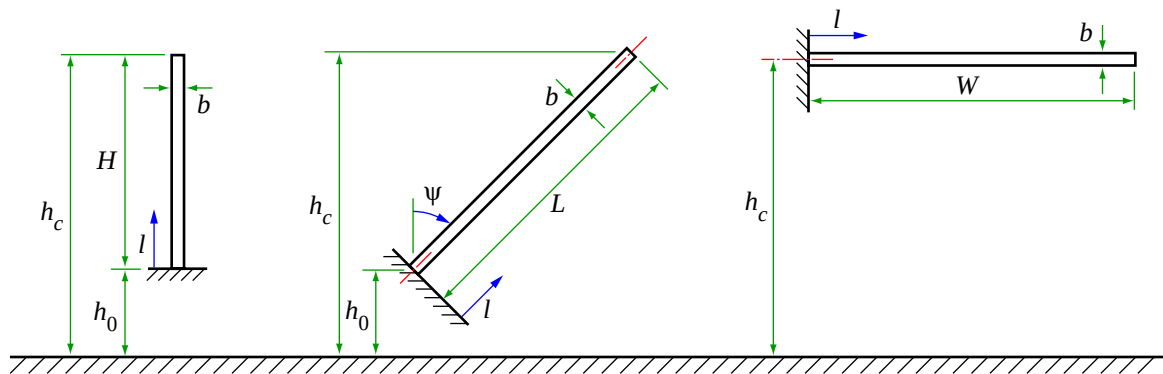
‡ 1 slug = 32.174 lb (pounds mass).

** Units depend on the influence function selected. For translational response: kg N/N or slug lbf/lbf for forces, kg Nm/N or slug lbf ft/lbf for moments. For torsional response: kg Nm/Nm or slug lbf ft/lbf ft.

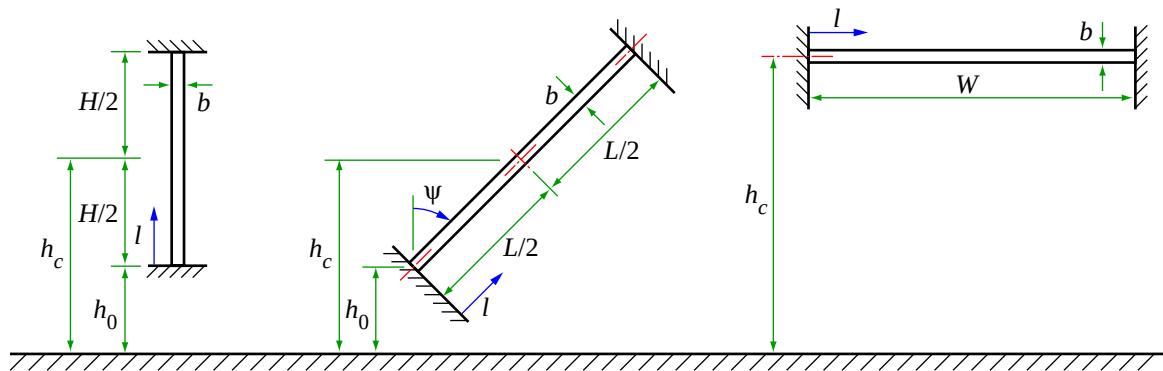
†† This mass should include the mass of any permanently installed equipment as well as the structure mass.

‡‡ the notation $|\mu_j|$ denotes the absolute value of μ_j .

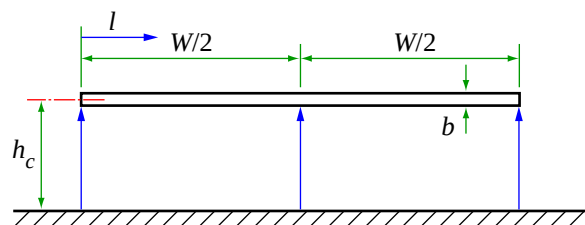
*** Units depend on load effect considered. For translational response: N/N or lbf/lbf for forces, Nm/N or lbf ft/lbf for moments. For torsional response: Nm/Nm or lbf ft/lbf ft.



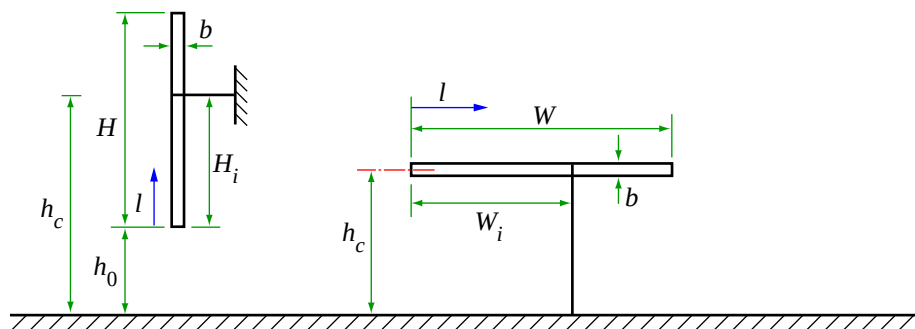
(a) Cantilever structures



(b) Single-span structures



(c) Two-span, bridge-type structures



(d) Single-support structures

Sketch 1.1 Geometrical parameters

2. INTRODUCTION

2.1 Background

Reference 1 discusses qualitatively a variety of mechanisms, such as vortex shedding, turbulence buffeting and galloping that may contribute to the overall response of a body in a fluid flow. Although in real situations interaction between the mechanisms may occur, the response is usually estimated from independent calculation for each mechanism.

The present Item is one of a group concerned with methods of calculating the loading and response due to buffeting by atmospheric turbulence. The loading on or response of a structure in the turbulent wind can be expressed in terms of a mean contribution, due to the hourly-mean wind speed, and a fluctuating contribution, due to the turbulent fluctuations in wind speed about that mean. For most practical cases a full dynamic analysis for the fluctuating contribution can be replaced by an approximate method that treats the fluctuating response as the sum of a background (quasi-static) contribution and separate resonant contributions for each response mode. The method is described in more detail in References 1 and 8. It forms the basis of the calculation methods in this group of Items where it is further simplified by the assumption that responses to each of the wind turbulence components, u , v and w , can be treated separately.

The present Item is concerned with response to across-wind, v and w , turbulence components. Response to the along-wind, u , component is considered in Reference 7. Computer program versions of these calculation procedures are described in Reference 4.

2.2 Response to Across-wind Turbulence Components

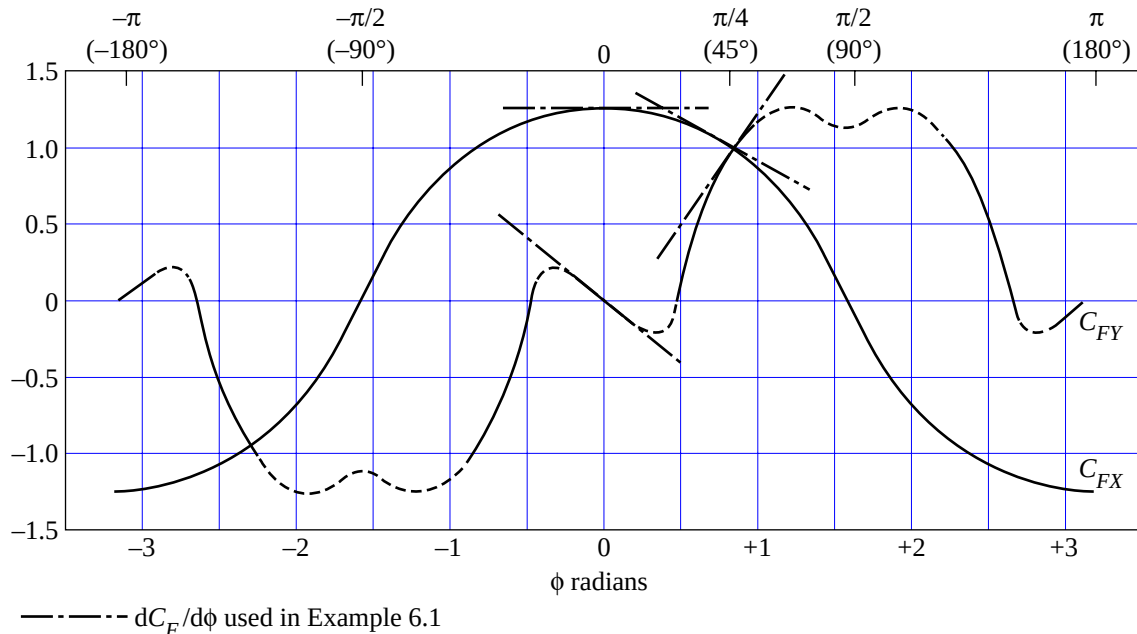
The present Item provides calculation procedures for the response of buildings and structures to buffeting by lateral components of atmospheric turbulence. The method of analysis assumes that the across-wind turbulent fluctuations in wind speed result in a fluctuating incidence of the wind which in turn produces a fluctuating loading on the structure. The analysis is given in Reference 8 and the resulting equations for the non-dimensional loading factors are given in Appendix B of the present Item. The response is calculated in terms of its principal-axis components (often coincident with the structural body axes). Variation in wind and structure properties is considered along one axis only of the structure, defined as the OR principal axis, and the responses calculated are translational, normal to that axis, or torsional, about that axis.

Section 4 of the Item gives step-by-step methods for calculating the loading or response in terms of a mean contribution and background and resonant fluctuating contributions, using loading factors precalculated for a range of wind conditions and mode shapes or response functions. If the fluctuating wind and the response to it are assumed to be stationary random processes, expected-maximum values of the loading or response due to across-wind turbulence buffeting can then be estimated. The loading factors required in the method are presented graphically and in equation form. Section 6 gives worked examples of the application of the method.

3. APPLICABILITY AND LIMITATIONS

The application of this Data Item is to isolated structures in the atmospheric wind. Such structures include buildings and towers, chimney stacks, masts, and some bridge, pipeline and gantry type structures.

Interaction between structures in groups may modify the buffeting response but this effect is not considered here. Interference effects may be expected to be small if a structure is spaced more than 3 structure widths upstream or to one side of a similar structure or more than 10 structure widths downstream of a similar structure.



Sketch 3.1 Body-axis force coefficients for 2:1 rectangular section

The method is applicable only to conditions and structures for which the slope of the load coefficient-vs-incidence curve can be approximated as constant. By way of example, Sketch 3.1 shows the force coefficients C_{FX} and C_{FY} for a 2:1 rectangular cross-sectioned structure⁹. The method will be applicable provided that the fluctuating incidence remains within one of the full-line regions of the curve. The required range is approximately $\pm \tan^{-1}$ (turbulence intensity) from mean incidence (about ± 0.17 rad, $\pm 10^\circ$, for 20 per cent turbulence intensity).

Only fundamental and some second mode responses are covered by the data presented but the method could be applied more generally by integration of the appropriate equations in Appendix B. Coupling between normal modes of response is not considered.

The effects of correlation between the turbulence components are not considered. The $\overline{uv}$ and $\overline{vw}$ co-variances are small and may be neglected without significant error; the $\overline{uw}$ co-variance is larger and neglecting this leads to underestimates of the w-component response for structures close to the ground.

For heights below about $1.5d$, the wind properties, especially wind speed and direction, are subject to strong local effects. For structures that are wholly or largely below this height, results obtained from this Item should be applied with caution unless derived with wind data specific to the location considered.

The ranges of parameters covered in generating Figures 2 and 3 are given in Appendix B. The curves in this Item reproduce the calculated background loading factor, J_B , to within about 10 per cent over a practical range of parameters and the resonant loading factor, J_R , to within about 20 per cent. The equations in Appendix A are within about 10 per cent for all computed data. This would allow the overall predicted response to be estimated to within about 10 per cent in the absence of other errors. The response calculated for cases where the admittance factor B_R^2 is less than unity is most reliable for components in or near an across-wind direction because B_R^2 is derived from tests with across-wind loading only. The uncertainty of the calculated response is influenced considerably by the uncertainty in the input data, which cannot be quantified here.

4. CALCULATION PROCEDURES

4.1 General

A calculation procedure is given in Section 4.3 for translational response of uniform, vertical or horizontal structures. Extension of the procedure to non-uniform structures, to structures inclined in the Oyz plane normal to the mean wind, to torsional responses and to bridge decks with yawed wind is discussed in Sections 4.4, 4.5, 4.6 and 4.7. A Calculation Sheet on which the results for the basic procedure may be recorded is given in Table 4.1. Worked examples of the application of the procedures are given in Section 6. A computer program version of the calculation procedures is available⁴.

The buffeting response will usually be required for several wind speeds including the design wind speed and values corresponding to serviceability criteria. The calculations in Steps (1) to (13) will be required for each wind-speed case although obviously some data will be common to all cases. It is also recommended that the effects of uncertainty in the input data, especially mass, damping and load coefficient and its slope, be considered.

Guidance on values and sources of data for some of the structural parameters required is given in Reference 2. The required wind property data can be obtained from the methods and data in Reference 5 or by use of the program E0108 (see ESDU 01008³). The appropriate load coefficients and slopes, C_F and $dC_F/d\phi$ or C_T , and $dC_T/d\phi$, are discussed in Section 4.2.

The required loading factors for a range of response functions and mode shapes are presented graphically in Figures 2 and 3 and by equations in Appendix A. If the required case is not covered exactly, the exact values should be used to evaluate E_{ref} and R_j and M_j , I_B and I_{Rj} with the nearest approximations in Figures 2, 3 and 7 (or the corresponding equations) used to derive J_B , J_R^2 and K_ζ .

The aerodynamic damping, calculated in Step (11) can be negative, in which case it reduces rather than increases the overall (structural + aerodynamic) damping. Because of the uncertainty in structural damping values, situations for which the overall damping is less than half the structural damping should generally be considered undesirable because of the increased possibility of a significant galloping response.

4.2 Load Coefficients and Slopes

4.2.1 General

The method requires input of an appropriate load coefficient, C_F or C_T , at the mean wind incidence* for calculation of the mean response.

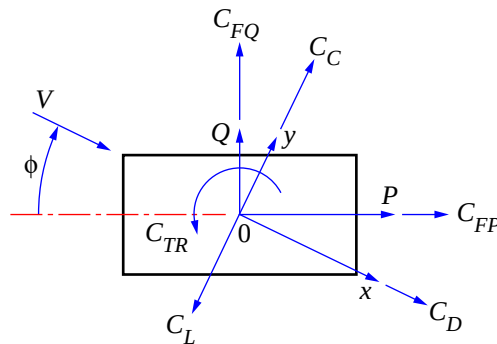
For calculation of the fluctuating components of response a representative value of the slope of the load coefficient-vs-incidence curve, $dC_F/d\phi$, or $dC_T/d\phi$ over the anticipated range of fluctuating incidence is required. The range is approximately $\pm \tan^{-1}$ (turbulence intensity) about the mean incidence.

Ideally, the variation of C_F and $dC_F/d\phi$ or C_T and $dC_T/d\phi$ along the structure is required but such information is rarely available and usually a mean value must be used. For structures such as lattice towers of varying section and composite towers of two or more cross-sections the variation can be approximated by constant values over each section of structure. The reference velocity used in the coefficients is V_{hc} , data should be adjusted to this basis if necessary. The reference torsional moment arm used for torsional response in this Data Item is $(bL)^{0.5}$; a consistent value must be used throughout the calculations.

Reference 6 gives information on mean force and moment coefficients for a range of structure shapes.

4.2.2 Vertical structures

In the case shown the OR axis is taken to coincide with the Oz wind-axis. For zero incidence, the OP axis is along the mean wind direction.



$$C_{FP} = C_D \cos \phi - C_L \sin \phi$$

$$C_{FQ} = -C_D \sin \phi - C_L \cos \phi$$

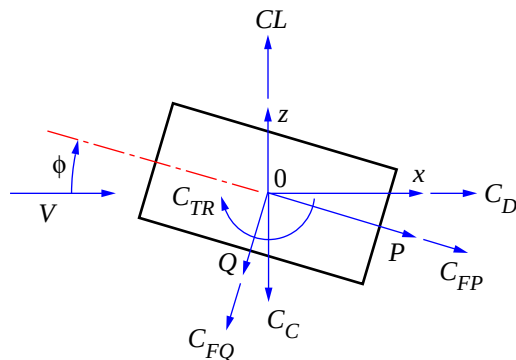
Sketch 4.1

Sketch 4.1 shows a plan view (along the negative OR -axis) of a vertical structure. The required loading coefficients and slopes are C_{FP} and $dC_{FP}/d\phi$ for translational response in the OP direction, C_{FQ} and $dC_{FQ}/d\phi$ for translational response in the OQ direction and C_{TR} and $dC_{TR}/d\phi$ about the OR axis for torsional response about that axis. The force coefficient data may be available as C_D and C_L (or C_C) which may be related to C_{FP} and C_{FQ} as shown in Sketch 4.1.

4.2.3 Horizontal structures

In the case shown the OR axis is taken to coincide with the Oy wind-axis. Again, for zero incidence, the OP axis is in the mean-wind direction.

* The term incidence has been used in this Item to denote the angle between mean wind direction and a reference direction in the OPQ plane, positive for clockwise rotation of the body relative to the wind direction, viewed along the OR axis.



$$C_{FP} = C_D \cos \phi - C_L \sin \phi$$

$$C_{FQ} = -C_D \sin \phi - C_L \cos \phi$$

Sketch 4.2

Sketch 4.2 shows a view along the OR -axis of a horizontal structure. The required loading coefficients and slopes are C_{FP} and $dC_{FP}/d\phi$ for translational response along the OP direction, C_{FQ} and $dC_{FQ}/d\phi$ for translational response along the OQ direction and C_{TR} and $dC_{TR}/d\phi$ about the OR -axis for torsional response about that axis. Data may also be expressed in terms of C_D and C_L (or C_C) which may be related to C_{FP} and C_{FQ} as shown in Sketch 4.2.

4.2.4 Isolated structures

Sections 4.2.2 and 4.2.3 refer to structures fixed at one or both ends. Some cases may occur where the structure can be approximated as an isolated structure, *e.g.* elevated storage tanks, and response in all three directions is required. In this case all three principal-axis force or moment coefficients and slopes will be required. The response will result from both v - and w -turbulence components.

4.2.5 Circular sections

For circular sections, $C_L = 0$ and the force coefficients are given by

$$C_{FP} = C_D \cos \phi \text{ and } C_{FQ} = C_D \sin \phi$$

and the slopes by

$$dC_{FP}/d\phi = -C_D \sin \phi \text{ and } dC_{FQ}/d\phi = C_D \cos \phi.$$

Unless structural asymmetries are present, the mean incidence can be taken as zero, for which $\cos \phi = 1$ and $\sin \phi = 0$.

4.3 Procedure for Structures of Constant Width and Cross-Section

4.3.1 Determine the input data

(1) The following input data are required:

L	$=$	Hor W	overall length of structure (Sketch 1.1)
b			overall width of structure (Sketch 1.1)
H_i or W_i			location at which response is required or at which support axis is located
M or m			mass or mass distribution
n_j			natural frequency
μ			mode shape function
β			response function
ζ_s			structural damping ratio
C_F			*reference load coefficient at mean incidence ϕ (radian) (Section 4.2)
$dC_F/d\phi$			*slope of load coefficient-vs-incidence curve (per radian) (Section 4.2)
h_c			†reference height (Sketch 1.1)
z_0			†terrain roughness at site and up-wind
d			†displacement height of wind-speed profile zero-plane
V_{hc}			hourly-mean wind speed at height h_c
V_z			hourly-mean wind-speed profile over vertical extent of structure
I_v or I_w			‡lateral turbulence intensity at height h_c
^xL_v and ^zL_v			‡turbulence length scales at height h_c
^xL_w and ^yL_w			

for each wind-speed case

* Values of these parameters are required for each response direction considered.

† These data are required in the calculation of the wind properties.

‡ For vertical structure use v components, for horizontal structures use w components.

(2) Calculate the overall mode-generalised mass from*

$$M_j = \int_0^L m \mu_j^2 dl = L \int_0^1 m \mu_j^2 d(l/L) \quad (4.1a)$$

which, for uniform mass distribution, reduces to

$$M_j = M \int_0^1 \mu_j^2 d(l/L). \quad (4.1b)$$

* A Simpson's rule approximation for these integrals will usually be adequate, see Section 6 for example application.

4.3.2 Calculate the mean contribution to the response

If the response to more than one turbulence component is calculated, the mean response in each direction should be included once only.

- (3) Calculate the mean load effect or the mean deflection in the required directions as appropriate. The mean acceleration will be zero.

Calculate the mean load effect from^{*}

$$E_{mean} = \int_0^L \frac{1}{2} \rho V_z^2 C_F \beta b dl = L \int_0^1 \frac{1}{2} \rho V_z^2 C_F \beta d(l/L), \quad (4.2a)$$

which, for constant width and C_F along the structure, reduces to

$$E_{mean} = \frac{1}{2} \rho V_{hc}^2 C_F b L \int_0^1 (V_z/V_{hc})^2 \beta d(l/L) \quad (4.2b)$$

where C_F is an appropriate mean load coefficient. For cantilever structures, if the load effect is required for $L_i \neq 0$, the integration need only be taken from L_i to L (or L_i/L to 1), since $\beta = 0$ for $l < L_i$.

For deflections, calculate the mean mode-generalised force from^{*}

$$F_j = \int_0^L \frac{1}{2} \rho V_z^2 C_F \mu_j b dl = L \int_0^1 \frac{1}{2} \rho V_z^2 C_F \mu_j b d(l/L) \quad (4.3a)$$

which, for constant width and C_F along the structure, reduces to

$$F_j = \frac{1}{2} \rho V^2 C_F b L \int_0^1 (V_z/V_{hc})^2 \mu_j d(l/L). \quad (4.3b)$$

The mean, mode-generalised, deflection is given by

$$P_{j,mean} \text{ or } Q_{j,mean} = \frac{F_j}{(2\pi n_j)^2 M_j}. \quad (4.4)$$

Repeat Step (3) if deflections for a second mode response ($j = 2$) are required.

^{*} A Simpson's rule approximation for these integrals will usually be adequate. If $\beta = 1$ or $\mu_j = 1$ the integrals reduce to $\int_0^1 (V_z/V_{hc})^2 d(l/L)$ or K_V^2 , which is given by Figure 1 for uniform upwind terrain.

4.3.3 Calculate the background or non-resonant contribution to response

- (4) Calculate the reference mean load effect from*

$$E_{ref} = \int_0^L \frac{1}{2} \rho V_z^2 |\beta| b dl = L \int_0^1 \frac{1}{2} \rho V_z^2 |\beta| b d(l/L), \quad (4.5a)$$

which, for constant width along the structure, reduces to

$$E_{ref} = \frac{1}{2} \rho V_{hc}^2 b L \int_0^1 (V_z/V_{hc})^2 |\beta| d(l/L), \quad (4.5b)$$

where $|\beta|$ denotes the modulus or absolute value[†] of β . For cantilever structures, if the load effect is required for $L_i \neq 0$, the integration need only be taken from L_i to L (or L_i/L to 1) since $\beta = 0$ for $l < L_i$.

- (5) Calculate $H/\bar{L}_v$ for vertical structures or $W/\bar{L}_w$ for horizontal structures and determine the background loading factor, J_B , from the appropriate part of Figure 2.
- (6) Calculate the background contribution to the load effect from

$$\sigma_{E,B} = E_{ref} \times |dC_F/d\phi| \times B_B I_i J_B \quad (4.6)$$

where the background admittance factor, B_B , is taken as unity and I_i is I_v , or I_w as appropriate. The absolute value of $dC_F/d\phi$ for the required response direction is used in this step.

If the deflection response is required, calculate $\sigma_{E,B}$ for $\beta = 1.0$ and the integral parameter I_B from*

$$I_B = \int_0^L m \mu_1 dl = L \int_0^1 m \mu_1 d(l/L) \quad (4.7a)$$

which, for uniform mass distribution, reduces to

$$I_B = M \int_0^1 \mu_1 d(l/L). \quad (4.7b)$$

Hence calculate the background contribution to the deflection, evaluated¹ as a mode-generalised deflection in mode 1, from

$$\sigma_{P,B} \text{ or } \sigma_{Q,B} = \sigma_{E,B} / (4\pi^2 n_1^2 I_B). \quad (4.8a)$$

* A Simpson's rule approximation for these integrals will usually be adequate, see Section 6 for example application. If $\beta = 1$, the integral in Equation (4.5b) reduces to K_V which is given by Figure 1 for uniform upwind terrain.

† If β has both positive and negative values, the mean load effect, E_{mean} , can become very small or even zero, leading to computational difficulties in the evaluation of J_B . Use of $|\beta|$ in E_{ref} avoids this problem.