

CONSTRUCTION OF INELASTIC STRESS-STRAIN CURVES FROM MINIMAL MATERIALS DATA

1. THEORY AND APPLICATION

1.1 Introduction

1.1.1 General notes

This Item provides a Fortran computer program for estimating the nominal or true inelastic stress-strain curve, tangent and secant moduli and the inelastic Poisson's ratio of a metallic material, from limited, standard, commonly available materials values. The program can be useful in predicting the behaviour of a structural element over its full inelastic range up to failure when the stress-strain curve is not known and only the modulus of elasticity and two points on the inelastic region of the curve are available. It assumes that the stress-strain curve may be represented by a smooth continuous generalised curve based on a reference stress, a material characteristic and a modulus of elasticity. The method is valid for both tension and compression stress-strain data.

The Data Item is provided in two parts. Section 1 describes the theory and its application. Section 2 describes the use of the Fortran program, its input, output and all messages, and gives example calculations. The program is available in two formats: (a) a Fortran source code with example input and output files, and (b) an executable program for PCs. Both versions can be found on the Compact Disc and on disk in the Sub-series Software Volume.

1.1.2 Scope of the program

The program provides a method of estimating the stress-strain behaviour of a metallic material within the assumptions that this behaviour may be represented by a smooth curve, as shown in Sketch 1.1. The curve is derived using the elastic modulus and two data points in the inelastic region. The curve follows the modulus in the elastic region and in the inelastic region the greatest accuracy is in the region of the two data points used to derive the curve. The shape of the knee of the stress-strain curve is often required and in these cases it is important to select two data points on the knee. However, the program will operate with widely spaced data points with consequent lack of accuracy in the regions remote from them. If more than two data points are available the program can be used to model different sections of the curve separately. It will give the stresses for particular individual strains or the strains for particular stresses at any point on the inelastic stress-strain curve. Alternatively, it will provide a tabulation of strains and their corresponding stresses at a specified number of intervals over the range of the stress-strain curve. For each calculated stress-strain data point the program also provides the inelastic Poisson's ratio, the tangent modulus and the secant modulus.

To define the stress-strain curve the program requires the modulus of elasticity and two parameters related to the stress-strain curve. The latter may be given as (a) two proof stresses and their corresponding permanent strains, (b) the material characteristic m and corresponding reference stress f_n or (c) two stresses and their corresponding total strains. The program will also output whichever of these options have not been put in. The tensile strength or a maximum stress or strain are required to limit the calculation.

The theoretical basis for the generalised stress-strain form used by the program is detailed in Data Item No. 76016*. A summary of the theory and equations used is given here in Section 1.3. Instructions for the use of the program and examples of input and output are given in Section 2 of this Item.

* Data Item No. 76016 "Generalisation of smooth continuous stress-strain curves for metallic materials" in the Structures Sub-series.

1.2 NOTATION

Both SI and British units are quoted but any coherent system of units may be used.

C	identifier for maximum stress or strain		
C_f	computational limit value for stress	N/m ²	lbf/in ²
C_ϵ	computational limit value for total strain		
$C_{p\epsilon}$	computational limit value for permanent strain		
E	modulus of elasticity (which may be different in tension and compression, see Section 1.4)	N/m ²	lbf/in ²
E_s	secant modulus	N/m ²	lbf/in ²
E_t	tangent modulus	N/m ²	lbf/in ²
f	stress*	N/m ²	lbf/in ²
f_n	reference stress at which $E_t = E/2^*$	N/m ²	lbf/in ²
f_t	tensile strength	N/m ²	lbf/in ²
f_A, f_B	stresses provided at any two points on stress-strain curve* ($f_A < f_B$)	N/m ²	lbf/in ²
m	material characteristic		
S	indicator for stress-strain curve input selection		
t	proof stress	N/m ²	lbf/in ²
t_A, t_B	proof stresses ($t_A < t_B$)	N/m ²	lbf/in ²
Z	number of output points required on stress-strain curve		
ϵ	total strain		
ϵ_A, ϵ_B	total strains appropriate to stresses f_A and f_B		
ϵ_p	permanent strain corresponding to proof stress t		
$\epsilon_{pA}, \epsilon_{pB}$	permanent strains corresponding to t_A and t_B respectively		
ν	Poisson's ratio		
ν_e	fully elastic value of Poisson's ratio		

For footnote, see end of notation

ν_p fully plastic value of Poisson's ratio

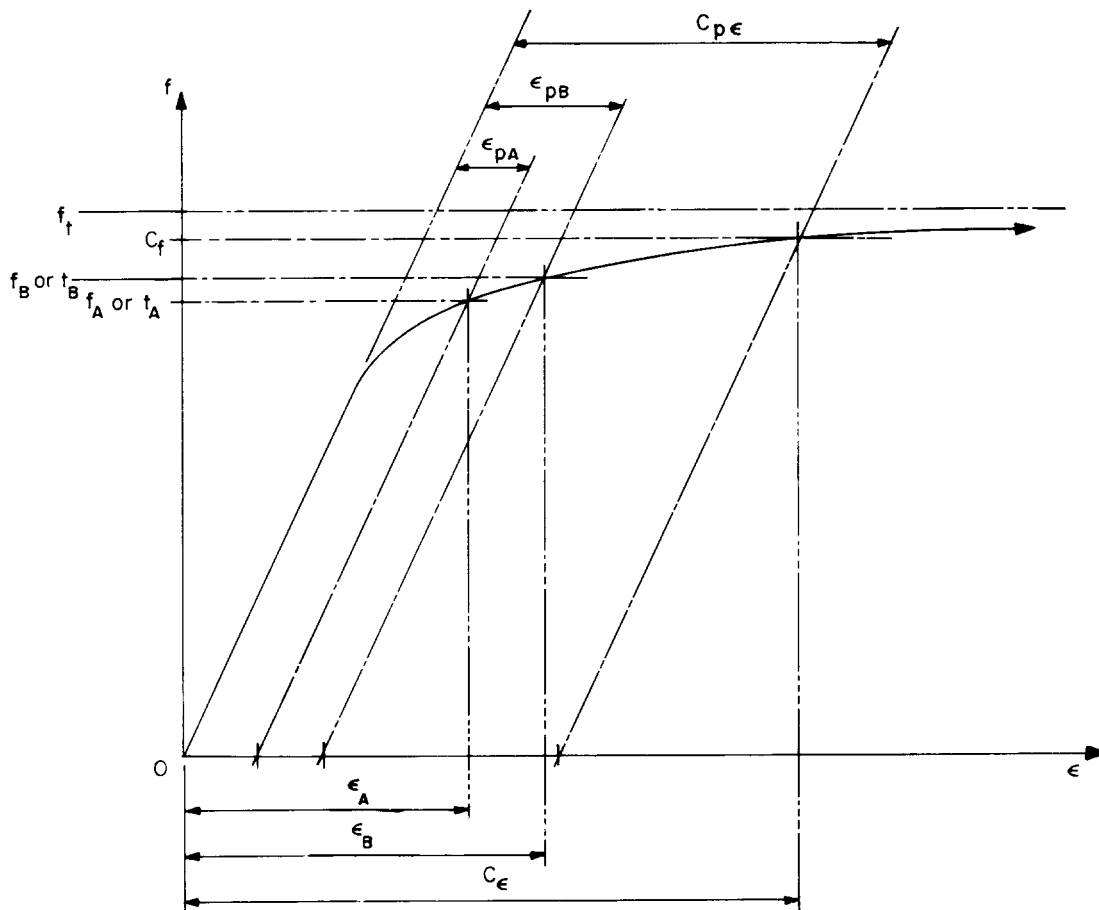
Suffixes

1,2,5 Typically suffixes *A* and *B* take the values 1, 2 and 5 for the 0.1, 0.2 and 0.5 per cent proof values of stresses or strains as applicable but any proof stress values may be used.

Notes

Lower case symbols may feature as upper case symbols in the program output.

* Stresses may be calculated on either a true or nominal basis provided they are used consistently throughout. However, the analysis will not convert from one basis to the other and will not reflect necking behaviour beyond the material nominal stress in a particular case.



Sketch 1.1 Generalised stress-strain curve

1.3 DERIVATION

1.3.1 Sources

The equations used in the program have been taken from Data Item No. 76016. The theory from which they evolved is presented in Derivations 1 and 2. The actual equations used are given in Section 1.3.2 below.

1.3.2 Theory

Derivation 2 shows that a smooth continuous uniaxial stress-strain curve can be represented by the equation

$$\frac{\varepsilon E}{f_n} = \frac{f}{f_n} + \frac{1}{m} \left(\frac{f}{f_n} \right)^m \quad (1.1)$$

which can be given alternatively as

$$\varepsilon = \frac{f}{E} \left[1 + \frac{1}{m} \left(\frac{f}{f_n} \right)^{m-1} \right]. \quad (1.2)$$

Equation (1.1) can be modified to give the tangent modulus in the form

$$E_t = E \left[1 + \left(\frac{f}{f_n} \right)^{m-1} \right]^{-1}. \quad (1.3)$$

The secant modulus is related to the tangent modulus and may be given as

$$E_s = E \left[1 + \frac{1}{m} \left(\frac{f}{f_n} \right)^{m-1} \right]^{-1}. \quad (1.4)$$

The value of Poisson's ratio varies in the inelastic range and is here based on the expression

$$\nu = \nu_p - (E_s/E)(\nu_p - \nu_e), \quad (1.5)$$

which is, alternatively,

$$\nu = \frac{\nu_e + \nu_p \frac{1}{m} \left(\frac{f}{f_n} \right)^{m-1}}{1 + \frac{1}{m} \left(\frac{f}{f_n} \right)^{m-1}}, \quad (1.6)$$

where, on the assumption of constant volume, the value of ν_p is taken to be 0.5.

The value of m , the material characteristic, can be found by standard curve fitting procedures if the full stress-strain curve is known. However, it may be estimated, provided that at least two points on the

non-proportional region of the curve are known, from the equation

$$m = \frac{\log(\epsilon_{pB}/\epsilon_{pA})}{\log(f_B/f_A)} \quad (1.7)$$

For a known value of m the reference stress, f_n , may be evaluated from the equation

$$f_n = f_B \left[\frac{m \epsilon_{pB} E}{f_B} \right]^{-1/(m-1)} \quad (1.8)$$

1.3.3 Derivation

This section lists selected sources that have assisted in the preparation of this Item.

1. RAMBERG, W. Description of stress-strain by three parameters. NACA Tech. Note 902, April 1943.
2. BARRETT, A.J. Generalised stress-strain data for aluminium alloys and certain other materials. *J. R. Aeronaut. Soc.*, Vol. 59, No. 539, pp. 152-158, February 1955.

1.4 APPLICATION OF THE PROGRAM

The program may be applied to provide estimates of the stresses, strains and other properties of any material for which the generalised curve form (see Sketch 1.1) may be considered to be a reasonable approximation. In practice while it can give a good approximation for some metallic materials such as aluminium alloys or alloy steels, it may best be used to provide an estimate over a limited range of the actual stress-strain curve.

Attention is drawn to the fact that compressive stress-strain curves may be significantly different from their tensile counterparts. Accordingly, it is usually necessary to analyse the tensile and compressive regions of the stress-strain curve separately and to use them appropriately.

In Figures 1 and 2 results from the analysis are compared with experimental stress-strain curves for a steel and an aluminium alloy respectively. In each figure the analytical results are presented for curves defined by three different sets of data, first t_1 and t_2 , then t_2 and t_5 , and finally two total strains ϵ_A and ϵ_B and their respective stresses f_A and f_B chosen from the experimental stress-strain curves. It can be seen that the best fit to the experimental curve is to be found in the region of these defined values in each case. The range of the analytical curve beyond the defined points is limited by the value of the tensile strength.

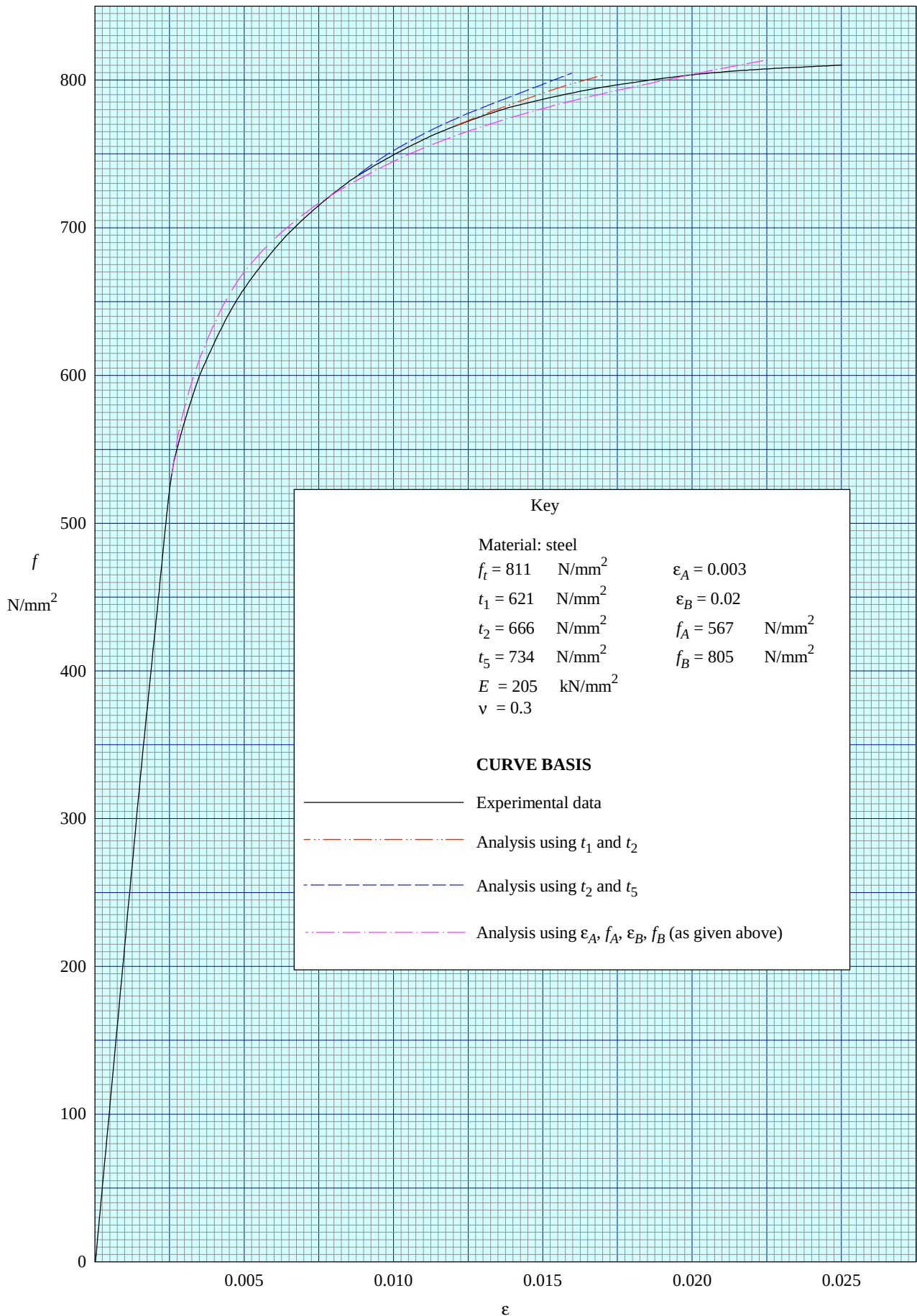


FIGURE 1 COMPARISON OF EXPERIMENTAL STRESS-STRAIN CURVE WITH THREE PROGRAM ANALYSES BASED ON SLIGHTLY DIFFERENT INPUT DATA AS SHOWN IN KEY