

STRUTS: STRENGTH UNDER FLEXURAL, LOCAL AND INTER-RIVET BUCKLING

1. DESCRIPTION OF SOLUTIONS AND THEIR DERIVATION

1.1 Introduction

This Data Item, together with ESDUpac A9002 (see Section 2), provides a Fortran computer program for calculating the buckling strength of uniform section struts subject to the following three modes of buckling.

- a) Failure in overall flexural buckling with an allowance for eccentricity of both the strut section (bow) and the applied load. This is based in part upon Data Item No. Struct 01.01.01 (Derivation 5).
- b) Local buckling of those parts of the strut section that are effectively long flat strips with their long edges either simply-supported or with one edge simply-supported and the other free. This is based upon Data Item No. Struct 01.01.08 (Derivation 2).
- c) Buckling between rivets of fabricated section struts. This is based upon Data Item No. Struct 02.01.08 (Derivation 3).

The strut is assumed to be manufactured from a single material or from materials having the same properties. In all of the above three buckling modes inelastic behaviour of the material is taken into account by using the generalised stress-strain curve equations of Data Item No. 76016 (Derivation 6). The local and inter-rivet buckling calculations of (b) and (c) above may be used to analyse structures other than struts that are subjected to compressive loading, see Derivations 2 and 3.

No account is taken of any interaction between the buckling modes for which solutions are provided. Data Item No. 78021* provides notes on the behaviour of struts together with a worked example. That Item should be consulted for guidance on the interpretation of buckling stresses when estimating the strength of struts.

1.2 Notation

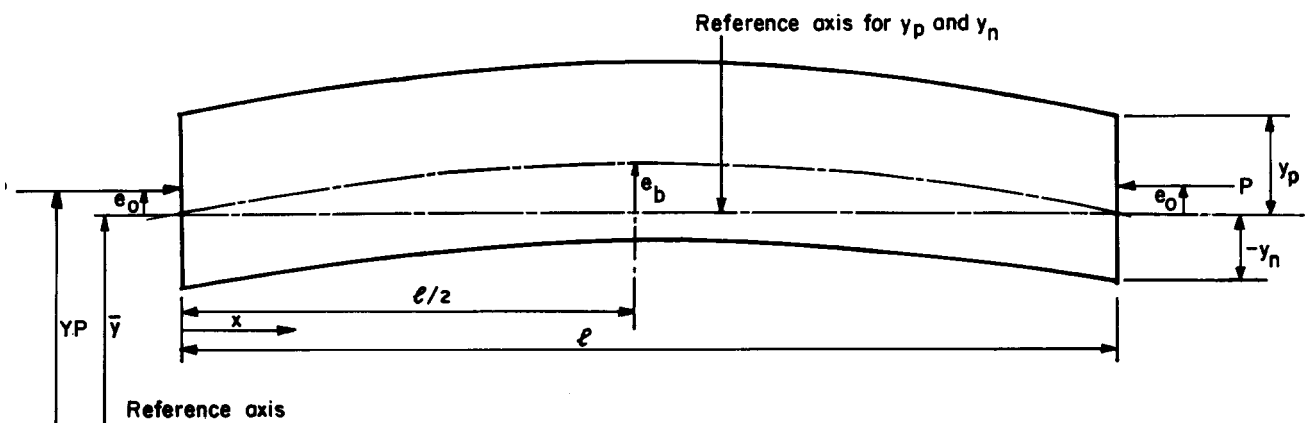
The following notation is identical to that of Derivations 2, 3, 5 and 6 except where the same symbol is used in two of the derivations for different parameters in which case one of the symbols is modified by the addition of a suffix. Both SI and British units are quoted but any coherent system of units may be used.

A	cross-sectional area of strut	m^2	in^2
b	dimension of element of strut cross-section (see Sketch 2.1)	m	in
d	dimension of rectangular element of strut cross section that is at θ to reference axis (see Sketch 2.1)	m	in
E	modulus of elasticity in tension and compression of strut material	N/m^2	lbf/in^2

* Data Item No. 78021 "Guide to Items on the strength and stability of struts".

E_t	tangent modulus in tension and compression of strut material, corresponding to average stress	N/m ²	lbf/in ²
e_b	eccentricity at centre of unloaded strut arising from nominal design bow (see Sketch 1.1)	m	in
e_o	maximum offset of line of loading from strut neutral axis at both ends of strut, $(YP - \bar{y})$ (see Sketch 1.1)	m	in
e_t	effective eccentricity at centre of unloaded strut resulting from manufacturing tolerances	m	in
f	stress	N/m ²	lbf/in ²
f_{all}	allowable stress in extreme fibres (compressive positive)	N/m ²	lbf/in ²
f_b	average stress at which local instability occurs (identical to f of Derivation 2)	N/m ²	lbf/in ²
f_e	edge stress at which sheet buckles between rivets	N/m ²	lbf/in ²
f_{fx}	average stress in strut for failure at point x distant from one end of the strut, P/A	N/m ²	lbf/in ²
f_n	stress at which $E_t = \frac{1}{2}E$ (see Derivation 6)	N/m ²	lbf/in ²
h	effective width (used for local buckling, see Section 1.3.3)	m	in
K	local buckling stress coefficient		
K_r	fixity coefficient at rivets (identical to K of Derivation 4)		
k	radius of gyration of section	m	in
l	actual length of strut	m	in
l'	equivalent length of strut	m	in
l_r	rivet pitch (identical to l of Derivation 3)	m	in
M_x	bending moment at point x along strut	N m	lbf in
m	material characteristic (see Derivation 6)		
P	failing end load	N	lbf
r	outer radius of curved element (see Sketch 2.1)	m	in
t	thickness (used for local and inter-rivet buckling, see Sections 1.3.3 and 1.3.4)	m	in
x	distance from one end of strut (see Sketch 1.1)	m	in

YC	distance to centre of curved element from reference axis (see Sketch 2.1)	m	in
YP	distance to load line from reference axis (see Sketch 1.1)	m	in
YR	distance to centroid of rectangular element from reference axis (see Sketch 2.1)	m	in
y	distance to fibre from strut neutral axis	m	in
$\bar{y}$	distance from reference axis to neutral axis of strut section at ends of strut (see Sketch 1.1)	m	in
y_p, y_n	distance from neutral axis of section to extreme positive and negative fibres (see Sketch 1.1)	m	in
β	angle subtended by curved element (see Sketch 2.1)	degree	deg
ν_e	fully elastic value of Poisson's ratio		
θ	angle of side of width d of rectangular element, with respect to reference axis (see Sketch 2.1)	degree	degree
ϕ	angle of edge of curved element with respect to reference axis (see Sketch 2.1)	degree	degree



Sketch 1.1 Showing sign convention. Sketch shows positive values of $\bar{y}$, YP , e_b and e_o (e_t not shown) and positive y_p and negative y_n

1.3 Theory and Assumptions

The solutions provided by the program are those set out in Section 1.3.2 and in Derivations 2, 3, 5 and 6. The derivations provide design curves for a range of geometries and materials and provide information on the basic theory and its limitations. The following sections set out the equations solved and draw attention to major aspects of the theory.

1.3.1 Section constants

Where the values of A , k and y_p and y_n have previously been calculated the program will accept these directly. Where they are not available the section constants can be calculated by the program. The program deals with rectangular elements and sectors of hollow or solid circle elements which can be combined to form the strut cross-section. The program requires the dimensions of each element and its location with respect to a single convenient reference axis parallel to the axis of flexure of the strut in overall flexural buckling. The calculation first estimates a new reference axis close to the centre of the section to minimise computational errors. The local second moment of area of each element is fully calculated so that elements of any geometric proportions can be used, that is, thin sheet and solid body elements (for example, bars, strips, circular rods or tubes). Where an element is inclined with respect to the axis of flexure, the local second moment of area is corrected to be parallel to the reference axis. By using negative values of dimension b for either rectangular or solid sector elements, holes or cut-outs in the section can easily be defined.

1.3.2 Overall flexural buckling

The program provides the minimum end load, P , that will give an extreme fibre stress of f_{all} in the strut section as a consequence of the direct load and bending resulting from eccentricity. The eccentricity can be composed of design bow, loading offset and/or manufacturing tolerances which can act together or in opposition (see Sketch 1.1). The solution is based upon engineer's theory of bending and results in the following equations. Note that these reduce to the equation given in Derivation 5 when $(e_b \pm e_t) = 0$.

$$f_{all} = f_{fx} \left\{ 1 + \frac{y}{k^2} \frac{M_x}{P} \right\}, \quad (1.1)$$

$$\text{where} \quad \frac{M_x}{P} = e_o \left\{ \frac{(1 - \cos \mu l') \sin \mu x}{\sin \mu l'} + \cos \mu x \right\} - \frac{(e_b \pm e_t) \sin(\pi x / l')}{1 - (\mu l' / \pi)^2} \quad (1.2)$$

$$\text{and in which} \quad \mu l' = \frac{l'}{k} \left(\frac{f_{fx}}{E_t} \right)^{1/2}. \quad (1.3)$$

The program provides the minimum value of $f_{fx} (= P/A)$ that satisfies the above equations and E_t is the value that applies at f_{fx} . To achieve this, the program searches through the full range of x and explores the extreme fibres at both $y = y_p$ and $y = y_n$ with f_{all} as a compressive or tensile allowable stress. The search is carried out with e_t added to and then subtracted from e_b to ensure that the limits of the possible range of eccentricities are explored.

Equations (1.1) to (1.3) refer only to flexural failure in which, on flexure, the cross-section translates normal to the axis to which k applies. The equations relate strictly to struts with both ends pinned. They may be used for struts with other end conditions by using the equivalent length l' . The equivalent lengths of a strut for simple conditions of restraint are given in Table 1.1, which is taken from Derivation 5. In the particular case of pinned/fixed ends use of the value $l' = 0.7l$ from Table 1.1 can give a conservative value of f_{fx} , that is, a lower value of f_{fx} than would be expected to occur.

TABLE 1.1

End conditions	Pinned	Pinned	Fixed	Fixed	Fixed
	Pinned	Fixed	Fixed	Free	Fixed in direction, free in translation
Equivalent length l'	$1.0 \times l$	$0.7 \times l^*$	$0.5 \times l$	$2.0 \times l$	$1.0 \times l$

* See notes above for this case.

The values of e_o , e_b and e_t in Equation (1.2) relate to the direction of flexural buckling defined by k . Their values should be the largest likely to arise in service. Either the bow or the offset of loading may be a feature of the design. The offset of the loading used must include the effect of manufacturing tolerances. The effect of manufacturing tolerances on bow are accounted for by e_t . Where a value for e_t is not known it can be estimated as the greater of $0.05k^2/(y_p|y_n|)^{1/2}$ and $0.001l$ as recommended in Derivation 5. These values of e_t will be computed by the program if required. Sketch 1.1 shows positive values of e_b , $\bar{y}$ and YP . The distances from the neutral axis to the extreme fibres are also shown. Note that y is taken positive in the same direction as the eccentricities but with respect to the neutral axis. Thus, as shown in Sketch 1.1, y_p is positive and y_n negative.

Equations (1.1) to (1.3) are based upon the assumption that the initial, unloaded, form of the strut is sinusoidal with $(e_b \pm e_t)$ applying at the centre. Other similar unloaded forms, for example a parabolic form, do not significantly influence the value of M_x . The equations are derived for conditions in which the slope of the loaded strut with respect to the reference axis is at an angle of less than about 15° .

If e_b , e_t and e_o are negligibly small then Equation (1.1) reduces to $f_{all} = f_{fx}$. In these circumstances the Euler buckling load in which E_t is used in place of E can be critical and if so this is output by the program. Also, if $\mu l' \rightarrow \pi$ the denominators of Equation (1.2) approach zero. In cases where $\mu l'$ lies between 3.1415 and π the program provides the Euler buckling stress (in which E is replaced by E_t).

1.3.3 Local buckling (Derivation 2)

The program provides the value of f_b in the following equation in which E_t is the value that applies at stress f_b .

$$f_b = \frac{K\pi^2 E_t}{3(1 - \nu_e^2)} \left(\frac{t}{h}\right)^2, \quad (1.4)$$

where $K = 0.16$ for strips with one edge simply-supported and the other edge free and $K = 1.0$ for strips with both edges simply-supported. Note that Equation (1.4) does not take account of interaction between attached strips which may increase the local buckling stress above the value obtained from the equation (see Section 2.6.2). If account of interaction is required reference should be made to Derivation 1.

For the majority of practical strut sections where local buckling can occur it is necessary to estimate an effective value of h for each strip. This should take account of the location of the support, or supports, along the edge, or edges, restraining movement of the strip out of its plane. Examples 1 and 2 of Section 2.6 illustrate a simple assumption often made when estimating h for sections formed from thin sheet. This is to assume h to be the dimension between the centre lines of the supporting strips or, when a free edge exists, the dimension between the strip edge and the centre line of the supporting strip. In open sections a lip is often provided on a web in order to raise K for the web from 0.16 to 1.0, so increasing its buckling stress. This normally requires a lip of width of at least five times the web thickness.

Equation (1.4) is derived using thin plate theory. For strips with a free edge with h less than about $4t$ or for strips simply supported on both sides with h less than about $10t$ the theory is not strictly valid and Equation (1.4) can give values of f_b that are in excess of the allowable fully stable compressive stress. In such cases the compressive strength of the strip is limited by the allowable stress and not by buckling.

The local buckling stresses of curved elements forming corners of strut cross sections are usually much higher than those of flat elements and are therefore usually limited by the allowable stable compressive stress. For circular tubes Data Item No. 83034* can be used to estimate local buckling.

* Data Item No. 83034 "Elastic local buckling stresses of thin-walled unstiffened circular cylinders under combined axial compression and internal pressure".

1.3.4 Inter-rivet buckling (Derivation 3)

The program provides the value of f_e in the following equation in which E_t is the value that applies at stress f_e .

$$f_e = \frac{K_r E_t \pi^2 t^2}{12 l_r^2} \quad (1.5)$$

The fixity coefficient at the rivets, K_r , depends on methods of fabrication and type and size of rivets or other attachments. However, limited test data indicate that, for conventional methods of attachment, values similar to those given in Table 1.2, which is taken from Derivation 3, are appropriate.

TABLE 1.2

Type of attachment	Fixity coefficient at rivets, K_r
Flathead rivets	4
Spotwelds	3½
Mushroom or snaphead rivets	3
Countersunk or dimpled rivets	1½

The local buckling wavelength of the riveted sections is assumed to be significantly different from the rivet pitch. Equation (1.5) takes no account of the restraints to inter-rivet buckling that may be provided by flanges or webs that run close to the rivet line. Derivation 4 shows how a set of design curves for a specific material and rivet type may be constructed.

1.3.5 Inelastic behaviour of the strut material (Derivation 6)

Equations (1.1) to (1.5) require a formulation for E_t . This is provided in Derivation 6 as

$$E_t = E \left\{ 1 + \left(\frac{f}{f_n} \right)^{m-1} \right\}^{-1}, \quad (1.6)$$

where f is the average stress under examination. Derivation 6 describes how f_n and m may be calculated from a pair of stresses and strains. If values of f_n and m are known they may be input directly into the program. Otherwise, it is necessary to input a pair of proof stresses and strains and the value of f_n and m are calculated by the program. Section 4 of Derivation 6 provides notes on the use of the material characteristic and reference stress in stressing procedures.

1.4 Derivation

1. ESDU Information on the use of Item No. 73007. Data Item No. Struct 02.01.28, ESDU International plc, London, Issue 1, June 1954.
2. ESDU Local instability of struts with flat sides. Item No. Struct 01.01.08, ESDU International plc, London, Issue 2, October 1962.
3. ESDU Buckling in compression of sheet between rivets. Data Item No. Struct 02.01.08, ESDU International plc, London, Issue 1, October 1962.

4. ESDU Inter-rivet buckling curves for specific materials. Data Item No. Struct 02.01.09, ESDU International plc, London, Issue 1, October 1962.
5. ESDU The strength of struts. Data Item No. Struct 01.01.01, ESDU International plc, London, Issue 3, November 1963.
6. ESDU Generalisation of smooth continuous stress-strain curves for metallic materials. Data Item No. 76016, ESDU International plc, London, Issue 1, August 1976.
7. BENNETT, J.K. Private communication. British Aerospace (Space Systems) Limited, July 1990.

1.5 Reference

8. CLARK, J.W. Eccentrically loaded aluminum columns. American Society of Civil Engineers, Proceedings Separate No. 299, October 1953.

2. ESDUpac A9002

2.1 Introduction

ESDUpac A9002 contains two Fortran computer programs, A9002A and A9002B, for use in calculating strut strength and buckling. The required calculations are carried out in program A9002B. This program accesses a data file containing all the necessary geometry and materials data. Program A9002A is used to build the data file. It is an interactive program that offers prompts for the required data in the correct order. The data file name is selected during this process. Each set of data in the data file is preceded by a statement describing it. Thus, editing the data file to alter some dimensions or a material property *etc*, is simple since each number is clearly identified. Program A9002B ignores all descriptions set out in the data file and uses only the numerical values provided.

The programs are available as Fortran source code with example input and output files on the Compact Disc and on disk in the Sub-series Software Volume.

2.2 Program Limitations

2.2.1 Program A9002A

Since the values input into this program are simply stored on file there are no numerical limits involved. However, in all cases where a numerical value is requested a number must be returned. All numbers are accepted in free format and they may be separated by either commas or spaces. Where options are offered and a y, indicating yes, or n, indicating no, is requested, either capital or lower case letters may be used. Eighty characters are provided for the name of the data file so that any number of characters from 1 to 80 may be used with the program. In practice most operating systems impose a limit on the number of characters for a file name which is well below 80 and characters above the operating system limit are usually ignored.

2.2.2 Program A9002B

Program A9002B can normally only be run using the special data file constructed by Program A9002A. This is because the data file is annotated to allow easy editing and Program A9002B is arranged so as to ignore all annotations in the data file (see Section 2.4 for running the program for a non-annotated file).

Only rectangular elements or sectors of hollow circle elements may be used to define the strut cross section and these are limited to a total of 50 elements of each type.

The number of local buckling conditions that can be examined in a run is limited to 20.

The number of inter-rivet buckling conditions that can be examined in a run is limited to 10.

The material characteristic m must be greater than 1.0 and positive non-zero values of material properties must be supplied for all but Poisson's ratio which can be zero. The value of the fully elastic Poisson's ratio input only influences the local buckling calculations.