

Struts: strength under flexural, local and inter-rivet buckling

Associated software: ESDUpac A9002

Endorsed by
The Institution of Mechanical Engineers

ESDU DATA ITEMS

Data Items provide validated information in engineering design and analysis for use by, or under the supervision of, professionally qualified engineers. The data are founded on an evaluation of all the relevant information, both published and unpublished, and are invariably supported by original work of ESDU staff engineers or consultants. The whole process is subject to independent review for which crucial support is provided by industrial companies, government research laboratories, universities and others from around the world through the participation of some of their leading experts on ESDU Technical Committees. This process ensures that the results of much valuable work (theoretical, experimental and operational), which may not be widely available or in a readily usable form, can be communicated concisely and accurately to the engineering community.

We are constantly striving to develop new work and review data already issued. Any comments arising out of your use of our data, or any suggestions for new topics or information that might lead to improvements, will help us to provide a better service.

THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item was monitored and guided by the Aerospace Structures Committee. This Committee first met in 1940 and now has the following membership:

Chairman	
Mr K. Fitzsimons	— Westland Helicopters Ltd
Vice-Chairman	—
Mr J.K. Bennett	— Independent
Members	
Prof. P. Bartholomew	— QinetiQ, Farnborough
Mr A. Dickson	— BAE SYSTEMS, Warton
Mr P.J. Mitchelmore	— Independent
Mr M.S. Pressnell	— Independent
Mr B. Popham	— MMS Space Systems Ltd
Prof. A. Rothwell	— Technische Universiteit Delft, The Netherlands
Mr P. Stocking	— Cranfield Aerospace Ltd
Mr J.H. van der Sloot	— Independent
Mr K. van Katwijk*	— European Space Agency, Noordwijk, The Netherlands.

* Corresponding Member

The construction and subsequent development of the Data Item was undertaken, under contract to ESDU, by

Mr J.H. van der Sloot.

The person with overall responsibility for the work in this subject area is Mr A.C. Quilter, Head of Strength Analysis Group.

STRUTS: STRENGTH UNDER FLEXURAL, LOCAL AND INTER-RIVET BUCKLING

CONTENTS

	Page
1. DESCRIPTION OF SOLUTIONS AND THEIR DERIVATION	1
1.1 Introduction	1
1.2 Notation	1
1.3 Theory and Assumptions	5
1.3.1 Section constants	5
1.3.2 Overall flexural failure	5
1.3.3 Local buckling (Derivation 9)	7
1.3.4 Inter-rivet buckling (Derivation 4)	8
1.3.5 Inelastic behaviour of the strut material (Derivation 7)	8
1.4 Derivation	9
1.5 Reference	9
2. ESDUPAC A9002	10
2.1 Introduction	10
2.2 Program Limitations	10
2.2.1 Program A9002A	10
2.2.2 Program A9002B	10
2.3 Data Input, Program A9002A	11
2.4 Input for Program A9002B	12
2.5 Program A9002B Output and Interpretation	13
2.5.1 Strut material properties	13
2.5.2 Strut geometry	13
2.5.3 Buckling stresses	13
2.5.4 Messages	13
2.5.4.1 Error messages	13
2.5.4.2 Warnings	15
2.5.5 Output with some inputs set to zero	15
2.6 EXAMPLES	16
2.6.1 Example 1	16
2.6.1.1 Screen prompts of program A9002A and responses for Example 1	18
2.6.1.2 Data file generated by program A9002A for Example 1	20
2.6.1.3 Output for Example 1	21
2.6.2 Example 2	23
2.6.2.1 Data file generated by program A9002A for Example 2	24
2.6.2.2 Output for Example 2	25

3.	COMPUTER PROGRAM	27
3.1	Introduction	27
3.2	Program Application	27
3.3	Description of Main Features of Algorithm	27
3.4	Program Testing	28

STRUTS: STRENGTH UNDER FLEXURAL, LOCAL AND INTER-RIVET BUCKLING

1. DESCRIPTION OF SOLUTIONS AND THEIR DERIVATION

1.1 Introduction

This Data Item, together with ESDUpac A9002 (see Section 2), provides a Fortran computer program for calculating the buckling strength of uniform section struts subject to the following three modes of buckling.

- (a) Failure in overall flexural buckling for struts of symmetric cross-section with an allowance for eccentricity of both the strut section (bow) and the applied load. This is based in part upon Data Item No. Struct 01.01.01 (Derivation 1).
- (b) Local buckling of those parts of the strut section that are effectively long flat strips with their long edges either simply-supported or with one edge simply-supported and the other free. This is based upon Appendix F of Derivation 9.
- (c) Buckling between rivets of fabricated section struts. This is based upon Data Item No. Struct 02.01.08 (Derivation 4).

The strut is assumed to be manufactured from a single material or from materials having the same properties. In all of the above three buckling modes inelastic behaviour of the material is taken into account by using the generalised stress-strain curve equations of Data Item No. 76016 (Derivation 7). The local and inter-rivet buckling calculations of (b) and (c) above may be used to analyse other structures that are subjected to compressive loading, see Derivations 3 and 4.

No account is taken of any interaction between the buckling modes for which solutions are provided. Data Item No. 78021* provides notes on the behaviour of struts together with a worked example. That Item should be consulted for guidance on the interpretation of buckling stresses when estimating the strength of struts.

1.2 Notation

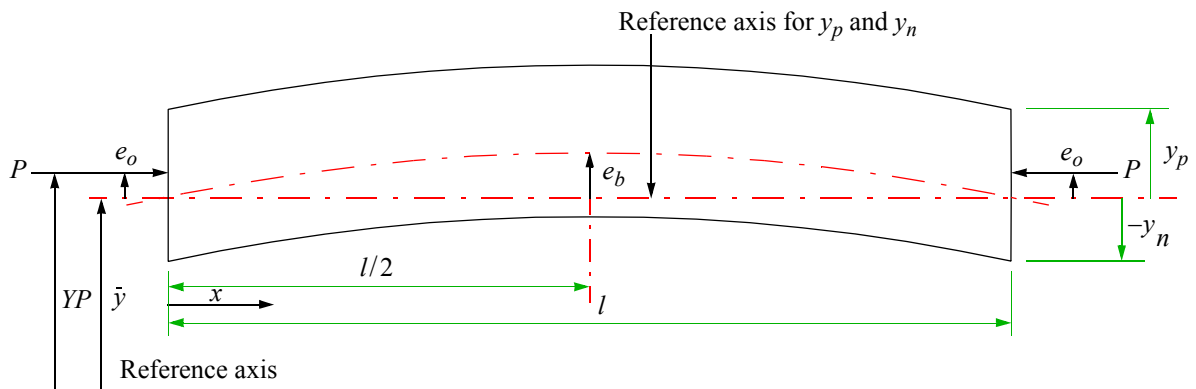
The following notation is identical to that of Derivations 3, 4, 5 and 7 except where the same symbol is used in two of the derivations for different parameters in which case one of the symbols is modified by the addition of a suffix. Both SI and British units are quoted but any coherent system of units may be used.

A	cross-sectional area of strut	m^2	in^2
b	dimension of element of strut cross-section (see Sketch 2.1)	m	in
d	dimension of rectangular element of strut cross-section that is at θ to reference axis (see Sketch 2.1)	m	in

* Data Item No. 78021 "Guide to Items on the strength and stability of struts".

E	modulus of elasticity in tension and compression of strut material	N/m ²	lbf/in ²	
E_s	secant modulus in tension and compression of strut material, corresponding to average stress	N/m ²	lbf/in ²	
E_t	tangent modulus in tension and compression of strut material, corresponding to average stress	N/m ²	lbf/in ²	
e_b	eccentricity at centre of unloaded strut arising from nominal design bow (see Sketch 1.1)	m	in	
e_o	maximum offset of line of loading from strut neutral axis at both ends of strut, $(YP - \bar{y})$ (see Sketch 1.1)	m	in	
e_t	effective eccentricity at centre of unloaded strut resulting from manufacturing tolerances	m	in	
f	stress	N/m ²	lbf/in ²	
f_{all}	allowable stress in extreme fibres (compressive positive)	N/m ²	lbf/in ²	
f_b	average stress at which local instability occurs (identical to f of Derivation 3)	N/m ²	lbf/in ²	
f_e	edge stress at which sheet buckles between rivets	N/m ²	lbf/in ²	
f_{fx}	average stress in strut for failure at point x distant from one end of the strut, P/A	N/m ²	lbf/in ²	
f_n	stress at which $E_t = \frac{1}{2}E$ (see Derivation 7)	N/m ²	lbf/in ²	
f_x	stress in x -direction	N/m ²	lbf/in ²	
f_y	stress in y -direction	N/m ²	lbf/in ²	
h	effective width (used for local buckling, see Section 1.3.3)	m	in	
K	local buckling stress coefficient			
K_r	fixity coefficient at rivets (identical to K of Derivation 5)			
k	radius of gyration of section	m	in	
l	actual length of strut	m	in	
l'	equivalent length of strut	m	in	
l_r	rivet pitch (identical to l of Derivation 4)	m	in	
L_{ij}	elasto-plastic moduli, see Equations (1.4a) and (1.4b)	N/m ²	lbf/in ²	
M_x	bending moment at point x along strut	N m	lbf in	
m	material characteristic (see Derivation 7)			

P	end load for failure	N	lbf
r	outer radius of curved element (see Sketch 2.1)	m	in
t	thickness (used for local and inter-rivet buckling, see Sections 1.3.3 and 1.3.4)	m	in
x	distance from one end of strut (see Sketch 1.1)	m	in
YC	distance to centre of curved element from reference axis (see Sketch 2.1)	m	in
YP	distance to load line from reference axis (see Sketch 1.1)	m	in
YR	distance to centroid of rectangular element from reference axis (see Sketch 2.1)	m	in
y	distance to fibre from strut neutral axis	m	in
$\bar{y}$	distance from reference axis to neutral axis of strut section at ends of strut (see Sketch 1.1)	m	in
y_p, y_n	distance from neutral axis of section to extreme positive and negative fibres (see Sketch 1.1)	m	in
β	angle subtended by curved element (see Sketch 2.1)	degrees	degrees
γ_{xy}	shear strain		
ϵ_x	strain in x -direction		
ϵ_y	strain in y -direction		
θ	angle of side of dimension d of rectangular element, with respect to reference axis (see Sketch 2.1)	degrees	degrees
ν	Poisson's ratio		
ν_e	fully elastic value of Poisson's ratio		
ν_p	fully plastic value of Poisson's ratio (see Derivation 7)		
τ_{xy}	shear stress	N/m ²	lbf/in ²
ϕ	angle of edge of curved element with respect to reference axis (see Sketch 2.1)	degrees	degrees



Sketch 1.1 Showing sign convention. Sketch shows positive values of $\bar{y}$, YP , e_b and e_o (e_t not shown) and positive y_p and negative y_n

1.3 Theory and Assumptions

The solutions provided by the program are those set out in Section 1.3.2 and in Derivations 3, 4, 1, 7 and 9. The derivations provide design curves for a range of geometries and materials and provide information on the basic theory and its limitations. The following sections set out the equations solved and draw attention to major aspects of the theory.

1.3.1 Section constants

If the values of A , k and y_p and y_n are available, they can be input into the program directly. If they are not available the section constants can be calculated by the program. The program deals with rectangular elements and sectors of hollow or solid circle elements which can be combined to form the strut cross-section. The program requires the dimensions of each element and its location with respect to a single convenient reference axis parallel to the axis of flexure of the strut in overall flexural buckling. The calculation first estimates the position of a new reference axis close to the centre of the section to minimise computational errors. The local second moment of area of each element is fully calculated so that elements of any geometric proportions can be used, that is, thin sheet and solid body elements (for example, bars, strips, circular rods or tubes). Where an element is inclined with respect to the axis of flexure, the local second moment of area is corrected to be parallel to the reference axis. By using negative values of dimension b for either rectangular or solid sector elements, holes or cut-outs in the section can easily be defined.

1.3.2 Overall flexural failure

The program provides the minimum end load, P , that will give an extreme fibre stress of f_{all} in the strut section as a consequence of the direct load and bending resulting from eccentricity. The eccentricity can be composed of design bow, loading offset and/or manufacturing tolerances which can act together or in opposition (see Sketch 1.1). The solution makes use of a beam-column analysis based upon engineer's theory of bending for struts of symmetric cross-section which results in the following equations. Note that these reduce to the equation given in Derivation 1 when $(e_b \pm e_t) = 0$.

$$f_{all} = f_{fx} \left\{ 1 + \frac{y}{k^2} \frac{M_x}{P} \right\}, \quad (1.1)$$

where

$$\frac{M_x}{P} = e_o \left\{ \frac{(1 - \cos \mu l') \sin \mu x}{\sin \mu l'} + \cos \mu x \right\} - \frac{(e_b \pm e_t) \sin \left(\frac{\pi x}{l'} \right)}{1 - \left(\frac{\mu l'}{\pi} \right)^2} \quad (1.2)$$

and in which

$$\mu l' = \frac{l'}{k} \left(\frac{f_{fx}}{E_t} \right)^{1/2}. \quad (1.3)$$

The program provides the minimum value of $f_{fx} (= P/A)$ that satisfies the above equations and E_t is the value that applies at f_{fx} . To achieve this, the program computes the stresses at the end of the strut, $x = 0$, and at the mid-span of the strut, $x = l'/2$, and explores the stresses in the extreme fibres at both $y = y_p$ and $y = y_n$ with f_{all} as a compressive or tensile allowable stress. The search is carried out with e_t added to and then subtracted from e_b to ensure that the limits of the possible range of eccentricities are explored.

Equations (1.1) to (1.3) refer only to flexural failure in which, on flexure, the cross-section translates normal to the axis to which k applies. The equations relate strictly to struts with both ends pinned and loaded symmetrically with respect to the mid-span. In order to estimate the elasto-plastic instability loads for struts

with other end conditions, the concept of the equivalent length, l' , is often used. The equivalent lengths of a strut for simple conditions of restraint are given in Table 1.1, which is taken from Derivation 1. Caution should therefore be exercised when employing the same concept in Equations (1.1) to (1.3) (and consequently in the program accompanying this Item) to estimate the maximum average compressive stress for failure in bending due to an allowable stress being exceeded. The reason for this is that the moment distributions are likely to differ considerably and, hence, so also will the extreme fibre stresses upon which the calculation of f_{fx} is based.

TABLE 1.1 Equivalent Lengths of Struts with Various End Conditions

End conditions	Pinned	Pinned	Fixed	Fixed	Fixed
	Pinned	Fixed	Fixed	Free	Fixed in rotation, free in translation
Equivalent length l'	$1.0 \times l$	$0.7 \times l$	$0.5 \times l$	$2.0 \times l$	$1.0 \times l$

The values of e_o , e_b and e_t in Equation (1.2) relate to the direction of flexural buckling defined by k . Their values should be the largest likely to arise in service. Either the bow or the offset of loading may be a feature of the design. The offset of the loading used must include the effect of manufacturing tolerances. The effect of manufacturing tolerances on bow are accounted for by e_t . Where a value for e_t is not known it can be estimated as the greater of $0.05k^2/(y_p|y_n|)^{1/2}$ and $0.001l$ as recommended in Derivation 1. These values of e_t will be computed by the program if required. Sketch 1.1 shows positive values of e_b , $\bar{y}$ and YP . The distances from the neutral axis to the extreme fibres are also shown. Note that y is taken positive in the same direction as the eccentricities but with respect to the neutral axis. Thus, as shown in Sketch 1.1, y_p is positive and y_n negative.

Equations (1.1) to (1.3) are based upon the assumption that the initial, unloaded, form of the strut is sinusoidal with $(e_b \pm e_t)$ applying at the centre. Other similar unloaded forms, for example a parabolic form, do not significantly influence the value of M_x . The equations are derived for conditions where the maximum slope of the loaded strut with respect to the reference axis is less than about 15° . The equations also show that the final rotation depends directly on the given values for e_o , e_b and e_t . When they amount to more than eight per cent of the effective length of the strut the user is warned in the output from the program and should interpret the results accordingly.

Equation (1.3) shows that plasticity is accounted for through the use of E_t . It is important to realise that this approach is a fairly crude approximation that does not account for the influence of loading history and the effects of a varying moment distribution and local stiffness reduction in computing average and extreme fibre stresses. In cases when the tangent modulus, E_t , becomes appreciably smaller than its elastic value, E , the 15° assumption is easily violated. In order to alert the user to such circumstances, a warning is issued when the program detects a neutral axis rotation of more than 0.25 radians (14.3°) and the user should be extremely careful in utilising the results as design allowables. Since no loading history is considered, the rotation could become very large. When a message warning of excessive neutral axis rotation is issued, it is recommended that the problem be re-run with either a modified cross-section or with a lower allowable extreme fibre stress, f_{all} , until no further such warnings are given.

If e_b , e_t and e_o are negligibly small then Equation (1.1) reduces to $f_{all} = f_{fx}$. In these circumstances the Euler buckling load in which E_t is used in place of E can be critical and if so this is output by the program. Also, if $\mu l' \rightarrow \pi$ the denominators of Equation (1.2) approach zero. In cases where $\mu l'$ lies between 3.1415 and π the program provides the Euler buckling stress (in which E is replaced by E_t).

1.3.3 Local buckling (Derivation 9)

The program provides the value of f_b in the following equation

$$f_b = \frac{\pi^2}{12} \left(\frac{t}{h} \right)^2 2 \left(\sqrt{L_{11} L_{22}} + (L_{12} + 2L_{33}) \right), \quad (1.4a)$$

in the case of a simply-supported plate, and the value given by

$$f_b = \frac{1}{12} \left(\frac{t}{h} \right)^2 \left(\pi^2 L_{11} \left(\frac{h}{l} \right)^2 + 12L_{33} \right), \quad (1.4b)$$

in the case of a flange with one edge simply supported and the other free.

In these relations, the constants L_{ij} are the elasto-plastic moduli as they appear in the stress-strain relation for the plate.

$$\begin{bmatrix} f_x \\ f_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} & 0 \\ L_{21} & L_{22} & 0 \\ 0 & 0 & L_{33} \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix}.$$

The constants L_{ij} are complicated functions of the material properties E_t and E_s at the stress under consideration and Poisson's ratio, ν , and are taken from Derivation 6 where the complete expressions for the constants can be found. The derivation of Equations (1.4a) and (1.4b) is presented in Derivation 9. Note that Equations (1.4a) and (1.4b) do not take account of interaction between attached strips which may increase the local buckling stress above the value obtained from the equations (see Section 2.6.2). If account of interaction is required reference should be made to Derivation 2 and Data Items Nos 98016* and 01001† which may be employed to perform the proper calculations.

For the majority of practical strut sections where local buckling can occur it is necessary to estimate an effective value of h for each strip. This should take account of the location of the support, or supports, along the edge, or edges, restraining movement of the strip out of its plane. Examples 1 and 2 of Section 2.6 illustrate a simple assumption often made when estimating h for sections formed from thin sheet. This is to assume h to be the dimension between the centre lines of the supporting strips or, when a free edge exists, the dimension between the strip edge and the centre line of the supporting strip. In open sections a lip is often provided on a web in order to raise K for the web from 0.16 to 1.0, so increasing its buckling stress. This normally requires a lip of width of at least five times the web thickness.

Equations (1.4a) and (1.4b) are derived using thin plate theory with approximate, but admissible, deflection functions for the buckling mode. For strips with a free edge with h less than about $4t$ or for strips simply supported on both sides with h less than about $10t$ the theory is not strictly valid and Equations (1.4a) and (1.4b) can give values of f_b that are in excess of the allowable fully stable compressive stress. In such cases the compressive strength of the strip is limited by the allowable stress and not by buckling.

* Data Item No. 98016 "Elastic buckling of flat isotropic stiffened panels and struts in compression".

† Data Item No. 01001 "Elasto-plastic buckling of flat isotropic stiffened panels and struts in compression".