



# ESDU 90005

Issued April 1990  
With Amendments A and B  
July 1991

## Frequencies of resonance in wind tunnels with ventilated walls and plenum chamber

ESDU Copyright material. For current status contact ESDU.

Endorsed by  
The Royal Aeronautical Society

**AVA** Documents  
Document Sharing Hub

## ESDU DATA ITEMS

Data Items provide validated information in engineering design and analysis for use by, or under the supervision of, professionally qualified engineers. The data are founded on an evaluation of all the relevant information, both published and unpublished, and are invariably supported by original work of ESDU staff engineers or consultants. The whole process is subject to independent review for which crucial support is provided by industrial companies, government research laboratories, universities and others from around the world through the participation of some of their leading experts on ESDU Technical Committees. This process ensures that the results of much valuable work (theoretical, experimental and operational), which may not be widely available or in a readily usable form, can be communicated concisely and accurately to the engineering community.

We are constantly striving to develop new work and review data already issued. Any comments arising out of your use of our data, or any suggestions for new topics or information that might lead to improvements, will help us to provide a better service.

## THE PREPARATION OF THIS DATA ITEM

The work on this particular Item was monitored and guided by the Aerodynamics Committee which first met in 1942 and now has the following membership:

|                    |                                                         |
|--------------------|---------------------------------------------------------|
| Chairman           |                                                         |
| Mr H.C. Garner     | — Independent                                           |
| Vice Chairman      |                                                         |
| Mr P.K. Jones      | — British Aerospace (Commercial Aircraft) Ltd, Woodford |
| Members            |                                                         |
| Mr G.E. Bean*      | — Boeing Aerospace Company, Seattle, Wash., USA         |
| Dr N.T. Birch      | — Rolls-Royce plc, Derby                                |
| Mr K. Burgin       | — Southampton University                                |
| Dr T.J. Cummings   | — Short Brothers plc                                    |
| Mr J.R.J. Dovey    | — Independent                                           |
| Mr L. Elmeland*    | — Saab-Scania, Linköping, Sweden                        |
| Dr J.W. Flower     | — Independent                                           |
| Dr K.P. Garry      | — Cranfield Institute of Technology                     |
| Mr P.G.C. Herring  | — Sowerby Research Centre, Bristol                      |
| Mr R. Jordan       | — Aircraft Research Association                         |
| Mr J.H. Krause*    | — Northrop Corporation, Hawthorne, Calif., USA          |
| Mr J.R.C. Pedersen | — Independent                                           |
| Mr R. Sanderson    | — Messerschmitt-Bölkow-Blohm GmbH, Bremen, Germany      |
| Mr A.E. Sewell*    | — McDonnell Douglas, Long Beach, Calif., USA            |
| Mr M.R. Smith      | — British Aerospace (Commercial Aircraft) Ltd, Bristol  |
| Miss J. Willaume   | — Aérospatiale, Toulouse, France.                       |

\* Corresponding Member

The technical work in the assessment of the available information and the construction and subsequent development of the Data Item was carried out by

Mr H.C. Garner — Independent.

FREQUENCIES OF RESONANCE IN WIND TUNNELS WITH VENTILATED WALLS  
AND PLENUM CHAMBER

CONTENTS

|                                                                                                                   | Page     |
|-------------------------------------------------------------------------------------------------------------------|----------|
| 1. NOTATION AND UNITS                                                                                             | 1        |
| 2. INTRODUCTION                                                                                                   | 2        |
| 3. THEORETICAL METHOD                                                                                             | 3        |
| 4. CALCULATED FREQUENCIES OF RESONANCE                                                                            | 7        |
| 5. DERIVATION AND REFERENCE                                                                                       | 9        |
| 5.1 Derivation                                                                                                    | 9        |
| 5.2 Reference                                                                                                     | 9        |
| 6. EXAMPLES                                                                                                       | 10       |
| 6.1 Example 1                                                                                                     | 10       |
| 6.2 Example 2                                                                                                     | 10       |
| 7. TABLES OF $\beta_p$                                                                                            | 12       |
| FIGURES 1 to 3                                                                                                    | 13 to 17 |
| APPENDIX A PROGRAM FOR CALCULATION OF FIRST RESONANCE FREQUENCY FOR<br>VENTILATED WIND TUNNELS AT SUBSONIC SPEEDS | 19       |
| A1. INTRODUCTION                                                                                                  | 19       |
| A2. INPUT                                                                                                         | 19       |
| A3. EXAMPLES OF INPUT AND OUTPUT                                                                                  | 20       |
| A3.1 Example 1                                                                                                    | 21       |
| A3.2 Example 2                                                                                                    | 23       |
| A3.2.1 Primary part of Example 2                                                                                  | 23       |
| A3.2.2 Rider to Example 2 for reduced                                                                             | 25       |
| A3.3 Example 3                                                                                                    | 27       |

ESDU Copyright material. For current status contact ESDU.

## FREQUENCIES OF RESONANCE IN WIND TUNNELS WITH VENTILATED WALLS AND PLENUM CHAMBER

### 1. NOTATION AND UNITS

|              |                                                                                           | <i>SI</i>         | <i>British</i>      |
|--------------|-------------------------------------------------------------------------------------------|-------------------|---------------------|
| $A$          | amplitude of oscillatory disturbance                                                      | m <sup>2</sup> /s | ft <sup>2</sup> /s  |
| $a$          | speed of sound                                                                            | m/s               | ft/s                |
| $b$          | width of slot                                                                             | m                 | ft                  |
| $D$          | diameter of perforation                                                                   | m                 | ft                  |
| $d$          | depth of plenum chamber                                                                   | m                 | ft                  |
| $F$          | geometric slot parameter, see Equation (3.7)                                              |                   |                     |
| $f$          | frequency of oscillation                                                                  | Hz                | Hz                  |
| $G$          | fall in $\beta p$ as $2kT/h$ increases from 0.1 to 0.7, see Equation (4.2)                |                   |                     |
| $g$          | mode in plenum chamber                                                                    | m <sup>2</sup> /s | ft <sup>2</sup> /s  |
| $h$          | height of tunnel                                                                          | m                 | ft                  |
| $i$          | $\sqrt{-1}$                                                                               |                   |                     |
| $k$          | porosity factor, $(1 - \sigma)/\sigma$                                                    |                   |                     |
| $M$          | Mach number                                                                               |                   |                     |
| $p$          | eigenvalue for response frequency, $\frac{1}{2}\lambda h$                                 |                   |                     |
| $\Delta p_w$ | pressure difference across ventilated wall, (plenum chamber pressure) – (tunnel pressure) | N/m <sup>2</sup>  | lbf/ft <sup>2</sup> |
| $Q$          | Mach number parameter in solution, see Equation (3.18)                                    |                   |                     |
| $T$          | effective perforation diameter, $(0.85D + Z \sec \theta)$ or $(0.85b + Z)$                | m                 | ft                  |
| $t$          | time                                                                                      | s                 | s                   |
| $V$          | velocity of stream                                                                        | m/s               | ft/s                |
| $x$          | streamwise distance                                                                       | m                 | ft                  |
| $Z$          | thickness of tunnel wall                                                                  | m                 | ft                  |
| $z$          | upward distance from tunnel centre-line                                                   | m                 | ft                  |
| $\alpha$     | eigenvalue factor for $x$                                                                 | m <sup>-1</sup>   | m <sup>-1</sup>     |

Issued April 1990

With Amendments A and B, July 1991 – 30 pages

|           |                                                             |          |             |
|-----------|-------------------------------------------------------------|----------|-------------|
| $\beta$   | $(1 - M^2)^{1/2}$                                           |          |             |
| $\gamma$  | interpolation factor, see Figure 3                          |          |             |
| $\theta$  | inclination of perforation axis to wall normal              | deg      | deg         |
| $\lambda$ | eigenvalue factor for $z$                                   | $m^{-1}$ | $ft^{-1}$   |
| $\rho$    | density of stream                                           | $kg/m^3$ | $slug/ft^3$ |
| $\sigma$  | open area ratio (open area / total area of ventilated wall) |          |             |
| $\phi$    | velocity potential of stream flow                           | $m^2/s$  | $ft^2/s$    |
| $\psi$    | velocity potential of plenum chamber flow                   | $m^2/s$  | $ft^2/s$    |
| $\omega$  | circular frequency, $2\pi f$ or $\beta da$                  | $s^{-1}$ | $s^{-1}$    |

## Subscripts

|                 |                                                     |
|-----------------|-----------------------------------------------------|
| $G_{max}$       | denotes value corresponding to maximum in $G$       |
| $\beta p_{max}$ | denotes value corresponding to maximum in $\beta p$ |
| 0.1             | denotes value corresponding to $2kT/h = 0.1$        |
| 0.7             | denotes value corresponding to $2kT/h = 0.7$        |

## 2. INTRODUCTION

Wind-tunnel resonance is a phenomenon of wall interference for which it is impossible to make any correction. It is a characteristic of subsonic flow, whereby transverse waves can develop in a uniform stream as a result of forced oscillations or buffeting at any one of a sequence of resonance frequencies dependent on the tunnel geometry. In practice, it is the lowest such frequency that imposes an upper limit for valid flutter testing and experiments on oscillating wings and control surfaces. It is important to know this first resonance frequency when planning dynamic measurements in a particular tunnel, for example to avoid a condition of resonance while the Mach number range is traversed and frequency is held constant.

In rectangular tunnels the phenomenon is essentially two-dimensional in character and concerns the roof and floor of the tunnel with the model located in the central horizontal plane  $z = 0$ . Before the consideration of ventilated tunnels with slotted or perforated walls in Section 3, the simple results for closed tunnels will be taken from Derivation 1. For wing motion out of the plane  $z = 0$  the perturbation in velocity potential is antisymmetric in  $z$  and the circular frequencies are

$$\omega = \frac{2\beta V p}{Mh} \text{ where } p = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots \quad (2.1)$$

For wing motion in the  $x$  direction, represented by sources and sinks, the velocity potential is symmetric in  $z$  and there is a second set of frequencies given by Equation (2.1) with  $p = \pi, 2\pi, 3\pi, \dots$ . The lowest critical frequency is therefore given by

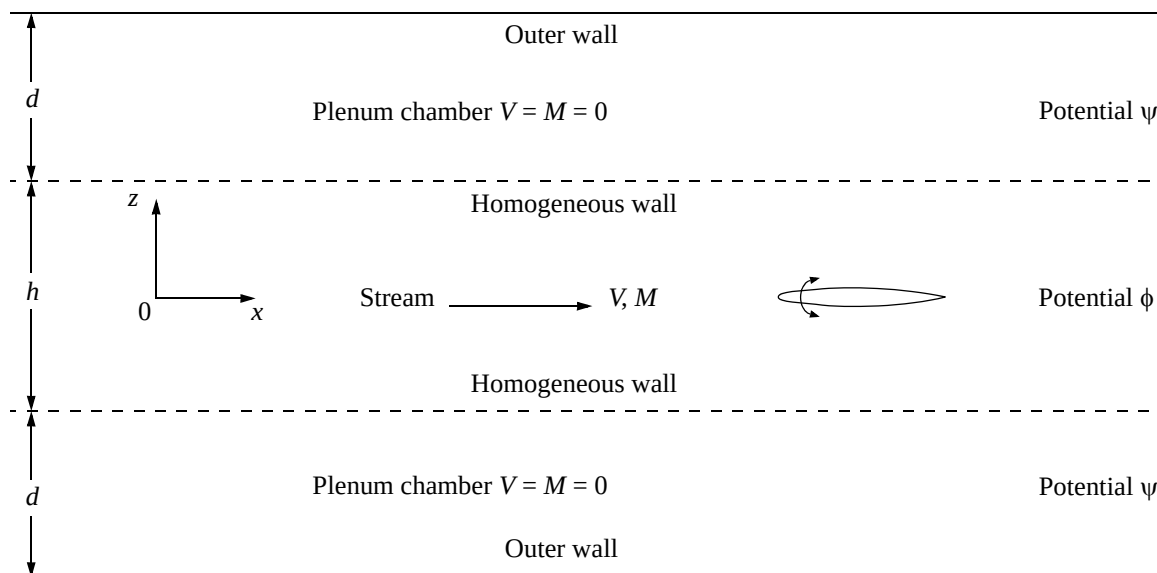
$$\omega = \frac{\pi\beta V}{Mh} = \frac{\pi\beta a}{h}, \quad (2.2)$$

which is too high to be of practical interest at low speeds, but it imposes an increasingly severe restriction on test frequencies at higher Mach numbers.

The basic theory for ventilated tunnels is taken from the work of Mabey in Derivation 6. His method is outlined and the key equations are described in Section 3. The numerical results have been specially calculated in terms of Mach number, plenum chamber depth and a single wall parameter and are presented and discussed in Section 4. After the Derivation and Reference two illustrative examples are given in Section 6. Appendix A provides details of a computer program for calculating the first resonance frequency.

### 3. THEORETICAL METHOD

To calculate resonance frequencies the two-dimensional model in Sketch 3.1 is used. The associated oscillatory flows are either symmetric or antisymmetric in  $z$ . The region  $-\frac{1}{2}h < z < \frac{1}{2}h$  contains the main stream of the tunnel with undisturbed velocity  $V$  and subsonic Mach number  $M$  in the  $x$  direction and a perturbation with velocity potential  $\phi$ . The regions  $-(\frac{1}{2}h + d) < z < -\frac{1}{2}h$  and  $\frac{1}{2}h < z < (\frac{1}{2}h + d)$  comprise the plenum chamber with still air  $V = M = 0$  and perturbation velocity potential  $\psi$ . The roof and floor,  $z = \pm\frac{1}{2}h$ , are modelled as homogeneous boundaries which admit normal flow. The outer walls,  $z = \pm(\frac{1}{2}h + d)$ , are solid.



Sketch 3.1 Representation of ventilated tunnel

For a full analysis of the theory the reader is referred to the Appendix of Derivation 6. The governing partial differential equations are

$$(1-M^2)\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial z^2} - \frac{2M^2}{V}\frac{\partial^2\phi}{\partial x\partial t} - \frac{M^2}{V^2}\frac{\partial^2\phi}{\partial t^2} = 0 \quad (3.1)$$

for the stream flow and

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial z^2} - \frac{1}{a^2}\frac{\partial^2\psi}{\partial t^2} = 0 \quad (3.2)$$

for the plenum chamber flow. The outer boundary conditions are

$$\frac{\partial\psi}{\partial z} = 0 \text{ on } z = \pm(\frac{1}{2}h + d). \quad (3.3)$$

At the homogeneous walls continuity of mass requires that

$$\frac{\partial\phi}{\partial z} = \frac{d\psi}{dz} \text{ on } z = \pm\frac{1}{2}h, \quad (3.4)$$

while across them there is a pressure difference

$$\Delta p_w = -\rho \left[ V \frac{\partial\phi}{\partial x} + \frac{\partial\phi}{\partial t} - \frac{\partial\psi}{\partial t} \right], \quad (3.5)$$

which can be related to wall geometry. For a slotted wall the steady-flow homogeneous boundary condition of Derivation 2 is extended, as in Derivation 3, to give

$$\Delta p_w = \mp \rho V(\frac{1}{2}Fh) \frac{\partial^2\phi}{\partial x\partial z} \text{ on } z = \pm\frac{1}{2}h, \quad (3.6)$$

where in terms of the slot width,  $b$ , and the open area ratio,  $\sigma$ ,

$$F = \frac{2b}{\pi\sigma h} \log_e \left[ \operatorname{cosec} \frac{\pi\sigma}{2} \right]. \quad (3.7)$$

For a perforated wall the homogeneous boundary condition in Derivation 6 is based on research in Derivations 4 and 5, from which for oscillatory perturbations the pressure difference can be identified as

$$\Delta p_w = \pm \rho k T \frac{\partial^2\phi}{\partial z\partial t} \text{ on } z = \pm\frac{1}{2}h \quad (3.8)$$

in terms of a porosity factor

$$k = (1-\sigma)/\sigma \quad (3.9)$$

and an effective diameter of perforation

$$T = 0.85D + Z \sec \theta, \quad (3.10)$$

where  $Z \sec \theta$  is the perforation length. When the expressions for  $\Delta p_w$  are equated, there are alternative boundary conditions on the roof and floor

$$\frac{\partial \phi}{\partial x} + \frac{1}{V} \left( \frac{\partial \phi}{\partial t} - \frac{\partial \psi}{\partial t} \right) \pm \frac{1}{2} F h \frac{\partial^2 \phi}{\partial x \partial z} = 0 \quad (3.11)$$

for a slotted wall and

$$\frac{\partial \phi}{\partial x} + \frac{1}{V} \left( \frac{\partial \phi}{\partial t} - \frac{\partial \psi}{\partial t} \right) \pm \frac{kT}{V} \frac{\partial^2 \phi}{\partial z \partial t} = 0 \quad (3.12)$$

for a perforated wall. The analysis in the Appendix of Derivation 6 then follows from Equations (3.1) to (3.4), (3.11) and (3.12). As regards wall geometry,  $b$  and  $\sigma$  for slotted walls or  $D$ ,  $Z$ ,  $\theta$  and  $\sigma$  for perforated walls are sufficient to determine the required parameters in Equation (3.7) or Equations (3.9) and (3.10).

The resonance mode of primary concern is of the form

$$\phi = A \sin(\lambda z) e^{i\alpha x} e^{i\omega t}, \quad (3.13)$$

which would be excited by oscillatory motion of the model involving vertical displacements. Additionally, it is possible that a symmetric mode

$$\phi = A \cos(\lambda z) e^{i\alpha x} e^{i\omega t} \quad (3.14)$$

could be excited by longitudinal motion of the model or unsteady flow in the tunnel. The three eigenvalues  $\lambda$ ,  $\alpha$  and  $\omega$  are related and, as found in Derivation 3, the resonance frequency satisfies the equations

$$\omega = \frac{V\beta^2\alpha}{M^2} = \frac{V\beta\lambda}{M}. \quad (3.15)$$

A compatible mode in the plenum chamber is of the form

$$\psi = g(z) e^{i\alpha x} e^{i\omega t}. \quad (3.16)$$

The analysis then leads to the differential equation for  $g(z)$

$$\frac{\partial^2 g}{\partial z^2} - \lambda^2 Q^2 g = 0, \quad (3.17)$$

where

$$Q = \left[ \frac{M^2}{\beta^2} - \beta^2 \right]^{1/2}. \quad (3.18)$$

The form of solution depends on whether  $Q$  is pure imaginary, zero or real and positive, *i.e.* on whether

$$M < , = \text{ or } > \frac{1}{2}(\sqrt{5} - 1) = 0.618. \quad (3.19)$$

For perforated walls the antisymmetric mode of Equation (3.13) is then shown to satisfy all the equations and boundary conditions for the set of resonance frequencies given by the following algebraic equations:

$$\tan p - (\beta^2/|Q|)\cot(2|Q|dp/h) + \beta^2(2kT/h)p = 0 \quad (M < 0.618) \quad (3.20)$$

$$\csc p = 0 \quad (M = 0.618) \quad (3.21)$$

$$\text{and} \quad \tan p + (\beta^2/Q)\coth(2Qdp/h) + \beta^2(2kT/h)p = 0 \quad (M > 0.618) \quad (3.22)$$

$$\text{where} \quad p = \frac{1}{2}\lambda h \quad (3.23)$$

$$\text{and} \quad |Q| = [\beta^2 - M^2/\beta^2]^{1/2}. \quad (3.24)$$

It is remarkable that these equations are of identical format for slotted walls, when the final terms in Equations (3.20) and (3.22) are replaced by  $M^2 Fp$ . Equations (3.20) to (3.22) have to be solved for  $p$ , which gives the resonance frequencies

$$f = \frac{\beta Vp}{\pi Mh} = \frac{a\beta p}{\pi h}. \quad (3.25)$$

These form the odd set of modes, first, third, fifth, etc. The even set is obtained when Equation (3.14) is used in place of Equation (3.13), but these are omitted as there does not appear to be any experimental evidence of them.

The theory has a number of empirical aspects including the rejection of the solution for slotted walls, which is considered to be unrealistic because the final term  $M^2 Fp$  gives a result for  $M = 0$  independent of slot geometry. Instead, Equations (3.20) and (3.22) are used with an effective perforation diameter

$$T = 0.85b + Z, \quad (3.26)$$

where  $b$  is the slot width. Although the theory is approximate, Derivation 6 provides some experimental support for the theoretical predictions of the odd resonance modes for both types of ventilated tunnel including inclined perforations. The comparisons do not reveal any systematic error in the theory.

More recently in Reference 7 a finite element method has been developed for predicting the resonance frequencies in slotted tunnels with plenum chambers, and this is likely to be more accurate than Mabey's theory. The only available comparisons in Figure 4 of Reference 7 indicate that his theory can give a pessimistically low resonance frequency for a slotted tunnel. Nevertheless, in combination with the experimental confirmation in Derivation 6, the comparisons are good enough to justify the detailed parametric calculations for the important first mode in Section 4.

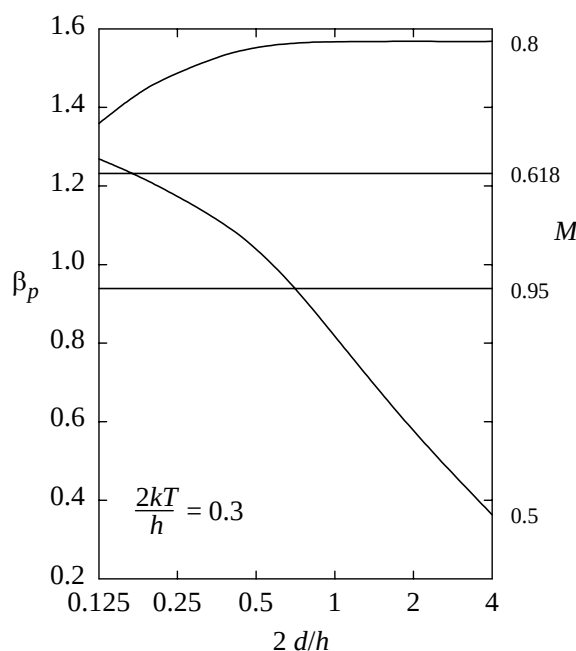
## 4. CALCULATED FREQUENCIES OF RESONANCE

The lowest eigenvalue  $p$  has been calculated iteratively from Equations (3.20) and (3.22) for the full range of three parameters,  $0 \leq M \leq 1$ ,  $0 \leq 2d/h \leq \infty$  and  $0 \leq 2kT/h \leq \infty$ . The resonance frequency is then obtained from Equation (3.25),

$$f = \frac{a\beta p}{\pi h}. \quad (4.1)$$

Here is one form of wall interference where it is disadvantageous to use a large tunnel; the greater the height of working section, the lower the resonance frequency. The practical range of plenum chamber depth is  $0 < 2d/h \leq 4$ , while that of the wall parameter is set at  $0 < 2kT/h \leq 1$ . Larger values of  $2kT/h$  correspond to very small open area ratios for which the homogeneous boundary condition in Equation (3.12) becomes unrealistic. The results are most conveniently presented in terms of  $\beta p$  and Tables 7.1 and 7.2 show the calculated values of  $\beta p$  for  $2kT/h = 0.1$  and  $0.7$  respectively.

The most interesting variations in  $\beta p$  are those against  $M$ . If there is a plenum chamber ( $d \neq 0$ ),  $\beta p$  either increases from its value at  $M = 0$  or falls to a minimum value in the lower part of the subsonic range. The resonance frequency peaks at a Mach number in the range  $0.68 < M < 0.82$  before falling to zero as  $M \rightarrow 1$ . The phenomenon disappears in supersonic flow.



**Sketch 4.1 Resonance frequency against plenum chamber depth**

In the absence of a plenum chamber  $p = \frac{1}{2}\pi$ . The effect of plenum chamber depth is illustrated in Sketch 4.1. For  $M < 0.618$  the effect of increasing  $2d/h$  is to reduce  $\beta p$ ; this seems to impose the most serious limitation on test frequencies. At  $M = 0.618$  the resonance frequency is invariant, with  $p = \frac{1}{2}\pi$ . At larger values of  $M$  the resonance frequency rises to an asymptotic limit as  $2d/h$  increases. It is significant that, as Mach number increases, the growth in  $\beta p$  is progressively more concentrated in the region of small  $2d/h$ , until at  $M = 0.95$  the asymptotic limit occurs virtually at  $2d/h = 0.25$ . To compress the upper range of  $2d/h$  it is convenient in graphical presentations to use the parameter  $\tan^{-1}(2d/h)$ .

The least interesting variations, and the easiest to handle, are those against the wall parameter  $2kT/h$  showing either constant or smoothly decreasing functions. It is sufficient to consider the range  $0 < 2kT/h \leq 1$ .

The quantity  $\beta p$  can be determined from the graphs given in Figures 1 to 3 as a function of  $M$ ,  $\tan^{-1}(2d/h)$  and  $2kT/h$ . Firstly, for  $2kT/h = 0.1$ , there is a carpet in Figure 1a for suitable values of  $\tan^{-1}(2d/h)$  and Mach numbers in the lower range  $M \leq 0.618$ . A fold in the carpet for  $M \geq 0.618$  makes it necessary to split Figure 1b into two parts, for  $M \leq (M)_{\beta p \max}$  and  $M \geq (M)_{\beta p \max}$  where  $(M)_{\beta p \max}$  decreases as  $2d/h$  increases. Secondly, to allow for the parameter  $2kT/h$ , there are carpets of the fall in  $\beta p$  as  $2kT/h$  is increased from 0.1 to 0.7 (Tables 7.1 and 7.2). Thus

$$G = (\beta p)_{0.1} - (\beta p)_{0.7} \quad (4.2)$$

which is plotted in Figure 2 against  $\tan^{-1}(2d/h)$  and  $M$ . In the range  $M \leq 0.618$  there are maxima in  $G$  against  $2d/h$ , and Figure 2a is subdivided into separate carpets for  $2d/h \leq (2d/h)_{G \max}$  and  $2d/h \geq (2d/h)_{G \max}$ . For the higher Mach numbers,  $M \geq 0.618$ , there are maxima in  $G$  against  $M$ , and Figure 2b is split into two parts for  $M \leq (M)_{G \max}$  and  $M \geq (M)_{G \max}$ . Finally, given the values of  $(\beta p)_{0.1}$  and  $G$ , it is sufficient to calculate

$$\beta p = (\beta p)_{0.1} - \gamma G, \quad (4.3)$$

where  $(\beta p)_{0.1}$  is given in Figure 1a, 1b(i) or 1b(ii),  $G$  is given in Figure 2a(i), 2a(ii), 2b(i) or 2b(ii), and  $\gamma$  may be taken as the single function of  $2kT/h$  given in Figure 3. This procedure should give  $\beta p$  to within  $\pm 0.01$  of its true calculated value.

As regards accuracy, the resulting values of  $\beta p$  are subject to the approximations in the theory. The evidence of the finite-element calculations in Reference 7 suggests that, for slotted tunnels, the values of  $\beta p$  may be somewhat pessimistically low. On the other hand, the experimental data in Derivation 6 for both perforated and slotted walls suggest that the present calculations are just as likely to overestimate as to underestimate  $\beta p$ . The lack of experimental evidence in the region  $M > 0.9$  casts some doubt on the manner in which  $\beta p$  tends to zero at  $M = 1$ , and on the significance of the results at very high subsonic Mach numbers. This adds to the uncertainties of dynamic wind-tunnel testing at near-sonic stream velocities. In practice it would seem prudent to allow for the possibility of an error of  $\pm 10\%$  in predictions of resonance frequency, or as much as  $\pm 20\%$  at the higher Mach numbers ( $M > 0.8$ , say). In particular, the calculated frequencies of resonance are not expected to apply when suction is introduced to induce mass flow from the working section into the plenum chamber.